

Computer algebra independent integration tests

6-Hyperbolic-functions/6.4-Hyperbolic-cotangent/6.4.1-c+d-x^m-a+b-coth^n

Nasser M. Abbasi

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3.54	$\int \frac{c+dx}{a+b \coth(e+fx)} dx$	297
3.55	$\int \frac{1}{(c+dx)(a+b \coth(e+fx))} dx$	301
3.56	$\int \frac{1}{(c+dx)^2(a+b \coth(e+fx))} dx$	304
3.57	$\int \frac{(c+dx)^3}{(a+b \coth(e+fx))^2} dx$	307
3.58	$\int \frac{(c+dx)^2}{(a+b \coth(e+fx))^2} dx$	318
3.59	$\int \frac{c+dx}{(a+b \coth(e+fx))^2} dx$	327
3.60	$\int \frac{1}{(c+dx)(a+b \coth(e+fx))^2} dx$	333
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Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from <https://rulebasedintegration.org>

The number of integrals in this report is [61]. This is test number [174].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or Hypergeometric2F1 functions. RootSum and RootOf are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (61)	% 0. (0)
Mathematica	% 98.36 (60)	% 1.64 (1)
Maple	% 95.08 (58)	% 4.92 (3)
Maxima	% 86.89 (53)	% 13.11 (8)
Fricas	% 100. (61)	% 0. (0)
Sympy	% 45.9 (28)	% 54.1 (33)
Giac	% 57.38 (35)	% 42.62 (26)

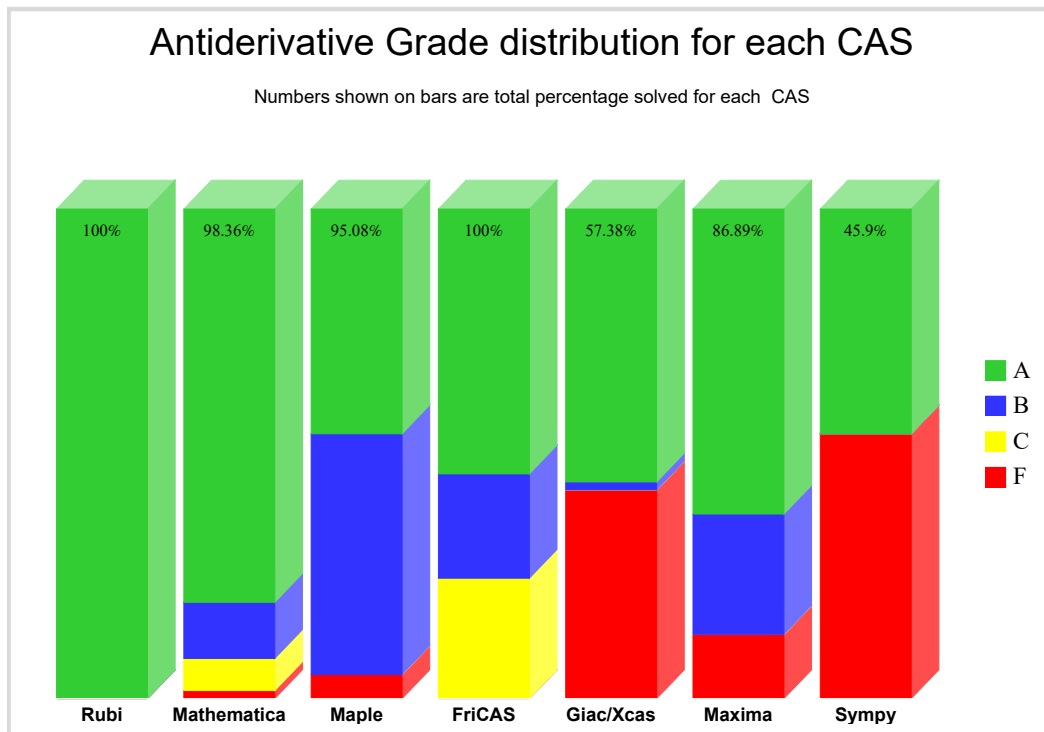
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons <ol style="list-style-type: none"> 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

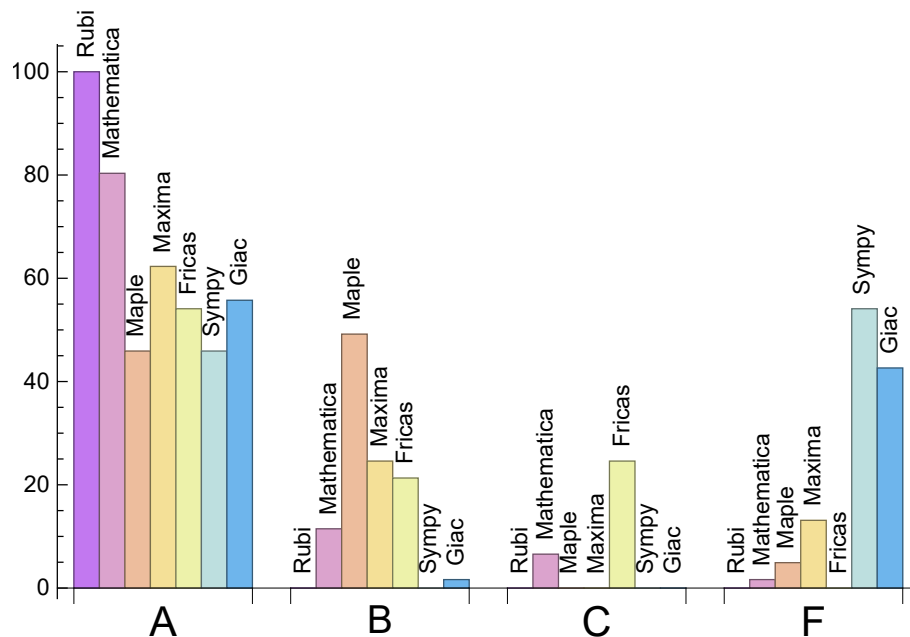
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	100.	0.	0.	0.
Mathematica	80.33	11.48	6.56	1.64
Maple	45.9	49.18	0.	4.92
Maxima	62.3	24.59	0.	13.11
Fricas	54.1	21.31	24.59	0.
Sympy	45.9	0.	0.	54.1
Giac	55.74	1.64	0.	42.62

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.31	150.31	0.7	114.	1.
Mathematica	13.12	253.67	1.15	177.	1.07
Maple	0.19	446.45	2.14	170.5	2.05
Maxima	2.07	298.64	1.47	155.	1.37
Fricas	1.69	1976.52	8.29	609.	3.88
Sympy	1.93	487.04	2.39	0.	0.
Giac	0.63	124.71	0.66	0.	0.

1.4 list of integrals that has no closed form antiderivative

{4, 5, 9, 10, 14, 15, 32, 33, 40, 41, 45, 46, 50, 51, 55, 56, 60, 61}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {7, 13, 31, 48, 59}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime')
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using `sagemath` (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See <https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/> for reference.

1.9.2 Important note about FriCAS and Giac/X-CAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the builtin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via `sagemath`) the following code is used

#see <https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in>

```
def tree(expr):
    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

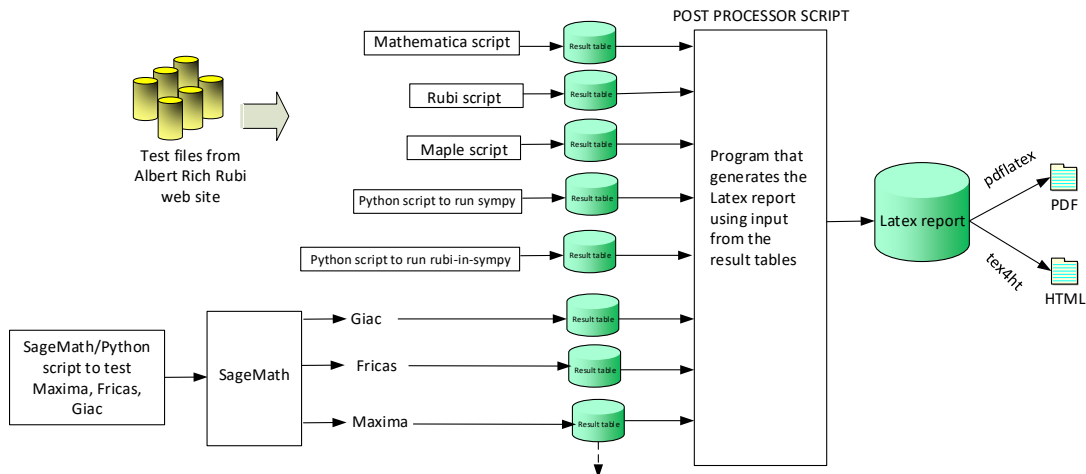
```
try:
    # 1.7 is a fudge factor since it is low side from actual leaf count
    leafCount = round(1.7*count_ops(anti))

except Exception as ee:
    leafCount =1
```

When these cas systems have a builtin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
13. String. The optimal antiderivative in CAS own syntax.

High level overview of the CAS independent integration test build system

Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 8, 9, 10, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 35, 37, 38, 39, 40, 41, 44, 45, 46, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 60, 61 }

B grade: { 6, 11, 12, 34, 42, 43, 47 }

C grade: { 7, 13, 48, 59 }

F grade: { 36 }

2.1.3 Maple

A grade: { 4, 5, 8, 9, 10, 14, 15, 19, 20, 21, 24, 25, 26, 27, 30, 31, 32, 33, 40, 41, 45, 46, 50, 51, 55, 56, 60, 61 }

B grade: { 1, 2, 3, 6, 7, 11, 12, 13, 16, 17, 18, 22, 23, 28, 29, 37, 38, 39, 42, 43, 44, 47, 48, 49, 52, 53, 54, 57, 58, 59 }

C grade: { }

F grade: { 34, 35, 36 }

2.1.4 Maxima

A grade: { 4, 5, 6, 7, 9, 10, 11, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 40, 41, 44, 45, 46, 50, 51, 55, 56, 57, 58, 60, 61 }

B grade: { 1, 2, 3, 8, 12, 13, 37, 38, 42, 43, 47, 48, 49, 52, 53 }

C grade: { }

F grade: { 32, 33, 34, 35, 36, 39, 54, 59 }

2.1.5 FriCAS

A grade: { 4, 5, 9, 10, 14, 15, 16, 17, 18, 19, 20, 21, 24, 25, 26, 29, 30, 31, 32, 33, 34, 35, 36, 40, 41, 45, 46, 50, 51, 55, 56, 60, 61 }

B grade: { 3, 7, 8, 13, 22, 23, 27, 28, 39, 44, 49, 54, 59 }

C grade: { 1, 2, 6, 11, 12, 37, 38, 42, 43, 47, 48, 52, 53, 57, 58 }

F grade: { }

2.1.6 Sympy

A grade: { 4, 5, 8, 9, 10, 14, 15, 16, 17, 18, 22, 23, 24, 27, 28, 29, 32, 33, 40, 41, 45, 46, 50, 51, 55, 56, 60, 61 }

B grade: { }

C grade: { }

F grade: { 1, 2, 3, 6, 7, 11, 12, 13, 19, 20, 21, 25, 26, 30, 31, 34, 35, 36, 37, 38, 39, 42, 43, 44, 47, 48, 49, 52, 53, 54, 57, 58, 59 }

2.1.7 Giac

A grade: { 4, 5, 9, 10, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 40, 41, 45, 46, 50, 51, 55, 56, 60, 61 }

B grade: { 8 }

C grade: { }

F grade: { 1, 2, 3, 6, 7, 11, 12, 13, 34, 35, 36, 37, 38, 39, 42, 43, 44, 47, 48, 49, 52, 53, 54, 57, 58, 59 }
}

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table, the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	91	200	230	644	0	0
normalized size	1	1.	1.05	2.3	2.64	7.4	0.	0.
time (sec)	N/A	0.159	0.01	0.101	1.048	2.541	0.	0.

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	66	166	186	489	0	0
normalized size	1	1.	1.05	2.63	2.95	7.76	0.	0.
time (sec)	N/A	0.135	0.01	0.03	1.105	2.194	0.	0.

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	47	122	132	336	0	0
normalized size	1	1.	1.04	2.71	2.93	7.47	0.	0.
time (sec)	N/A	0.085	0.006	0.026	1.051	2.158	0.	0.

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.016	12.211	0.085	0.	0.	0.	0.

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.017	18.118	0.055	0.	0.	0.	0.

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	204	198	197	1643	0	0
normalized size	1	1.	2.34	2.28	2.26	18.89	0.	0.
time (sec)	N/A	0.178	3.921	0.028	1.379	2.244	0.	0.

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	A	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	65	65	163	156	146	1211	0	0
normalized size	1	1.	2.51	2.4	2.25	18.63	0.	0.
time (sec)	N/A	0.119	5.09	0.029	1.35	2.212	0.	0.

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	46	54	155	477	80	132
normalized size	1	1.	1.48	1.74	5.	15.39	2.58	4.26
time (sec)	N/A	0.027	0.153	0.029	1.184	2.147	4.74	1.178

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.028	20.161	0.04	0.	0.	0.	0.

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.029	19.436	0.063	0.	0.	0.	0.

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	390	375	408	5023	0	0
normalized size	1	1.	2.18	2.09	2.28	28.06	0.	0.
time (sec)	N/A	0.33	4.598	0.049	1.298	2.406	0.	0.

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	295	246	305	3729	0	0
normalized size	1	1.	2.59	2.16	2.68	32.71	0.	0.
time (sec)	N/A	0.207	3.258	0.043	1.263	2.278	0.	0.

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	B	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	82	82	232	164	201	2569	0	0
normalized size	1	1.	2.83	2.	2.45	31.33	0.	0.
time (sec)	N/A	0.122	6.132	0.034	1.223	2.215	0.	0.

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.028	14.899	0.29	0.	0.	0.	0.

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.029	11.14	0.227	0.	0.	0.	0.

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	244	959	247	640	864	261
normalized size	1	1.	1.44	5.67	1.46	3.79	5.11	1.54
time (sec)	N/A	0.185	0.401	0.077	1.375	2.083	5.251	1.184

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	169	469	167	419	522	166
normalized size	1	1.	1.39	3.84	1.37	3.43	4.28	1.36
time (sec)	N/A	0.117	0.253	0.065	1.254	2.084	2.181	1.152

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	81	175	97	239	250	88
normalized size	1	1.	1.09	2.36	1.31	3.23	3.38	1.19
time (sec)	N/A	0.051	0.262	0.058	1.166	2.117	1.45	1.19

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	122	61	65	166	0	68
normalized size	1	1.	0.78	0.39	0.41	1.06	0.	0.43
time (sec)	N/A	0.251	0.25	0.182	1.752	2.181	0.	1.15

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	206	91	76	498	0	149
normalized size	1	1.	1.3	0.57	0.48	3.13	0.	0.94
time (sec)	N/A	0.213	0.764	0.203	2.258	2.134	0.	1.333

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	211	265	210	92	748	0	242
normalized size	1	1.	1.26	1.	0.44	3.55	0.	1.15
time (sec)	N/A	0.301	1.073	0.219	3.059	2.17	0.	1.149

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	230	230	420	2200	401	1257	2200	517
normalized size	1	1.	1.83	9.57	1.74	5.47	9.57	2.25
time (sec)	N/A	0.268	1.023	0.108	2.493	2.171	10.887	1.167

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	207	1087	258	811	1358	306
normalized size	1	1.	1.22	6.39	1.52	4.77	7.99	1.8
time (sec)	N/A	0.192	0.946	0.097	2.028	2.266	3.463	1.143

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	114	74	144	454	700	147
normalized size	1	1.	0.86	0.56	1.08	3.41	5.26	1.11
time (sec)	N/A	0.129	0.522	0.122	1.51	2.037	2.846	1.145

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	297	297	199	106	109	304	0	105
normalized size	1	1.	0.67	0.36	0.37	1.02	0.	0.35
time (sec)	N/A	0.704	0.439	0.27	3.381	2.193	0.	1.191

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	420	420	442	164	135	1413	0	255
normalized size	1	1.	1.05	0.39	0.32	3.36	0.	0.61
time (sec)	N/A	0.733	1.419	0.282	6.056	2.242	0.	1.928

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	336	336	615	368	548	1877	3925	774
normalized size	1	1.	1.83	1.1	1.63	5.59	11.68	2.3
time (sec)	N/A	0.37	2.263	0.183	7.633	2.225	13.282	1.241

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	246	246	371	2098	343	1215	2448	447
normalized size	1	1.	1.51	8.53	1.39	4.94	9.95	1.82
time (sec)	N/A	0.258	1.372	0.125	5.602	2.132	5.252	1.259

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	185	791	186	709	1290	204
normalized size	1	1.	1.01	4.32	1.02	3.87	7.05	1.11
time (sec)	N/A	0.205	0.701	0.107	3.252	2.09	4.7	1.169

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	437	437	312	151	154	439	0	146
normalized size	1	1.	0.71	0.35	0.35	1.	0.	0.33
time (sec)	N/A	1.765	0.649	0.477	10.932	2.23	0.	1.17

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	692	692	796	239	189	2685	0	358
normalized size	1	1.	1.15	0.35	0.27	3.88	0.	0.52
time (sec)	N/A	1.762	3.331	0.408	23.914	2.339	0.	2.382

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.05	23.682	0.095	0.	0.	0.	0.

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.027	13.574	0.085	0.	0.	0.	0.

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	186	0	0	366	0	0
normalized size	1	1.	2.11	0.	0.	4.16	0.	0.
time (sec)	N/A	0.117	1.13	0.139	0.	2.238	0.	0.

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	194	0	0	609	0	0
normalized size	1	1.	1.28	0.	0.	4.01	0.	0.
time (sec)	N/A	0.172	8.102	0.128	0.	2.342	0.	0.

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	A	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	223	223	0	0	0	853	0	0
normalized size	1	1.	0.	0.	0.	3.83	0.	0.
time (sec)	N/A	0.233	180.002	0.161	0.	2.287	0.	0.

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	249	748	566	1207	0	0
normalized size	1	1.	1.87	5.62	4.26	9.08	0.	0.
time (sec)	N/A	0.26	0.358	0.153	1.414	2.359	0.	0.

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	149	447	324	782	0	0
normalized size	1	1.	1.48	4.43	3.21	7.74	0.	0.
time (sec)	N/A	0.217	0.217	0.055	1.382	2.243	0.	0.

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	87	201	0	435	0	0
normalized size	1	1.	1.16	2.68	0.	5.8	0.	0.
time (sec)	N/A	0.126	0.056	0.043	0.	2.119	0.	0.

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.029	4.994	0.14	0.	0.	0.	0.

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.029	20.337	0.134	0.	0.	0.	0.

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	271	271	857	1393	1054	7128	0	0
normalized size	1	1.	3.16	5.14	3.89	26.3	0.	0.
time (sec)	N/A	0.546	9.889	0.168	1.491	2.836	0.	0.

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	478	793	667	4251	0	0
normalized size	1	1.	2.29	3.79	3.19	20.34	0.	0.
time (sec)	N/A	0.404	10.212	0.088	1.438	2.665	0.	0.

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	192	318	329	2142	0	0
normalized size	1	1.	1.51	2.5	2.59	16.87	0.	0.
time (sec)	N/A	0.184	1.9	0.082	1.41	2.674	0.	0.

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.055	42.653	0.376	0.	0.	0.	0.

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.053	30.126	0.589	0.	0.	0.	0.

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	556	556	2043	2777	2067	23616	0	0
normalized size	1	1.	3.67	4.99	3.72	42.47	0.	0.
time (sec)	N/A	1.049	14.001	0.239	1.661	4.18	0.	0.

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	B	C	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	401	401	2031	1542	1346	13835	0	0
normalized size	1	1.	5.06	3.85	3.36	34.5	0.	0.
time (sec)	N/A	0.698	12.915	0.166	1.659	3.196	0.	0.

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	261	261	266	651	713	6711	0	0
normalized size	1	1.	1.02	2.49	2.73	25.71	0.	0.
time (sec)	N/A	0.35	3.457	0.121	1.504	2.63	0.	0.

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.055	62.508	0.599	0.	0.	0.	0.

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.053	59.527	0.924	0.	0.	0.	0.

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	210	239	1145	703	1808	0	0
normalized size	1	1.	1.14	5.45	3.35	8.61	0.	0.
time (sec)	N/A	0.345	3.366	0.203	1.803	2.373	0.	0.

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	191	722	448	1258	0	0
normalized size	1	1.	1.22	4.63	2.87	8.06	0.	0.
time (sec)	N/A	0.284	3.596	0.147	1.634	2.305	0.	0.

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	144	357	0	798	0	0
normalized size	1	1.	1.33	3.31	0.	7.39	0.	0.
time (sec)	N/A	0.163	2.922	0.131	0.	2.292	0.	0.

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.063	14.072	0.155	0.	0.	0.	0.

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.06	22.471	0.156	0.	0.	0.	0.

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	638	638	646	2039	1426	12840	0	0
normalized size	1	1.	1.01	3.2	2.24	20.13	0.	0.
time (sec)	N/A	2.274	12.299	0.301	2.775	3.555	0.	0.

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	475	475	509	1217	1014	7876	0	0
normalized size	1	1.	1.07	2.56	2.13	16.58	0.	0.
time (sec)	N/A	1.714	8.295	0.263	2.791	2.905	0.	0.

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	196	196	477	620	0	4058	0	0
normalized size	1	1.	2.43	3.16	0.	20.7	0.	0.
time (sec)	N/A	0.296	6.004	0.234	0.	2.559	0.	0.

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.062	144.035	0.752	0.	0.	0.	0.

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.059	115.397	0.697	0.	0.	0.	0.

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [11] had the largest ratio of [0.8333]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	6	6	1.	10	0.6

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
2	A	5	5	1.	10	0.5
3	A	4	4	1.	8	0.5
4	A	0	0	0.	0	0.
5	A	0	0	0.	0	0.
6	A	7	7	1.	12	0.583
7	A	6	6	1.	12	0.5
8	A	3	3	1.	10	0.3
9	A	0	0	0.	0	0.
10	A	0	0	0.	0	0.
11	A	13	10	1.	12	0.833
12	A	9	8	1.	12	0.667
13	A	7	7	1.	10	0.7
14	A	0	0	0.	0	0.
15	A	0	0	0.	0	0.
16	A	5	3	1.	20	0.15
17	A	4	3	1.	20	0.15
18	A	3	3	1.	18	0.167
19	A	7	4	1.	20	0.2
20	A	7	4	1.	20	0.2
21	A	8	5	1.	20	0.25
22	A	10	3	1.	20	0.15
23	A	8	3	1.	20	0.15
24	A	7	3	1.	18	0.167
25	A	21	5	1.	20	0.25
26	A	24	7	1.	20	0.35
27	A	14	3	1.	20	0.15
28	A	11	3	1.	20	0.15
29	A	11	3	1.	18	0.167
30	A	53	7	1.	20	0.35
31	A	60	9	1.	20	0.45
32	A	0	0	0.	0	0.
33	A	0	0	0.	0	0.

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
34	A	2	2	1.	20	0.1
35	A	4	2	1.	20	0.1
36	A	5	2	1.	20	0.1
37	A	8	7	1.	18	0.389
38	A	7	6	1.	18	0.333
39	A	6	5	1.	16	0.312
40	A	0	0	0.	0	0.
41	A	0	0	0.	0	0.
42	A	15	9	1.	20	0.45
43	A	13	10	1.	20	0.5
44	A	9	7	1.	18	0.389
45	A	0	0	0.	0	0.
46	A	0	0	0.	0	0.
47	A	28	11	1.	20	0.55
48	A	22	11	1.	20	0.55
49	A	16	9	1.	18	0.5
50	A	0	0	0.	0	0.
51	A	0	0	0.	0	0.
52	A	6	6	1.	20	0.3
53	A	5	5	1.	20	0.25
54	A	4	4	1.	18	0.222
55	A	0	0	0.	0	0.
56	A	0	0	0.	0	0.
57	A	28	10	1.	20	0.5
58	A	24	11	1.	20	0.55
59	A	5	5	1.	18	0.278
60	A	0	0	0.	0	0.
61	A	0	0	0.	0	0.

Chapter 3

Listing of integrals

3.1 $\int x^3 \coth(a + bx) dx$

Optimal. Leaf size=87

$$\frac{3x^2 \text{PolyLog}(2, e^{2(a+bx)})}{2b^2} - \frac{3x \text{PolyLog}(3, e^{2(a+bx)})}{2b^3} + \frac{3 \text{PolyLog}(4, e^{2(a+bx)})}{4b^4} + \frac{x^3 \log(1 - e^{2(a+bx)})}{b} - \frac{x^4}{4}$$

[Out] $-x^4/4 + (x^3 \text{Log}[1 - E^{(2*(a + b*x))}])/b + (3*x^2*\text{PolyLog}[2, E^{(2*(a + b*x))}])/(2*b^2) - (3*x*\text{PolyLog}[3, E^{(2*(a + b*x))}])/(2*b^3) + (3*\text{PolyLog}[4, E^{(2*(a + b*x))}])/(4*b^4)$

Rubi [A] time = 0.158773, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.6$, Rules used = {3716, 2190, 2531, 6609, 2282, 6589}

$$\frac{3x^2 \text{PolyLog}(2, e^{2(a+bx)})}{2b^2} - \frac{3x \text{PolyLog}(3, e^{2(a+bx)})}{2b^3} + \frac{3 \text{PolyLog}(4, e^{2(a+bx)})}{4b^4} + \frac{x^3 \log(1 - e^{2(a+bx)})}{b} - \frac{x^4}{4}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^3*\text{Coth}[a + b*x], x]$

[Out] $-x^4/4 + (x^3 \text{Log}[1 - E^{(2*(a + b*x))}])/b + (3*x^2*\text{PolyLog}[2, E^{(2*(a + b*x))}])/(2*b^2) - (3*x*\text{PolyLog}[3, E^{(2*(a + b*x))}])/(2*b^3) + (3*\text{PolyLog}[4, E^{(2*(a + b*x))}])/(4*b^4)$

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)
*(x_)], x_Symbol] := -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-I*
e) + f*fz*x))/E^(2*I*k*Pi))], x], x] /; FreeQ[{c, d, e, f, fz}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_.) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a]]/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.)
*(x_))))^(p_.)], x_Symbol] := Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]]/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

, e, n, p}, x] && EqQ[b*d, a*e]

Rubi steps

$$\begin{aligned}
 \int x^3 \coth(a + bx) dx &= -\frac{x^4}{4} - 2 \int \frac{e^{2(a+bx)} x^3}{1 - e^{2(a+bx)}} dx \\
 &= -\frac{x^4}{4} + \frac{x^3 \log(1 - e^{2(a+bx)})}{b} - \frac{3 \int x^2 \log(1 - e^{2(a+bx)}) dx}{b} \\
 &= -\frac{x^4}{4} + \frac{x^3 \log(1 - e^{2(a+bx)})}{b} + \frac{3x^2 \text{Li}_2(e^{2(a+bx)})}{2b^2} - \frac{3 \int x \text{Li}_2(e^{2(a+bx)}) dx}{b^2} \\
 &= -\frac{x^4}{4} + \frac{x^3 \log(1 - e^{2(a+bx)})}{b} + \frac{3x^2 \text{Li}_2(e^{2(a+bx)})}{2b^2} - \frac{3x \text{Li}_3(e^{2(a+bx)})}{2b^3} + \frac{3 \int \text{Li}_3(e^{2(a+bx)}) dx}{2b^3} \\
 &= -\frac{x^4}{4} + \frac{x^3 \log(1 - e^{2(a+bx)})}{b} + \frac{3x^2 \text{Li}_2(e^{2(a+bx)})}{2b^2} - \frac{3x \text{Li}_3(e^{2(a+bx)})}{2b^3} + \frac{3 \text{Subst}\left(\int \frac{\text{Li}_3(x)}{x} dx, x, e^{2(a+bx)}\right)}{4b^4} \\
 &= -\frac{x^4}{4} + \frac{x^3 \log(1 - e^{2(a+bx)})}{b} + \frac{3x^2 \text{Li}_2(e^{2(a+bx)})}{2b^2} - \frac{3x \text{Li}_3(e^{2(a+bx)})}{2b^3} + \frac{3 \text{Li}_4(e^{2(a+bx)})}{4b^4}
 \end{aligned}$$

Mathematica [A] time = 0.0097344, size = 91, normalized size = 1.05

$$\frac{3x^2 \text{PolyLog}(2, e^{2a+2bx})}{2b^2} - \frac{3x \text{PolyLog}(3, e^{2a+2bx})}{2b^3} + \frac{3 \text{PolyLog}(4, e^{2a+2bx})}{4b^4} + \frac{x^3 \log(1 - e^{2a+2bx})}{b} - \frac{x^4}{4}$$

Antiderivative was successfully verified.

[In] Integrate[x^3*Coth[a + b*x], x]

[Out] -x^4/4 + (x^3*Log[1 - E^(2*a + 2*b*x)])/b + (3*x^2*PolyLog[2, E^(2*a + 2*b*x)])/(2*b^2) - (3*x*PolyLog[3, E^(2*a + 2*b*x)])/(2*b^3) + (3*PolyLog[4, E^(2*a + 2*b*x)])/(4*b^4)

Maple [B] time = 0.101, size = 200, normalized size = 2.3

$$-\frac{x^4}{4} - \frac{a^3 \ln(e^{bx+a} - 1)}{b^4} + 2 \frac{a^3 \ln(e^{bx+a})}{b^4} + \frac{\ln(1 - e^{bx+a}) a^3}{b^4} + 3 \frac{\text{polylog}(2, -e^{bx+a}) x^2}{b^2} - 6 \frac{\text{polylog}(3, -e^{bx+a}) x}{b^3} + \frac{\text{polylog}(4, -e^{bx+a})}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*coth(b*x+a),x)`

[Out] $-1/4*x^4-1/b^4*a^3*\ln(\exp(b*x+a)-1)+2/b^4*a^3*\ln(\exp(b*x+a))+1/b^4*\ln(1-\exp(b*x+a))*a^3+3/b^2*\text{polylog}(2,-\exp(b*x+a))*x^2-6/b^3*\text{polylog}(3,-\exp(b*x+a))*x+1/b*\ln(1-\exp(b*x+a))*x^3+3/b^2*\text{polylog}(2,\exp(b*x+a))*x^2-6/b^3*\text{polylog}(3,\exp(b*x+a))*x+1/b*\ln(1+\exp(b*x+a))*x^3+6/b^4*\text{polylog}(4,\exp(b*x+a))+6/b^4*\text{polylog}(4,-\exp(b*x+a))-2/b^3*a^3*x-3/2/b^4*a^4$

Maxima [B] time = 1.04828, size = 230, normalized size = 2.64

$$\frac{1}{4}x^4 \coth(bx+a) - \frac{1}{2} \left(\frac{x^4}{be^{(2bx+2a)} - b} + \frac{x^4}{b} - \frac{2(b^3x^3 \log(e^{(bx+a)} + 1) + 3b^2x^2 \text{Li}_2(-e^{(bx+a)}) - 6bx \text{Li}_3(-e^{(bx+a)}) + 6 \text{Li}_4(-e^{(bx+a)}))}{b^5} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*coth(b*x+a),x, algorithm="maxima")`

[Out] $1/4*x^4*\coth(b*x+a) - 1/2*(x^4/(b*e^{(2*b*x+2*a)} - b) + x^4/b - 2*(b^3*x^3*\log(e^{(b*x+a)} + 1) + 3*b^2*x^2*\text{dilog}(-e^{(b*x+a)}) - 6*b*x*\text{polylog}(3,-e^{(b*x+a)}) + 6*\text{polylog}(4,-e^{(b*x+a)}))/b^5 - 2*(b^3*x^3*\log(-e^{(b*x+a)} + 1) + 3*b^2*x^2*\text{dilog}(e^{(b*x+a)}) - 6*b*x*\text{polylog}(3,e^{(b*x+a)}) + 6*\text{polylog}(4,e^{(b*x+a)}))/b^5)*b$

Fricas [C] time = 2.54141, size = 644, normalized size = 7.4

$$b^4x^4 - 4b^3x^3 \log(\cosh(bx+a) + \sinh(bx+a) + 1) - 12b^2x^2 \text{Li}_2(\cosh(bx+a) + \sinh(bx+a)) - 12b^2x^2 \text{Li}_2(-\cosh(bx+a) - \sinh(bx+a))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*coth(b*x+a),x, algorithm="fricas")`

[Out] $-1/4*(b^4*x^4 - 4*b^3*x^3*\log(\cosh(b*x+a) + \sinh(b*x+a) + 1) - 12*b^2*x^2*\text{dilog}(\cosh(b*x+a) + \sinh(b*x+a)) - 12*b^2*x^2*\text{dilog}(-\cosh(b*x+a) - \sinh(b*x+a)) + 4*a^3*\log(\cosh(b*x+a) + \sinh(b*x+a) - 1) + 24*b*x*\text{polylog}(3,\cosh(b*x+a) + \sinh(b*x+a)) + 24*b*x*\text{polylog}(3,-\cosh(b*x+a) - \sinh(b*x+a)) - 4*(b^3*x^3 + a^3)*\log(-\cosh(b*x+a) - \sinh(b*x+a) + 1) - 24*\text{polylog}(4,\cosh(b*x+a) + \sinh(b*x+a)) - 24*\text{polylog}(4,-\cosh(b*x+a) - \sinh(b*x+a))$

a) $-\sinh(b*x + a))/b^4$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \coth(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*coth(b*x+a),x)`

[Out] `Integral(x**3*coth(a + b*x), x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \coth(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*coth(b*x+a),x, algorithm="giac")`

[Out] `integrate(x^3*coth(b*x + a), x)`

3.2 $\int x^2 \coth(a + bx) dx$

Optimal. Leaf size=63

$$\frac{x \operatorname{PolyLog}(2, e^{2(a+bx)})}{b^2} - \frac{\operatorname{PolyLog}(3, e^{2(a+bx)})}{2b^3} + \frac{x^2 \log(1 - e^{2(a+bx)})}{b} - \frac{x^3}{3}$$

[Out] $-x^3/3 + (x^2 \cdot \operatorname{Log}[1 - E^{(2(a + b*x))}])/b + (x \cdot \operatorname{PolyLog}[2, E^{(2(a + b*x))}])/b^2 - \operatorname{PolyLog}[3, E^{(2(a + b*x))}]/(2 \cdot b^3)$

Rubi [A] time = 0.13454, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {3716, 2190, 2531, 2282, 6589}

$$\frac{x \operatorname{PolyLog}(2, e^{2(a+bx)})}{b^2} - \frac{\operatorname{PolyLog}(3, e^{2(a+bx)})}{2b^3} + \frac{x^2 \log(1 - e^{2(a+bx)})}{b} - \frac{x^3}{3}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[x^2 \cdot \operatorname{Coth}[a + b \cdot x], x]$

[Out] $-x^3/3 + (x^2 \cdot \operatorname{Log}[1 - E^{(2(a + b*x))}])/b + (x \cdot \operatorname{PolyLog}[2, E^{(2(a + b*x))}])/b^2 - \operatorname{PolyLog}[3, E^{(2(a + b*x))}]/(2 \cdot b^3)$

Rule 3716

$\operatorname{Int}[\frac{((c_.) + (d_.) \cdot (x_.)^{(m_.)}) \cdot \tan[(e_.) + \operatorname{Pi} \cdot (k_.) + (\operatorname{Complex}[0, fz_]) \cdot (f_.) \cdot (x_.)]}{x_Symbol}], x] \rightarrow -\operatorname{Simp}[\frac{I \cdot (c + d \cdot x)^{(m+1)}}{d \cdot (m+1)}, x] + \operatorname{Dist}[2 \cdot I, \operatorname{Int}[\frac{(c + d \cdot x)^m \cdot E^{(2 \cdot (-I \cdot e) + f \cdot fz \cdot x))}}{(E^{(2 \cdot I \cdot k \cdot \operatorname{Pi})} \cdot (1 + E^{(2 \cdot (-I \cdot e) + f \cdot fz \cdot x))}) / E^{(2 \cdot I \cdot k \cdot \operatorname{Pi})})}], x], x] /;$ FreeQ[{c, d, e, f, fz}, x] && IntegerQ[4*k] && IGtQ[m, 0]

Rule 2190

$\operatorname{Int}[\frac{((F_.)^{((g_.) \cdot ((e_.) + (f_.) \cdot (x_.)^{(n_.)}))^{(n_.)}) \cdot ((c_.) + (d_.) \cdot (x_.)^{(m_.)}))}{((a_.) + (b_.) \cdot ((F_.)^{((g_.) \cdot ((e_.) + (f_.) \cdot (x_.)^{(n_.)}))^{(n_.)})})^{(n_.)}}, x_Symbol] \rightarrow \operatorname{Simp}[\frac{(c + d \cdot x)^m \cdot \operatorname{Log}[1 + (b \cdot (F^{(g \cdot (e + f \cdot x)))^n) / a]}{b \cdot f \cdot g \cdot n \cdot \operatorname{Log}[F]}], x] - \operatorname{Dist}[\frac{d \cdot m}{b \cdot f \cdot g \cdot n \cdot \operatorname{Log}[F]}, \operatorname{Int}[(c + d \cdot x)^{(m-1)} \cdot \operatorname{Log}[1 + (b \cdot (F^{(g \cdot (e + f \cdot x)))^n) / a}], x], x] /;$ FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*(a_.) + (b_.)*(x_)))^(n_.)]*((f_.) + (g_.)
*(x_)^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*(a_.) + (b_.)*x))*
(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int x^2 \coth(a + bx) dx &= -\frac{x^3}{3} - 2 \int \frac{e^{2(a+bx)} x^2}{1 - e^{2(a+bx)}} dx \\
&= -\frac{x^3}{3} + \frac{x^2 \log(1 - e^{2(a+bx)})}{b} - \frac{2 \int x \log(1 - e^{2(a+bx)}) dx}{b} \\
&= -\frac{x^3}{3} + \frac{x^2 \log(1 - e^{2(a+bx)})}{b} + \frac{x \operatorname{Li}_2(e^{2(a+bx)})}{b^2} - \frac{\int \operatorname{Li}_2(e^{2(a+bx)}) dx}{b^2} \\
&= -\frac{x^3}{3} + \frac{x^2 \log(1 - e^{2(a+bx)})}{b} + \frac{x \operatorname{Li}_2(e^{2(a+bx)})}{b^2} - \frac{\operatorname{Subst}\left(\int \frac{\operatorname{Li}_2(x)}{x} dx, x, e^{2(a+bx)}\right)}{2b^3} \\
&= -\frac{x^3}{3} + \frac{x^2 \log(1 - e^{2(a+bx)})}{b} + \frac{x \operatorname{Li}_2(e^{2(a+bx)})}{b^2} - \frac{\operatorname{Li}_3(e^{2(a+bx)})}{2b^3}
\end{aligned}$$

Mathematica [A] time = 0.0095108, size = 66, normalized size = 1.05

$$\frac{x \operatorname{PolyLog}\left(2, e^{2a+2bx}\right)}{b^2} - \frac{\operatorname{PolyLog}\left(3, e^{2a+2bx}\right)}{2b^3} + \frac{x^2 \log\left(1 - e^{2a+2bx}\right)}{b} - \frac{x^3}{3}$$

Antiderivative was successfully verified.

[In] Integrate[x^2*Coth[a + b*x],x]

[Out] $-x^3/3 + (x^2*\text{Log}[1 - E^{(2*a + 2*b*x)}])/b + (x*\text{PolyLog}[2, E^{(2*a + 2*b*x)}])/b^2 - \text{PolyLog}[3, E^{(2*a + 2*b*x)}]/(2*b^3)$

Maple [B] time = 0.03, size = 166, normalized size = 2.6

$$-\frac{x^3}{3} + \frac{a^2 \ln(e^{bx+a} - 1)}{b^3} - 2 \frac{a^2 \ln(e^{bx+a})}{b^3} + 2 \frac{a^2 x}{b^2} + \frac{4a^3}{3b^3} + \frac{\ln(1 + e^{bx+a}) x^2}{b} + 2 \frac{\text{polylog}(2, -e^{bx+a}) x}{b^2} - 2 \frac{\text{polylog}(3, -e^{bx+a})}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*coth(b*x+a),x)

[Out] $-1/3*x^3+1/b^3*a^2*\ln(\exp(b*x+a)-1)-2/b^3*a^2*\ln(\exp(b*x+a))+2/b^2*a^2*x+4/3/b^3*a^3+1/b*\ln(1+\exp(b*x+a))*x^2+2/b^2*\text{polylog}(2,-\exp(b*x+a))*x-2/b^3*\text{polylog}(3,-\exp(b*x+a))+1/b*\ln(1-\exp(b*x+a))*x^2-1/b^3*\ln(1-\exp(b*x+a))*a^2+2/b^2*\text{polylog}(2,\exp(b*x+a))*x-2/b^3*\text{polylog}(3,\exp(b*x+a))$

Maxima [B] time = 1.10507, size = 186, normalized size = 2.95

$$\frac{1}{3} x^3 \coth(bx + a) - \frac{1}{3} \left(\frac{2x^3}{be^{(2bx+2a)} - b} + \frac{2x^3}{b} - \frac{3(b^2x^2 \log(e^{(bx+a)} + 1) + 2bx\text{Li}_2(-e^{(bx+a)}) - 2\text{Li}_3(-e^{(bx+a)}))}{b^4} - \frac{3(b^2x^2 \log(e^{(bx+a)} - 1) + 2bx\text{Li}_2(-e^{(bx+a)}) - 2\text{Li}_3(-e^{(bx+a)}))}{b^4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*coth(b*x+a),x, algorithm="maxima")

[Out] $1/3*x^3*\coth(b*x + a) - 1/3*(2*x^3/(b*e^{(2*b*x + 2*a)} - b) + 2*x^3/b - 3*(b^2*x^2*\log(e^{(b*x + a)} + 1) + 2*b*x*\text{dilog}(-e^{(b*x + a)}) - 2*\text{polylog}(3, -e^{(b*x + a)}))/b^4 - 3*(b^2*x^2*\log(-e^{(b*x + a)} + 1) + 2*b*x*\text{dilog}(e^{(b*x + a)}) - 2*\text{polylog}(3, e^{(b*x + a)}))/b^4)*b$

Fricas [C] time = 2.19446, size = 489, normalized size = 7.76

$$b^3x^3 - 3b^2x^2 \log(\cosh(bx + a) + \sinh(bx + a) + 1) - 6bx\text{Li}_2(\cosh(bx + a) + \sinh(bx + a)) - 6bx\text{Li}_2(-\cosh(bx + a) - \sinh(bx + a)) - 3b^2x^2 \log(\cosh(bx + a) - \sinh(bx + a) + 1) - 6bx\text{Li}_2(\cosh(bx + a) - \sinh(bx + a)) - 6bx\text{Li}_2(-\cosh(bx + a) + \sinh(bx + a)) - 3b^2x^2 \log(\cosh(bx + a) - \sinh(bx + a) - 1) - 6bx\text{Li}_2(\cosh(bx + a) - \sinh(bx + a)) - 6bx\text{Li}_2(-\cosh(bx + a) + \sinh(bx + a)) - 3b^2x^2 \log(\cosh(bx + a) + \sinh(bx + a) - 1) - 6bx\text{Li}_2(\cosh(bx + a) + \sinh(bx + a)) - 6bx\text{Li}_2(-\cosh(bx + a) + \sinh(bx + a))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*coth(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/3*(b^3*x^3 - 3*b^2*x^2*log(cosh(b*x + a) + sinh(b*x + a) + 1) - 6*b*x*dilog(cosh(b*x + a) + sinh(b*x + a)) - 6*b*x*dilog(-cosh(b*x + a) - sinh(b*x + a)) - 3*a^2*log(cosh(b*x + a) + sinh(b*x + a) - 1) - 3*(b^2*x^2 - a^2)*log(-cosh(b*x + a) - sinh(b*x + a) + 1) + 6*polylog(3, cosh(b*x + a) + sinh(b*x + a)) + 6*polylog(3, -cosh(b*x + a) - sinh(b*x + a)))/b^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \coth(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*coth(b*x+a),x)
```

```
[Out] Integral(x**2*coth(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \coth(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*coth(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^2*coth(b*x + a), x)
```

3.3 $\int x \coth(a + bx) dx$

Optimal. Leaf size=45

$$\frac{\text{PolyLog}(2, e^{2(a+bx)})}{2b^2} + \frac{x \log(1 - e^{2(a+bx)})}{b} - \frac{x^2}{2}$$

[Out] $-x^2/2 + (x*\text{Log}[1 - E^{(2*(a + b*x))}])/b + \text{PolyLog}[2, E^{(2*(a + b*x))}]/(2*b^2)$

Rubi [A] time = 0.0847552, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {3716, 2190, 2279, 2391}

$$\frac{\text{PolyLog}(2, e^{2(a+bx)})}{2b^2} + \frac{x \log(1 - e^{2(a+bx)})}{b} - \frac{x^2}{2}$$

Antiderivative was successfully verified.

[In] `Int[x*Coth[a + b*x], x]`

[Out] $-x^2/2 + (x*\text{Log}[1 - E^{(2*(a + b*x))}])/b + \text{PolyLog}[2, E^{(2*(a + b*x))}]/(2*b^2)$

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c + d*x)^m*E^(2*(-I*e) + f*fz*x))]/(E^(2*I*k*Pi)*(1 + E^(2*(-I*e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a]]/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)))^n)/a]], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_)))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned} \int x \coth(a + bx) dx &= -\frac{x^2}{2} - 2 \int \frac{e^{2(a+bx)} x}{1 - e^{2(a+bx)}} dx \\ &= -\frac{x^2}{2} + \frac{x \log(1 - e^{2(a+bx)})}{b} - \frac{\int \log(1 - e^{2(a+bx)}) dx}{b} \\ &= -\frac{x^2}{2} + \frac{x \log(1 - e^{2(a+bx)})}{b} - \frac{\text{Subst}\left(\int \frac{\log(1-x)}{x} dx, x, e^{2(a+bx)}\right)}{2b^2} \\ &= -\frac{x^2}{2} + \frac{x \log(1 - e^{2(a+bx)})}{b} + \frac{\text{Li}_2(e^{2(a+bx)})}{2b^2} \end{aligned}$$

Mathematica [A] time = 0.0060498, size = 47, normalized size = 1.04

$$\frac{\text{PolyLog}\left(2, e^{2a+2bx}\right)}{2b^2} + \frac{x \log\left(1 - e^{2a+2bx}\right)}{b} - \frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Coth[a + b*x], x]
```

```
[Out] -x^2/2 + (x*Log[1 - E^(2*a + 2*b*x)])/b + PolyLog[2, E^(2*a + 2*b*x)]/(2*b^2)
```

Maple [B] time = 0.026, size = 122, normalized size = 2.7

$$-\frac{x^2}{2} - 2\frac{ax}{b} - \frac{a^2}{b^2} + \frac{\ln(1 + e^{bx+a})x}{b} + \frac{\text{polylog}(2, -e^{bx+a})}{b^2} + \frac{\ln(1 - e^{bx+a})x}{b} + \frac{\ln(1 - e^{bx+a})a}{b^2} + \frac{\text{polylog}(2, e^{bx+a})}{b^2} -$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*coth(b*x+a),x)`

[Out] $-1/2*x^2-2/b*a*x-a^2/b^2+1/b*\ln(1+\exp(b*x+a))*x+1/b^2*\text{polylog}(2,-\exp(b*x+a))+1/b*\ln(1-\exp(b*x+a))*x+1/b^2*\ln(1-\exp(b*x+a))*a+1/b^2*\text{polylog}(2,\exp(b*x+a))-1/b^2*a*\ln(\exp(b*x+a)-1)+2/b^2*a*\ln(\exp(b*x+a))$

Maxima [B] time = 1.05142, size = 132, normalized size = 2.93

$$\frac{1}{2}x^2 \coth(bx + a) - b \left(\frac{x^2}{be^{(2bx+2a)} - b} + \frac{x^2}{b} - \frac{bx \log(e^{(bx+a)} + 1) + \text{Li}_2(-e^{(bx+a)})}{b^3} - \frac{bx \log(-e^{(bx+a)} + 1) + \text{Li}_2(e^{(bx+a)})}{b^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*coth(b*x+a),x, algorithm="maxima")`

[Out] $1/2*x^2*\coth(b*x + a) - b*(x^2/(b*e^{(2*b*x + 2*a)} - b) + x^2/b - (b*x*\log(e^{(b*x + a)} + 1) + \text{dilog}(-e^{(b*x + a)}))/b^3 - (b*x*\log(-e^{(b*x + a)} + 1) + \text{dilog}(e^{(b*x + a)}))/b^3)$

Fricas [B] time = 2.15813, size = 336, normalized size = 7.47

$$\frac{b^2x^2 - 2bx \log(\cosh(bx + a) + \sinh(bx + a) + 1) + 2a \log(\cosh(bx + a) + \sinh(bx + a) - 1) - 2(bx + a) \log(-\cosh(bx + a) + \sinh(bx + a))}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*coth(b*x+a),x, algorithm="fricas")`

[Out] $-1/2*(b^2*x^2 - 2*b*x*\log(\cosh(b*x + a) + \sinh(b*x + a) + 1) + 2*a*\log(\cosh(b*x + a) + \sinh(b*x + a) - 1) - 2*(b*x + a)*\log(-\cosh(b*x + a) + \sinh(b*x + a) + 1) - 2*\text{dilog}(\cosh(b*x + a) + \sinh(b*x + a)) - 2*\text{dilog}(-\cosh(b*x + a) + \sinh(b*x + a)))/b^2$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x \coth(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*coth(b*x+a),x)
```

```
[Out] Integral(x*coth(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \coth (bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*coth(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x*coth(b*x + a), x)
```

$$3.4 \quad \int \frac{\coth(ax+bx)}{x} dx$$

Optimal. Leaf size=12

$$\text{Unintegrable}\left(\frac{\coth(a+bx)}{x}, x\right)$$

[Out] Unintegrable[Coth[a + b*x]/x, x]

Rubi [A] time = 0.0164414, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\coth(a+bx)}{x} dx$$

Verification is Not applicable to the result.

[In] Int[Coth[a + b*x]/x, x]

[Out] Defer[Int][Coth[a + b*x]/x, x]

Rubi steps

$$\int \frac{\coth(a+bx)}{x} dx = \int \frac{\coth(a+bx)}{x} dx$$

Mathematica [A] time = 12.2113, size = 0, normalized size = 0.

$$\int \frac{\coth(a+bx)}{x} dx$$

Verification is Not applicable to the result.

[In] Integrate[Coth[a + b*x]/x, x]

[Out] Integrate[Coth[a + b*x]/x, x]

Maple [A] time = 0.085, size = 0, normalized size = 0.

$$\int \frac{\coth(bx + a)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(b*x+a)/x,x)

[Out] int(coth(b*x+a)/x,x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{xe^{(bx+a)} + x} dx + \int \frac{1}{xe^{(bx+a)} - x} dx + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(b*x+a)/x,x, algorithm="maxima")

[Out] -integrate(1/(x*e^(b*x + a) + x), x) + integrate(1/(x*e^(b*x + a) - x), x) + log(x)

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\coth(bx + a)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(b*x+a)/x,x, algorithm="fricas")

[Out] integral(coth(b*x + a)/x, x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(a + bx)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)/x,x)
```

```
[Out] Integral(coth(a + b*x)/x, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(bx + a)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)/x,x, algorithm="giac")
```

```
[Out] integrate(coth(b*x + a)/x, x)
```


$$3.5 \quad \int \frac{\coth(ax+bx)}{x^2} dx$$

Optimal. Leaf size=12

$$\text{Unintegrable}\left(\frac{\coth(a+bx)}{x^2}, x\right)$$

[Out] Unintegrable[Coth[a + b*x]/x^2, x]

Rubi [A] time = 0.016917, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\coth(a+bx)}{x^2} dx$$

Verification is Not applicable to the result.

[In] Int[Coth[a + b*x]/x^2, x]

[Out] Defer[Int][Coth[a + b*x]/x^2, x]

Rubi steps

$$\int \frac{\coth(a+bx)}{x^2} dx = \int \frac{\coth(a+bx)}{x^2} dx$$

Mathematica [A] time = 18.1177, size = 0, normalized size = 0.

$$\int \frac{\coth(a+bx)}{x^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[Coth[a + b*x]/x^2, x]

[Out] Integrate[Coth[a + b*x]/x^2, x]

Maple [A] time = 0.055, size = 0, normalized size = 0.

$$\int \frac{\coth(bx + a)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(b*x+a)/x^2,x)

[Out] int(coth(b*x+a)/x^2,x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{x} - \int \frac{1}{x^2 e^{(bx+a)} + x^2} dx + \int \frac{1}{x^2 e^{(bx+a)} - x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(b*x+a)/x^2,x, algorithm="maxima")

[Out] -1/x - integrate(1/(x^2*e^(b*x + a) + x^2), x) + integrate(1/(x^2*e^(b*x + a) - x^2), x)

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\coth(bx + a)}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(b*x+a)/x^2,x, algorithm="fricas")

[Out] integral(coth(b*x + a)/x^2, x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(a + bx)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)/x**2,x)
```

```
[Out] Integral(coth(a + b*x)/x**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(bx + a)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(b*x+a)/x^2,x, algorithm="giac")
```

```
[Out] integrate(coth(b*x + a)/x^2, x)
```

3.6 $\int x^3 \coth^2(a + bx) dx$

Optimal. Leaf size=87

$$\frac{3x \operatorname{PolyLog}(2, e^{2(a+bx)})}{b^3} - \frac{3 \operatorname{PolyLog}(3, e^{2(a+bx)})}{2b^4} + \frac{3x^2 \log(1 - e^{2(a+bx)})}{b^2} - \frac{x^3 \coth(a + bx)}{b} - \frac{x^3}{b} + \frac{x^4}{4}$$

```
[Out] -(x^3/b) + x^4/4 - (x^3*Coth[a + b*x])/b + (3*x^2*Log[1 - E^(2*(a + b*x))])
/b^2 + (3*x*PolyLog[2, E^(2*(a + b*x))])/b^3 - (3*PolyLog[3, E^(2*(a + b*x)
)])/(2*b^4)
```

Rubi [A] time = 0.177601, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {3720, 3716, 2190, 2531, 2282, 6589, 30}

$$\frac{3x \operatorname{PolyLog}(2, e^{2(a+bx)})}{b^3} - \frac{3 \operatorname{PolyLog}(3, e^{2(a+bx)})}{2b^4} + \frac{3x^2 \log(1 - e^{2(a+bx)})}{b^2} - \frac{x^3 \coth(a + bx)}{b} - \frac{x^3}{b} + \frac{x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Coth[a + b*x]^2,x]
```

```
[Out] -(x^3/b) + x^4/4 - (x^3*Coth[a + b*x])/b + (3*x^2*Log[1 - E^(2*(a + b*x))])
/b^2 + (3*x*PolyLog[2, E^(2*(a + b*x))])/b^3 - (3*PolyLog[3, E^(2*(a + b*x)
)])/(2*b^4)
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol]
:> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Dist
[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol]
:> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*e) + f*fz*x)))/E^(2*I*k*Pi))], x], x] /; FreeQ[{c, d, e, f, fz}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_)*(e_) + (f_)*(x_)))^(n_))*((c_) + (d_)*(x_))^(m_)]/
((a_) + (b_)*((F_)^((g_)*(e_) + (f_)*(x_)))^(n_)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_)*((F_)^((c_)*((a_) + (b_)*(x_)))^(n_))]*((f_) + (g_)
*(x_))^(m_), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
))^n])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]), x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_)*((a_) + (b_)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_)*((a_) + (b_)*(x_))^(p_)]/((d_) + (e_)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 30

```
Int[(x_)^(m_), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int x^3 \coth^2(a + bx) dx &= -\frac{x^3 \coth(a + bx)}{b} + \frac{3 \int x^2 \coth(a + bx) dx}{b} + \int x^3 dx \\
&= -\frac{x^3}{b} + \frac{x^4}{4} - \frac{x^3 \coth(a + bx)}{b} - \frac{6 \int \frac{e^{2(a+bx)} x^2}{1 - e^{2(a+bx)}} dx}{b} \\
&= -\frac{x^3}{b} + \frac{x^4}{4} - \frac{x^3 \coth(a + bx)}{b} + \frac{3x^2 \log(1 - e^{2(a+bx)})}{b^2} - \frac{6 \int x \log(1 - e^{2(a+bx)}) dx}{b^2} \\
&= -\frac{x^3}{b} + \frac{x^4}{4} - \frac{x^3 \coth(a + bx)}{b} + \frac{3x^2 \log(1 - e^{2(a+bx)})}{b^2} + \frac{3x \operatorname{Li}_2(e^{2(a+bx)})}{b^3} - \frac{3 \int \operatorname{Li}_2(e^{2(a+bx)}) dx}{b^3} \\
&= -\frac{x^3}{b} + \frac{x^4}{4} - \frac{x^3 \coth(a + bx)}{b} + \frac{3x^2 \log(1 - e^{2(a+bx)})}{b^2} + \frac{3x \operatorname{Li}_2(e^{2(a+bx)})}{b^3} - \frac{3 \operatorname{Subst}\left(\int \frac{\operatorname{Li}_2(x)}{x} dx, e^{2(a+bx)}\right)}{2b^4} \\
&= -\frac{x^3}{b} + \frac{x^4}{4} - \frac{x^3 \coth(a + bx)}{b} + \frac{3x^2 \log(1 - e^{2(a+bx)})}{b^2} + \frac{3x \operatorname{Li}_2(e^{2(a+bx)})}{b^3} - \frac{3 \operatorname{Li}_3(e^{2(a+bx)})}{2b^4}
\end{aligned}$$

Mathematica [B] time = 3.92103, size = 204, normalized size = 2.34

$$\frac{e^{2a} \left(6 \left(1 - e^{-2a} \right) \left(bx \operatorname{PolyLog} \left(2, -e^{-a-bx} \right) + \operatorname{PolyLog} \left(3, -e^{-a-bx} \right) \right) + 6 \left(1 - e^{-2a} \right) \left(bx \operatorname{PolyLog} \left(2, e^{-a-bx} \right) + \operatorname{PolyLog} \left(3, e^{-a-bx} \right) \right) \right)}{\left(e^{2a} - 1 \right) b^4}$$

Antiderivative was successfully verified.

[In] Integrate[x^3*Coth[a + b*x]^2,x]

[Out] $x^4/4 - (E^{(2*a)}*((2*b^3*x^3)/E^{(2*a)} - 3*b^2*(1 - E^{(-2*a)})*x^2*\log[1 - E^{(-a - b*x)}] - 3*b^2*(1 - E^{(-2*a)})*x^2*\log[1 + E^{(-a - b*x)}] + 6*(1 - E^{(-2*a)})*(b*x*\operatorname{PolyLog}[2, -E^{(-a - b*x)}] + \operatorname{PolyLog}[3, -E^{(-a - b*x)}]) + 6*(1 - E^{(-2*a)})*(b*x*\operatorname{PolyLog}[2, E^{(-a - b*x)}] + \operatorname{PolyLog}[3, E^{(-a - b*x)}])))/(b^4*(-1 + E^{(2*a)})) + (x^3*\operatorname{Csch}[a]*\operatorname{Csch}[a + b*x]*\operatorname{Sinh}[b*x])/b$

Maple [B] time = 0.028, size = 198, normalized size = 2.3

$$\frac{x^4}{4} - 2 \frac{x^3}{b(e^{2bx+2a} - 1)} + 3 \frac{a^2 \ln(e^{bx+a} - 1)}{b^4} - 6 \frac{a^2 \ln(e^{bx+a})}{b^4} - 2 \frac{x^3}{b} + 6 \frac{a^2 x}{b^3} + 4 \frac{a^3}{b^4} + 3 \frac{\ln(1 + e^{bx+a}) x^2}{b^2} + 6 \frac{\operatorname{polylog}(2, e^{bx+a})}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*coth(b*x+a)^2,x)

```
[Out] 1/4*x^4-2*x^3/b/(exp(2*b*x+2*a)-1)+3/b^4*a^2*ln(exp(b*x+a)-1)-6/b^4*a^2*ln(
exp(b*x+a))-2*x^3/b+6/b^3*a^2*x+4/b^4*a^3+3/b^2*ln(1+exp(b*x+a))*x^2+6/b^3*
polylog(2,-exp(b*x+a))*x-6/b^4*polylog(3,-exp(b*x+a))+3/b^2*ln(1-exp(b*x+a)
)*x^2-3/b^4*ln(1-exp(b*x+a))*a^2+6/b^3*polylog(2,exp(b*x+a))*x-6/b^4*polylo
g(3,exp(b*x+a))
```

Maxima [A] time = 1.379, size = 197, normalized size = 2.26

$$-\frac{2x^3}{b} + \frac{bx^4e^{2bx+2a} - bx^4 - 8x^3}{4(be^{2bx+2a} - b)} + \frac{3(b^2x^2 \log(e^{bx+a} + 1) + 2bx\text{Li}_2(-e^{bx+a}) - 2\text{Li}_3(-e^{bx+a}))}{b^4} + \frac{3(b^2x^2 \log(-e^{bx+a}))}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*coth(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -2*x^3/b + 1/4*(b*x^4*e^(2*b*x + 2*a) - b*x^4 - 8*x^3)/(b*e^(2*b*x + 2*a) -
b) + 3*(b^2*x^2*log(e^(b*x + a) + 1) + 2*b*x*dilog(-e^(b*x + a)) - 2*polylo
g(3, -e^(b*x + a)))/b^4 + 3*(b^2*x^2*log(-e^(b*x + a) + 1) + 2*b*x*dilog(e
^(b*x + a)) - 2*polylog(3, e^(b*x + a)))/b^4
```

Fricas [C] time = 2.24419, size = 1643, normalized size = 18.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*coth(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(b^4*x^4 - 8*a^3 - (b^4*x^4 - 8*b^3*x^3 - 8*a^3)*cosh(b*x + a)^2 - 2*(
b^4*x^4 - 8*b^3*x^3 - 8*a^3)*cosh(b*x + a)*sinh(b*x + a) - (b^4*x^4 - 8*b^3
*x^3 - 8*a^3)*sinh(b*x + a)^2 - 24*(b*x*cosh(b*x + a)^2 + 2*b*x*cosh(b*x +
a)*sinh(b*x + a) + b*x*sinh(b*x + a)^2 - b*x)*dilog(cosh(b*x + a) + sinh(b*
x + a)) - 24*(b*x*cosh(b*x + a)^2 + 2*b*x*cosh(b*x + a)*sinh(b*x + a) + b*x
*sinh(b*x + a)^2 - b*x)*dilog(-cosh(b*x + a) - sinh(b*x + a)) - 12*(b^2*x^2
*cosh(b*x + a)^2 + 2*b^2*x^2*cosh(b*x + a)*sinh(b*x + a) + b^2*x^2*sinh(b*x
+ a)^2 - b^2*x^2)*log(cosh(b*x + a) + sinh(b*x + a) + 1) - 12*(a^2*cosh(b*
x + a)^2 + 2*a^2*cosh(b*x + a)*sinh(b*x + a) + a^2*sinh(b*x + a)^2 - a^2)*l
og(cosh(b*x + a) + sinh(b*x + a) - 1) + 12*(b^2*x^2 - (b^2*x^2 - a^2)*cosh(
b*x + a)^2 - 2*(b^2*x^2 - a^2)*cosh(b*x + a)*sinh(b*x + a) - (b^2*x^2 - a^2
```

```
) * sinh(b*x + a)^2 - a^2) * log(-cosh(b*x + a) - sinh(b*x + a) + 1) + 24 * (cosh
(b*x + a)^2 + 2 * cosh(b*x + a) * sinh(b*x + a) + sinh(b*x + a)^2 - 1) * polylog(
3, cosh(b*x + a) + sinh(b*x + a)) + 24 * (cosh(b*x + a)^2 + 2 * cosh(b*x + a) * s
inh(b*x + a) + sinh(b*x + a)^2 - 1) * polylog(3, -cosh(b*x + a) - sinh(b*x +
a)) / (b^4 * cosh(b*x + a)^2 + 2 * b^4 * cosh(b*x + a) * sinh(b*x + a) + b^4 * sinh(b*
x + a)^2 - b^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \coth^2(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*coth(b*x+a)**2,x)
```

```
[Out] Integral(x**3*coth(a + b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \coth(bx + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*coth(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^3*coth(b*x + a)^2, x)
```


3.7 $\int x^2 \coth^2(a + bx) dx$

Optimal. Leaf size=65

$$\frac{\text{PolyLog}\left(2, e^{2(a+bx)}\right)}{b^3} + \frac{2x \log\left(1 - e^{2(a+bx)}\right)}{b^2} - \frac{x^2 \coth(a + bx)}{b} - \frac{x^2}{b} + \frac{x^3}{3}$$

[Out] $-(x^2/b) + x^3/3 - (x^2 \coth[a + b*x])/b + (2*x*\text{Log}[1 - E^{2*(a + b*x)}])/b^2 + \text{PolyLog}[2, E^{2*(a + b*x)}]/b^3$

Rubi [A] time = 0.119435, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {3720, 3716, 2190, 2279, 2391, 30}

$$\frac{\text{PolyLog}\left(2, e^{2(a+bx)}\right)}{b^3} + \frac{2x \log\left(1 - e^{2(a+bx)}\right)}{b^2} - \frac{x^2 \coth(a + bx)}{b} - \frac{x^2}{b} + \frac{x^3}{3}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^2 \coth[a + b*x]^2, x]$

[Out] $-(x^2/b) + x^3/3 - (x^2 \coth[a + b*x])/b + (2*x*\text{Log}[1 - E^{2*(a + b*x)}])/b^2 + \text{PolyLog}[2, E^{2*(a + b*x)}]/b^3$

Rule 3720

$\text{Int}[\left((c_.) + (d_.)*(x_.)\right)^{(m_.)} * \left((b_.)*\tan[(e_.) + (f_.)*(x_.)]\right)^{(n_.)}, x_Symbol] \rightarrow \text{Simp}[(b*(c + d*x)^m*(b*\text{Tan}[e + f*x])^{n-1})/(f*(n-1)), x] + (-\text{Dist}[(b*d*m)/(f*(n-1)), \text{Int}[(c + d*x)^{m-1}*(b*\text{Tan}[e + f*x])^{n-1}, x], x] - \text{Dist}[b^2, \text{Int}[(c + d*x)^m*(b*\text{Tan}[e + f*x])^{n-2}, x], x]) /; \text{FreeQ}\{b, c, d, e, f\}, x] \&\& \text{GtQ}[n, 1] \&\& \text{GtQ}[m, 0]$

Rule 3716

$\text{Int}[\left((c_.) + (d_.)*(x_.)\right)^{(m_.)} * \tan[(e_.) + \text{Pi}*(k_.) + (\text{Complex}[0, fz_])*(f_.)*(x_.)], x_Symbol] \rightarrow -\text{Simp}[(I*(c + d*x)^{(m+1)})/(d*(m+1)), x] + \text{Dist}[2*I, \text{Int}[\left((c + d*x)^m * E^{2*(-(I*e) + f*fz*x)}\right)/(E^{2*I*k*Pi}*(1 + E^{2*(-(I*e) + f*fz*x)})/E^{2*I*k*Pi})], x], x] /; \text{FreeQ}\{c, d, e, f, fz\}, x] \&\& \text{IntegerQ}[4*k] \&\& \text{IGtQ}[m, 0]$

Rule 2190

```
Int[(((F_)^((g_)*((e_) + (f_)*(x_))))^(n_))*((c_) + (d_)*(x_))^(m_)]/
((a_) + (b_)*((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a]]/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_))))^(n_)], x_Symbol]
:= Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 30

```
Int[(x_)^(m_), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int x^2 \coth^2(a + bx) dx &= -\frac{x^2 \coth(a + bx)}{b} + \frac{2 \int x \coth(a + bx) dx}{b} + \int x^2 dx \\ &= -\frac{x^2}{b} + \frac{x^3}{3} - \frac{x^2 \coth(a + bx)}{b} - \frac{4 \int \frac{e^{2(a+bx)} x}{1 - e^{2(a+bx)}} dx}{b} \\ &= -\frac{x^2}{b} + \frac{x^3}{3} - \frac{x^2 \coth(a + bx)}{b} + \frac{2x \log(1 - e^{2(a+bx)})}{b^2} - \frac{2 \int \log(1 - e^{2(a+bx)}) dx}{b^2} \\ &= -\frac{x^2}{b} + \frac{x^3}{3} - \frac{x^2 \coth(a + bx)}{b} + \frac{2x \log(1 - e^{2(a+bx)})}{b^2} - \frac{\text{Subst}\left(\int \frac{\log(1-x)}{x} dx, x, e^{2(a+bx)}\right)}{b^3} \\ &= -\frac{x^2}{b} + \frac{x^3}{3} - \frac{x^2 \coth(a + bx)}{b} + \frac{2x \log(1 - e^{2(a+bx)})}{b^2} + \frac{\text{Li}_2(e^{2(a+bx)})}{b^3} \end{aligned}$$

Mathematica [C] time = 5.09045, size = 163, normalized size = 2.51

$$-\text{PolyLog}\left(2, e^{-2(\tanh^{-1}(\tanh(a))+bx)}\right) - b^2 x^2 e^{-\tanh^{-1}(\tanh(a))} \coth(a) \sqrt{\text{sech}^2(a)} + 2bx \log\left(1 - e^{-2(\tanh^{-1}(\tanh(a))+bx)}\right) + 2t$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^2*Coth[a + b*x]^2,x]

[Out] $x^3/3 + (I*b*Pi*x - I*Pi*Log[1 + E^{(2*b*x)}] + 2*b*x*Log[1 - E^{-2*(b*x + ArcTanh[Tanh[a]])}] + I*Pi*Log[Cosh[b*x]] + 2*ArcTanh[Tanh[a]]*(b*x + Log[1 - E^{-2*(b*x + ArcTanh[Tanh[a]])}] - Log[I*Sinh[b*x + ArcTanh[Tanh[a]]])] - PolyLog[2, E^{-2*(b*x + ArcTanh[Tanh[a]])}] - (b^2*x^2*Coth[a]*Sqrt[Sech[a]^2])/E^{ArcTanh[Tanh[a]])/b^3 + (x^2*Csch[a]*Csch[a + b*x]*Sinh[b*x])/b$

Maple [B] time = 0.029, size = 156, normalized size = 2.4

$$\frac{x^3}{3} - 2 \frac{x^2}{b(e^{2bx+2a} - 1)} - 2 \frac{x^2}{b} - 4 \frac{ax}{b^2} - 2 \frac{a^2}{b^3} + 2 \frac{\ln(1 + e^{bx+a})x}{b^2} + 2 \frac{\text{polylog}(2, -e^{bx+a})}{b^3} + 2 \frac{\ln(1 - e^{bx+a})x}{b^2} + 2 \frac{\ln(1 - e^{bx+a})}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*coth(b*x+a)^2,x)

[Out] $1/3*x^3 - 2*x^2/b / (\exp(2*b*x+2*a) - 1) - 2*x^2/b - 4/b^2*a*x - 2/b^3*a^2 + 2/b^2*\ln(1 + \exp(b*x+a))*x + 2/b^3*\text{polylog}(2, -\exp(b*x+a)) + 2/b^2*\ln(1 - \exp(b*x+a))*x + 2/b^3*\ln(1 - \exp(b*x+a))*a + 2/b^3*\text{polylog}(2, \exp(b*x+a)) - 2/b^3*a*\ln(\exp(b*x+a) - 1) + 4/b^3*a*\ln(\exp(b*x+a))$

Maxima [A] time = 1.34962, size = 146, normalized size = 2.25

$$-\frac{2x^2}{b} + \frac{bx^3e^{(2bx+2a)} - bx^3 - 6x^2}{3(be^{(2bx+2a)} - b)} + \frac{2(bx \log(e^{(bx+a)} + 1) + \text{Li}_2(-e^{(bx+a)}))}{b^3} + \frac{2(bx \log(-e^{(bx+a)} + 1) + \text{Li}_2(e^{(bx+a)}))}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*coth(b*x+a)^2,x, algorithm="maxima")

[Out] $-2*x^2/b + 1/3*(b*x^3*e^{(2*b*x + 2*a)} - b*x^3 - 6*x^2)/(b*e^{(2*b*x + 2*a)} - b) + 2*(b*x*\log(e^{(b*x + a)} + 1) + \text{dilog}(-e^{(b*x + a)}))/b^3 + 2*(b*x*\log(-e^{(b*x + a)} + 1) + \text{dilog}(e^{(b*x + a)}))/b^3$

Fricas [B] time = 2.2115, size = 1211, normalized size = 18.63

$$\frac{b^3 x^3 - (b^3 x^3 - 6 b^2 x^2 + 6 a^2) \cosh(bx + a)^2 - 2(b^3 x^3 - 6 b^2 x^2 + 6 a^2) \cosh(bx + a) \sinh(bx + a) - (b^3 x^3 - 6 b^2 x^2 + 6 a^2) \sinh(bx + a)^2}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*coth(b*x+a)^2,x, algorithm="fricas")

[Out]
$$-1/3*(b^3*x^3 - (b^3*x^3 - 6*b^2*x^2 + 6*a^2)*\cosh(b*x + a)^2 - 2*(b^3*x^3 - 6*b^2*x^2 + 6*a^2)*\cosh(b*x + a)*\sinh(b*x + a) - (b^3*x^3 - 6*b^2*x^2 + 6*a^2)*\sinh(b*x + a)^2 + 6*a^2 - 6*(\cosh(b*x + a)^2 + 2*\cosh(b*x + a)*\sinh(b*x + a) + \sinh(b*x + a)^2 - 1)*\operatorname{dilog}(\cosh(b*x + a) + \sinh(b*x + a)) - 6*(\cosh(b*x + a)^2 + 2*\cosh(b*x + a)*\sinh(b*x + a) + \sinh(b*x + a)^2 - 1)*\operatorname{dilog}(-\cosh(b*x + a) - \sinh(b*x + a)) - 6*(b*x*\cosh(b*x + a)^2 + 2*b*x*\cosh(b*x + a)*\sinh(b*x + a) + b*x*\sinh(b*x + a)^2 - b*x)*\log(\cosh(b*x + a) + \sinh(b*x + a) + 1) + 6*(a*\cosh(b*x + a)^2 + 2*a*\cosh(b*x + a)*\sinh(b*x + a) + a*\sinh(b*x + a)^2 - a)*\log(\cosh(b*x + a) + \sinh(b*x + a) - 1) - 6*((b*x + a)*\cosh(b*x + a)^2 + 2*(b*x + a)*\cosh(b*x + a)*\sinh(b*x + a) + (b*x + a)*\sinh(b*x + a)^2 - b*x - a)*\log(-\cosh(b*x + a) - \sinh(b*x + a) + 1))/(b^3*\cosh(b*x + a)^2 + 2*b^3*\cosh(b*x + a)*\sinh(b*x + a) + b^3*\sinh(b*x + a)^2 - b^3)$$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \coth^2(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*coth(b*x+a)**2,x)

[Out] Integral(x**2*coth(a + b*x)**2, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \coth(bx + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*coth(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*coth(b*x + a)^2, x)
```

3.8 $\int x \coth^2(a + bx) dx$

Optimal. Leaf size=31

$$\frac{\log(\sinh(a + bx))}{b^2} - \frac{x \coth(a + bx)}{b} + \frac{x^2}{2}$$

[Out] $x^2/2 - (x*\text{Coth}[a + b*x])/b + \text{Log}[\text{Sinh}[a + b*x]]/b^2$

Rubi [A] time = 0.0267439, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$, Rules used = {3720, 3475, 30}

$$\frac{\log(\sinh(a + bx))}{b^2} - \frac{x \coth(a + bx)}{b} + \frac{x^2}{2}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x*\text{Coth}[a + b*x]^2, x]$

[Out] $x^2/2 - (x*\text{Coth}[a + b*x])/b + \text{Log}[\text{Sinh}[a + b*x]]/b^2$

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol]
:= Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Dist[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x], x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Log[RemoveContent[Cos[c + d*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]
```

Rubi steps

$$\int x \coth^2(a + bx) dx = -\frac{x \coth(a + bx)}{b} + \frac{\int \coth(a + bx) dx}{b} + \int x dx$$

$$= \frac{x^2}{2} - \frac{x \coth(a + bx)}{b} + \frac{\log(\sinh(a + bx))}{b^2}$$

Mathematica [A] time = 0.153167, size = 46, normalized size = 1.48

$$\frac{-2bx \coth(a) + 2 \log(\sinh(a + bx)) + 2bx \operatorname{csch}(a) \sinh(bx) \operatorname{csch}(a + bx) + b^2 x^2}{2b^2}$$

Antiderivative was successfully verified.

[In] Integrate[x*Coth[a + b*x]^2,x]

[Out] (b^2*x^2 - 2*b*x*Coth[a] + 2*Log[Sinh[a + b*x]] + 2*b*x*Csch[a]*Csch[a + b*x]*Sinh[b*x])/(2*b^2)

Maple [A] time = 0.029, size = 54, normalized size = 1.7

$$\frac{x^2}{2} - 2\frac{x}{b} - 2\frac{a}{b^2} - 2\frac{x}{b(e^{2bx+2a}-1)} + \frac{\ln(e^{2bx+2a}-1)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*coth(b*x+a)^2,x)

[Out] 1/2*x^2-2*x/b-2*a/b^2-2*x/b/(exp(2*b*x+2*a)-1)+1/b^2*ln(exp(2*b*x+2*a)-1)

Maxima [B] time = 1.184, size = 155, normalized size = 5.

$$-\frac{x e^{(2bx+2a)}}{b e^{(2bx+2a)} - b} - \frac{bx^2 - (bx^2 e^{(2a)} - 2x e^{(2a)}) e^{(2bx)}}{2(b e^{(2bx+2a)} - b)} + \frac{\log((e^{(bx+a)} + 1)e^{(-a)})}{b^2} + \frac{\log((e^{(bx+a)} - 1)e^{(-a)})}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*coth(b*x+a)^2,x, algorithm="maxima")

```
[Out] -x*e^(2*b*x + 2*a)/(b*e^(2*b*x + 2*a) - b) - 1/2*(b*x^2 - (b*x^2*e^(2*a) -
2*x*e^(2*a))*e^(2*b*x))/(b*e^(2*b*x + 2*a) - b) + log((e^(b*x + a) + 1)*e^(-
a))/b^2 + log((e^(b*x + a) - 1)*e^(-a))/b^2
```

Fricas [B] time = 2.14701, size = 477, normalized size = 15.39

$$\frac{b^2x^2 - (b^2x^2 - 4bx) \cosh(bx + a)^2 - 2(b^2x^2 - 4bx) \cosh(bx + a) \sinh(bx + a) - (b^2x^2 - 4bx) \sinh(bx + a)^2 - 2(\cosh(bx + a) \sinh(bx + a) - 1) \log(2 \sinh(bx + a))}{2(b^2 \cosh(bx + a)^2 + 2b^2 \cosh(bx + a) \sinh(bx + a) + b^2 \sinh(bx + a)^2 - b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*coth(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(b^2*x^2 - (b^2*x^2 - 4*b*x)*cosh(b*x + a)^2 - 2*(b^2*x^2 - 4*b*x)*cos
h(b*x + a)*sinh(b*x + a) - (b^2*x^2 - 4*b*x)*sinh(b*x + a)^2 - 2*(cosh(b*x
+ a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)*log(2*sinh(b*
x + a)/(cosh(b*x + a) - sinh(b*x + a)))/(b^2*cosh(b*x + a)^2 + 2*b^2*cosh(
b*x + a)*sinh(b*x + a) + b^2*sinh(b*x + a)^2 - b^2)
```

Sympy [A] time = 4.73998, size = 80, normalized size = 2.58

$$\begin{cases} \infty x^2 + \frac{\infty x}{b} & \text{for } a = \log(-e^{-bx}) \\ \infty x^2 & \text{for } a = \log(e^{-bx}) \\ \frac{x^2 \coth^2(a)}{2} & \text{for } b = 0 \\ \frac{x^2}{2} + \frac{x}{b} - \frac{x}{b \tanh(a+bx)} - \frac{\log(\tanh(a+bx)+1)}{b^2} + \frac{\log(\tanh(a+bx))}{b^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*coth(b*x+a)**2,x)
```

```
[Out] Piecewise((zoo*x**2 + zoo*x/b, Eq(a, log(-exp(-b*x)))), (zoo*x**2, Eq(a, lo
g(exp(-b*x)))), (x**2*coth(a)**2/2, Eq(b, 0)), (x**2/2 + x/b - x/(b*tanh(a
+ b*x)) - log(tanh(a + b*x) + 1)/b**2 + log(tanh(a + b*x))/b**2, True))
```


Giac [B] time = 1.17756, size = 132, normalized size = 4.26

$$\frac{b^2 x^2 e^{(2bx+2a)} - b^2 x^2 - 4bx e^{(2bx+2a)} + 2e^{(2bx+2a)} \log(e^{(2bx+2a)} - 1) - 2 \log(e^{(2bx+2a)} - 1)}{2(b^2 e^{(2bx+2a)} - b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*coth(b*x+a)^2,x, algorithm="giac")

[Out] 1/2*(b^2*x^2*e^(2*b*x + 2*a) - b^2*x^2 - 4*b*x*e^(2*b*x + 2*a) + 2*e^(2*b*x + 2*a)*log(e^(2*b*x + 2*a) - 1) - 2*log(e^(2*b*x + 2*a) - 1))/(b^2*e^(2*b*x + 2*a) - b^2)

3.9

$$\int \frac{\coth^2(a+bx)}{x} dx$$

Optimal. Leaf size=14

$$\text{Unintegrable}\left(\frac{\coth^2(a+bx)}{x}, x\right)$$

[Out] Unintegrable[Coth[a + b*x]^2/x, x]

Rubi [A] time = 0.0279736, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\coth^2(a+bx)}{x} dx$$

Verification is Not applicable to the result.

[In] Int[Coth[a + b*x]^2/x, x]

[Out] Defer[Int][Coth[a + b*x]^2/x, x]

Rubi steps

$$\int \frac{\coth^2(a+bx)}{x} dx = \int \frac{\coth^2(a+bx)}{x} dx$$

Mathematica [A] time = 20.1614, size = 0, normalized size = 0.

$$\int \frac{\coth^2(a+bx)}{x} dx$$

Verification is Not applicable to the result.

[In] Integrate[Coth[a + b*x]^2/x, x]

[Out] Integrate[Coth[a + b*x]^2/x, x]

Maple [A] time = 0.04, size = 0, normalized size = 0.

$$\int \frac{(\coth(bx + a))^2}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(b*x+a)^2/x,x)

[Out] int(coth(b*x+a)^2/x,x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{2}{bx e^{2bx+2a} - bx} + \int \frac{1}{bx^2 e^{bx+a} + bx^2} dx - \int \frac{1}{bx^2 e^{bx+a} - bx^2} dx + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(b*x+a)^2/x,x, algorithm="maxima")

[Out] -2/(b*x*e^(2*b*x + 2*a) - b*x) + integrate(1/(b*x^2*e^(b*x + a) + b*x^2), x) - integrate(1/(b*x^2*e^(b*x + a) - b*x^2), x) + log(x)

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\coth(bx + a)^2}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(b*x+a)^2/x,x, algorithm="fricas")

[Out] integral(coth(b*x + a)^2/x, x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth^2(a + bx)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(b*x+a)**2/x,x)

[Out] Integral(coth(a + b*x)**2/x, x)

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth^2(bx + a)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(b*x+a)^2/x,x, algorithm="giac")

[Out] integrate(coth(b*x + a)^2/x, x)

$$3.10 \quad \int \frac{\coth^2(a+bx)}{x^2} dx$$

Optimal. Leaf size=14

$$\text{Unintegrable}\left(\frac{\coth^2(a+bx)}{x^2}, x\right)$$

[Out] Unintegrable[Coth[a + b*x]^2/x^2, x]

Rubi [A] time = 0.0289372, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\coth^2(a+bx)}{x^2} dx$$

Verification is Not applicable to the result.

[In] Int[Coth[a + b*x]^2/x^2,x]

[Out] Defer[Int][Coth[a + b*x]^2/x^2, x]

Rubi steps

$$\int \frac{\coth^2(a+bx)}{x^2} dx = \int \frac{\coth^2(a+bx)}{x^2} dx$$

Mathematica [A] time = 19.4357, size = 0, normalized size = 0.

$$\int \frac{\coth^2(a+bx)}{x^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[Coth[a + b*x]^2/x^2,x]

[Out] Integrate[Coth[a + b*x]^2/x^2, x]

Maple [A] time = 0.063, size = 0, normalized size = 0.

$$\int \frac{(\coth(bx + a))^2}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(b*x+a)^2/x^2,x)

[Out] int(coth(b*x+a)^2/x^2,x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{bx e^{(2bx+2a)} - bx + 2}{bx^2 e^{(2bx+2a)} - bx^2} + 2 \int \frac{1}{bx^3 e^{(bx+a)} + bx^3} dx - 2 \int \frac{1}{bx^3 e^{(bx+a)} - bx^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(b*x+a)^2/x^2,x, algorithm="maxima")

[Out] -(b*x*e^(2*b*x + 2*a) - b*x + 2)/(b*x^2*e^(2*b*x + 2*a) - b*x^2) + 2*integrate(1/(b*x^3*e^(b*x + a) + b*x^3), x) - 2*integrate(1/(b*x^3*e^(b*x + a) - b*x^3), x)

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\coth(bx + a)^2}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(b*x+a)^2/x^2,x, algorithm="fricas")

[Out] integral(coth(b*x + a)^2/x^2, x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth^2(a + bx)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(b*x+a)**2/x**2,x)

[Out] Integral(coth(a + b*x)**2/x**2, x)

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth^2(bx + a)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(b*x+a)^2/x^2,x, algorithm="giac")

[Out] integrate(coth(b*x + a)^2/x^2, x)

3.11 $\int x^3 \coth^3(a + bx) dx$

Optimal. Leaf size=179

$$\frac{3x^2 \text{PolyLog}\left(2, e^{2(a+bx)}\right)}{2b^2} - \frac{3x \text{PolyLog}\left(3, e^{2(a+bx)}\right)}{2b^3} + \frac{3 \text{PolyLog}\left(2, e^{2(a+bx)}\right)}{2b^4} + \frac{3 \text{PolyLog}\left(4, e^{2(a+bx)}\right)}{4b^4} - \frac{3x^2 \coth(a + bx)}{2b^2}$$

[Out] $(-3x^2)/(2b^2) + x^3/(2b) - x^4/4 - (3x^2 \text{Coth}[a + bx])/(2b^2) - (x^3 \text{Coth}[a + bx]^2)/(2b) + (3x \text{Log}[1 - E^{2(a + bx)}])/b^3 + (x^3 \text{Log}[1 - E^{2(a + bx)}])/b + (3 \text{PolyLog}[2, E^{2(a + bx)}])/(2b^4) + (3x^2 \text{PolyLog}[2, E^{2(a + bx)}])/(2b^2) - (3x \text{PolyLog}[3, E^{2(a + bx)}])/(2b^3) + (3 \text{PolyLog}[4, E^{2(a + bx)}])/(4b^4)$

Rubi [A] time = 0.329702, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {3720, 3716, 2190, 2279, 2391, 30, 2531, 6609, 2282, 6589}

$$\frac{3x^2 \text{PolyLog}\left(2, e^{2(a+bx)}\right)}{2b^2} - \frac{3x \text{PolyLog}\left(3, e^{2(a+bx)}\right)}{2b^3} + \frac{3 \text{PolyLog}\left(2, e^{2(a+bx)}\right)}{2b^4} + \frac{3 \text{PolyLog}\left(4, e^{2(a+bx)}\right)}{4b^4} - \frac{3x^2 \coth(a + bx)}{2b^2}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^3 \text{Coth}[a + bx]^3, x]$

[Out] $(-3x^2)/(2b^2) + x^3/(2b) - x^4/4 - (3x^2 \text{Coth}[a + bx])/(2b^2) - (x^3 \text{Coth}[a + bx]^2)/(2b) + (3x \text{Log}[1 - E^{2(a + bx)}])/b^3 + (x^3 \text{Log}[1 - E^{2(a + bx)}])/b + (3 \text{PolyLog}[2, E^{2(a + bx)}])/(2b^4) + (3x^2 \text{PolyLog}[2, E^{2(a + bx)}])/(2b^2) - (3x \text{PolyLog}[3, E^{2(a + bx)}])/(2b^3) + (3 \text{PolyLog}[4, E^{2(a + bx)}])/(4b^4)$

Rule 3720

$\text{Int}[(c + d \cdot x)^m \cdot (b \cdot \tan[e + f \cdot x])^{n-1} / (f \cdot (n-1)), x] + (-\text{Dist}[(b \cdot d \cdot m) / (f \cdot (n-1)), \text{Int}[(c + d \cdot x)^{m-1} \cdot (b \cdot \tan[e + f \cdot x])^{n-1}, x], x] - \text{Dist}[b^2, \text{Int}[(c + d \cdot x)^m \cdot (b \cdot \tan[e + f \cdot x])^{n-2}, x], x]) / ; \text{FreeQ}\{b, c, d, e, f, x\} \ \&\& \text{GtQ}[n, 1] \ \&\& \text{GtQ}[m, 0]$

Rule 3716

$\text{Int}[(c + d \cdot x)^m \cdot \tan[e + \text{Pi} \cdot (k + \text{Complex}[0, fz]) \cdot (f \cdot x)], x_{\text{Symbol}}] \text{ :> } -\text{Simp}[(I \cdot (c + d \cdot x)^{m+1}) / (d \cdot (m+1)), x] + \text{Dist}[2$

*I, Int[((c + d*x)^m*E^(2*(-I*e) + f*fz*x))/(E^(2*I*k*Pi)*(1 + E^(2*(-I*e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && IntegerQ[4*k] && IGtQ[m, 0]

Rule 2190

Int[(((F_)^((g_)*(e_) + (f_)*(x_)))^(n_))*((c_) + (d_)*(x_))^(m_)]/((a_) + (b_)*((F_)^((g_)*(e_) + (f_)*(x_)))^(n_)), x_Symbol] :> Simp[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a]]/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

Rule 2279

Int[Log[(a_) + (b_)*((F_)^((e_)*(c_) + (d_)*(x_)))^(n_)], x_Symbol] :> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

Rule 2391

Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] :> -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]

Rule 30

Int[(x_)^(m_), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 2531

Int[Log[1 + (e_)*((F_)^((c_)*((a_) + (b_)*(x_)))^(n_))]*(f_) + (g_)*(x_)^(m_), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m - 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

Rule 6609

Int[(((e_) + (f_)*(x_))^(m_))*PolyLog[n_, (d_)*((F_)^((c_)*((a_) + (b_)*(x_)))^(p_)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p]]/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c, d, e, f, n, p}, x] && GtQ[m, 0]

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x],
Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int x^3 \coth^3(a + bx) dx &= -\frac{x^3 \coth^2(a + bx)}{2b} + \frac{3 \int x^2 \coth^2(a + bx) dx}{2b} + \int x^3 \coth(a + bx) dx \\
&= -\frac{x^4}{4} - \frac{3x^2 \coth(a + bx)}{2b^2} - \frac{x^3 \coth^2(a + bx)}{2b} - 2 \int \frac{e^{2(a+bx)} x^3}{1 - e^{2(a+bx)}} dx + \frac{3 \int x \coth(a + bx) dx}{b^2} + \frac{3}{b} \int \frac{e^{2(a+bx)} x}{1 - e^{2(a+bx)}} dx \\
&= -\frac{3x^2}{2b^2} + \frac{x^3}{2b} - \frac{x^4}{4} - \frac{3x^2 \coth(a + bx)}{2b^2} - \frac{x^3 \coth^2(a + bx)}{2b} + \frac{x^3 \log(1 - e^{2(a+bx)})}{b} - \frac{6 \int \frac{e^{2(a+bx)} x}{1 - e^{2(a+bx)}} dx}{b^2} \\
&= -\frac{3x^2}{2b^2} + \frac{x^3}{2b} - \frac{x^4}{4} - \frac{3x^2 \coth(a + bx)}{2b^2} - \frac{x^3 \coth^2(a + bx)}{2b} + \frac{3x \log(1 - e^{2(a+bx)})}{b^3} + \frac{x^3 \log(1 - e^{2(a+bx)})}{b} \\
&= -\frac{3x^2}{2b^2} + \frac{x^3}{2b} - \frac{x^4}{4} - \frac{3x^2 \coth(a + bx)}{2b^2} - \frac{x^3 \coth^2(a + bx)}{2b} + \frac{3x \log(1 - e^{2(a+bx)})}{b^3} + \frac{x^3 \log(1 - e^{2(a+bx)})}{b} \\
&= -\frac{3x^2}{2b^2} + \frac{x^3}{2b} - \frac{x^4}{4} - \frac{3x^2 \coth(a + bx)}{2b^2} - \frac{x^3 \coth^2(a + bx)}{2b} + \frac{3x \log(1 - e^{2(a+bx)})}{b^3} + \frac{x^3 \log(1 - e^{2(a+bx)})}{b} \\
&= -\frac{3x^2}{2b^2} + \frac{x^3}{2b} - \frac{x^4}{4} - \frac{3x^2 \coth(a + bx)}{2b^2} - \frac{x^3 \coth^2(a + bx)}{2b} + \frac{3x \log(1 - e^{2(a+bx)})}{b^3} + \frac{x^3 \log(1 - e^{2(a+bx)})}{b}
\end{aligned}$$

Mathematica [B] time = 4.59822, size = 390, normalized size = 2.18

$$\frac{1}{4} \left(-\frac{2e^{2a} (6(1 - e^{-2a}) (b^2 x^2 \text{PolyLog}(2, -e^{-a-bx}) + 2(bx \text{PolyLog}(3, -e^{-a-bx}) + \text{PolyLog}(4, -e^{-a-bx}))) + 6(1 - e^{-2a}) (b^2 x^2 \text{PolyLog}(2, -e^{-a-bx}) + 2(bx \text{PolyLog}(3, -e^{-a-bx}) + \text{PolyLog}(4, -e^{-a-bx})))}{4} \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^3*Coth[a + b*x]^3,x]

```
[Out] (x^4*Coth[a] - (2*x^3*Csch[a + b*x]^2)/b - (2*E^(2*a)*((6*b^2*x^2)/E^(2*a)
+ (b^4*x^4)/E^(2*a) - 6*b*(1 - E^(-2*a))*x*Log[1 - E^(-a - b*x)] - (2*b^3*(
-1 + E^(2*a))*x^3*Log[1 - E^(-a - b*x)])/E^(2*a) - 6*b*(1 - E^(-2*a))*x*Log
[1 + E^(-a - b*x)] - (2*b^3*(-1 + E^(2*a))*x^3*Log[1 + E^(-a - b*x)])/E^(2*
a) + 6*(1 - E^(-2*a))*PolyLog[2, -E^(-a - b*x)] + 6*(1 - E^(-2*a))*PolyLog[
2, E^(-a - b*x)] + 6*(1 - E^(-2*a))*(b^2*x^2*PolyLog[2, -E^(-a - b*x)] + 2*
(b*x*PolyLog[3, -E^(-a - b*x)] + PolyLog[4, -E^(-a - b*x)])) + 6*(1 - E^(-2
*a))*(b^2*x^2*PolyLog[2, E^(-a - b*x)] + 2*(b*x*PolyLog[3, E^(-a - b*x)] +
PolyLog[4, E^(-a - b*x)])))/((b^4*(-1 + E^(2*a))) + (6*x^2*Csch[a]*Csch[a +
b*x]*Sinh[b*x])/b^2)/4
```

Maple [B] time = 0.049, size = 375, normalized size = 2.1

$$3 \frac{\text{polylog}\left(2, e^{bx+a}\right)}{b^4} - 3 \frac{a^2}{b^4} + 3 \frac{\text{polylog}\left(2, -e^{bx+a}\right)}{b^4} - \frac{x^2 \left(2 b x e^{2 b x+2 a} + 3 e^{2 b x+2 a} - 3\right)}{b^2 \left(e^{2 b x+2 a} - 1\right)^2} - 3 \frac{a \ln \left(e^{b x+a} - 1\right)}{b^4} + 6 \frac{a \ln \left(e^{b x+a}\right)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*coth(b*x+a)^3,x)
```

```
[Out] 3/b^4*polylog(2,exp(b*x+a))-3/b^4*a^2+3/b^4*polylog(2,-exp(b*x+a))-x^2*(2*b
*x*exp(2*b*x+2*a)+3*exp(2*b*x+2*a)-3)/b^2/(exp(2*b*x+2*a)-1)^2-3/b^4*a*ln(e
xp(b*x+a)-1)+6/b^4*a*ln(exp(b*x+a))-3/2/b^4*a^4+6/b^4*polylog(4,exp(b*x+a))
+6/b^4*polylog(4,-exp(b*x+a))-2/b^3*a^3*x-1/b^4*a^3*ln(exp(b*x+a)-1)+2/b^4*
a^3*ln(exp(b*x+a))+1/b^4*ln(1-exp(b*x+a))*a^3+3/b^2*polylog(2,-exp(b*x+a))*
x^2-6/b^3*polylog(3,-exp(b*x+a))*x+1/b*ln(1-exp(b*x+a))*x^3+3/b^2*polylog(2
,exp(b*x+a))*x^2-6/b^3*polylog(3,exp(b*x+a))*x+1/b*ln(1+exp(b*x+a))*x^3-6/b
^3*a*x+3/b^4*a*ln(1-exp(b*x+a))+3/b^3*ln(1+exp(b*x+a))*x+3/b^3*ln(1-exp(b*x
+a))*x-3*x^2/b^2-1/4*x^4
```

Maxima [A] time = 1.298, size = 408, normalized size = 2.28

$$\frac{b^2 x^4 e^{4 b x+4 a} + b^2 x^4 + 12 x^2 - 2 \left(b^2 x^4 e^{2 a} + 4 b x^3 e^{2 a} + 6 x^2 e^{2 a}\right) e^{2 b x}}{4 \left(b^2 e^{4 b x+4 a} - 2 b^2 e^{2 b x+2 a} + b^2\right)} - \frac{b^4 x^4 + 6 b^2 x^2}{2 b^4} + \frac{b^3 x^3 \log \left(e^{b x+a} + 1\right) + 3 b^2 x^2}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*coth(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(b^2*x^4*e^(4*b*x + 4*a) + b^2*x^4 + 12*x^2 - 2*(b^2*x^4*e^(2*a) + 4*b*x^3*e^(2*a) + 6*x^2*e^(2*a))*e^(2*b*x))/(b^2*e^(4*b*x + 4*a) - 2*b^2*e^(2*b*x + 2*a) + b^2) - 1/2*(b^4*x^4 + 6*b^2*x^2)/b^4 + (b^3*x^3*log(e^(b*x + a) + 1) + 3*b^2*x^2*dilog(-e^(b*x + a)) - 6*b*x*polylog(3, -e^(b*x + a)) + 6*polylog(4, -e^(b*x + a)))/b^4 + (b^3*x^3*log(-e^(b*x + a) + 1) + 3*b^2*x^2*dilog(e^(b*x + a)) - 6*b*x*polylog(3, e^(b*x + a)) + 6*polylog(4, e^(b*x + a)))/b^4 + 3*(b*x*log(e^(b*x + a) + 1) + dilog(-e^(b*x + a)))/b^4 + 3*(b*x*log(-e^(b*x + a) + 1) + dilog(e^(b*x + a)))/b^4
```

Fricas [C] time = 2.40589, size = 5023, normalized size = 28.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*coth(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(b^4*x^4 + (b^4*x^4 - 2*a^4 + 12*b^2*x^2 - 12*a^2)*cosh(b*x + a)^4 + 4*(b^4*x^4 - 2*a^4 + 12*b^2*x^2 - 12*a^2)*cosh(b*x + a)*sinh(b*x + a)^3 + (b^4*x^4 - 2*a^4 + 12*b^2*x^2 - 12*a^2)*sinh(b*x + a)^4 - 2*a^4 - 2*(b^4*x^4 - 4*b^3*x^3 - 2*a^4 + 6*b^2*x^2 - 12*a^2)*cosh(b*x + a)^2 - 2*(b^4*x^4 - 4*b^3*x^3 - 2*a^4 + 6*b^2*x^2 - 3*(b^4*x^4 - 2*a^4 + 12*b^2*x^2 - 12*a^2)*cosh(b*x + a)^2 - 12*a^2)*sinh(b*x + a)^2 - 12*a^2 - 12*((b^2*x^2 + 1)*cosh(b*x + a)^4 + 4*(b^2*x^2 + 1)*cosh(b*x + a)*sinh(b*x + a)^3 + (b^2*x^2 + 1)*sinh(b*x + a)^4 + b^2*x^2 - 2*(b^2*x^2 + 1)*cosh(b*x + a)^2 - 2*(b^2*x^2 - 3*(b^2*x^2 + 1)*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 4*((b^2*x^2 + 1)*cosh(b*x + a)^3 - (b^2*x^2 + 1)*cosh(b*x + a))*sinh(b*x + a) + 1)*dilog(cosh(b*x + a) + sinh(b*x + a)) - 12*((b^2*x^2 + 1)*cosh(b*x + a)^4 + 4*(b^2*x^2 + 1)*cosh(b*x + a)*sinh(b*x + a)^3 + (b^2*x^2 + 1)*sinh(b*x + a)^4 + b^2*x^2 - 2*(b^2*x^2 + 1)*cosh(b*x + a)^2 - 2*(b^2*x^2 - 3*(b^2*x^2 + 1)*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 4*((b^2*x^2 + 1)*cosh(b*x + a)^3 - (b^2*x^2 + 1)*cosh(b*x + a))*sinh(b*x + a) + 1)*dilog(-cosh(b*x + a) - sinh(b*x + a)) - 4*(b^3*x^3 + (b^3*x^3 + 3*b*x)*cosh(b*x + a)^4 + 4*(b^3*x^3 + 3*b*x)*cosh(b*x + a)*sinh(b*x + a)^3 + (b^3*x^3 + 3*b*x)*sinh(b*x + a)^4 - 2*(b^3*x^3 + 3*b*x)*cosh(b*x + a)^2 - 2*(b^3*x^3 - 3*(b^3*x^3 + 3*b*x)*cosh(b*x + a)^2 + 3*b*x)*sinh(b*x + a)^2 + 3*b*x + 4*((b^3*x^3 + 3*b*x)*cosh(b*x + a)^3 - (b^3*x^3 + 3*b*x)*cosh(b*x + a))*sinh(b*x + a))*log(cosh(b*x + a) + sinh(b*x + a) + 1) + 4*((a^3 + 3*a)*cosh(b*x + a)^4 + 4*(a^3 + 3*a)*cosh(b*x + a)*sinh(b*x + a)^3 + (a^3 + 3*a)*sinh(b*x + a)^4 + a^3 - 2*(a^3 + 3*a)*cosh(b*x + a)^2 - 2*(a^3 - 3*(a^3 + 3*a)*cosh(b*x + a)^2 + 3*a)*sinh(b*x + a)^2 + 4*((a^3 + 3*a)*cosh(b*x + a)^3 - (a^3 + 3*a)*cosh(b*x + a))*sinh(b*x + a) + 3*a)*log(cosh(b*x + a) + sinh(b*x + a) - 1) - 4*(b^3*x^3 + (b^3*x^3 + a^3 +
```

```

3*b*x + 3*a)*cosh(b*x + a)^4 + 4*(b^3*x^3 + a^3 + 3*b*x + 3*a)*cosh(b*x +
a)*sinh(b*x + a)^3 + (b^3*x^3 + a^3 + 3*b*x + 3*a)*sinh(b*x + a)^4 + a^3 -
2*(b^3*x^3 + a^3 + 3*b*x + 3*a)*cosh(b*x + a)^2 - 2*(b^3*x^3 + a^3 - 3*(b^3
*x^3 + a^3 + 3*b*x + 3*a)*cosh(b*x + a)^2 + 3*b*x + 3*a)*sinh(b*x + a)^2 +
3*b*x + 4*((b^3*x^3 + a^3 + 3*b*x + 3*a)*cosh(b*x + a)^3 - (b^3*x^3 + a^3 +
3*b*x + 3*a)*cosh(b*x + a))*sinh(b*x + a) + 3*a)*log(-cosh(b*x + a) - sinh
(b*x + a) + 1) - 24*(cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + si
nh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)
^2 + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*polylog(4, cosh
(b*x + a) + sinh(b*x + a)) - 24*(cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x
+ a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*c
osh(b*x + a)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*pol
ylog(4, -cosh(b*x + a) - sinh(b*x + a)) + 24*(b*x*cosh(b*x + a)^4 + 4*b*x*c
osh(b*x + a)*sinh(b*x + a)^3 + b*x*sinh(b*x + a)^4 - 2*b*x*cosh(b*x + a)^2
+ 2*(3*b*x*cosh(b*x + a)^2 - b*x)*sinh(b*x + a)^2 + b*x + 4*(b*x*cosh(b*x +
a)^3 - b*x*cosh(b*x + a))*sinh(b*x + a))*polylog(3, cosh(b*x + a) + sinh(b
*x + a)) + 24*(b*x*cosh(b*x + a)^4 + 4*b*x*cosh(b*x + a)*sinh(b*x + a)^3 +
b*x*sinh(b*x + a)^4 - 2*b*x*cosh(b*x + a)^2 + 2*(3*b*x*cosh(b*x + a)^2 - b*
x)*sinh(b*x + a)^2 + b*x + 4*(b*x*cosh(b*x + a)^3 - b*x*cosh(b*x + a))*sinh
(b*x + a))*polylog(3, -cosh(b*x + a) - sinh(b*x + a)) + 4*((b^4*x^4 - 2*a^4
+ 12*b^2*x^2 - 12*a^2)*cosh(b*x + a)^3 - (b^4*x^4 - 4*b^3*x^3 - 2*a^4 + 6*
b^2*x^2 - 12*a^2)*cosh(b*x + a))*sinh(b*x + a))/(b^4*cosh(b*x + a)^4 + 4*b^
4*cosh(b*x + a)*sinh(b*x + a)^3 + b^4*sinh(b*x + a)^4 - 2*b^4*cosh(b*x + a)
^2 + b^4 + 2*(3*b^4*cosh(b*x + a)^2 - b^4)*sinh(b*x + a)^2 + 4*(b^4*cosh(b*
x + a)^3 - b^4*cosh(b*x + a))*sinh(b*x + a))

```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \coth^3(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*coth(b*x+a)**3,x)

[Out] Integral(x**3*coth(a + b*x)**3, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \coth(bx + a)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*coth(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x^3*coth(b*x + a)^3, x)
```

3.12 $\int x^2 \coth^3(a + bx) dx$

Optimal. Leaf size=114

$$\frac{x \operatorname{PolyLog}\left(2, e^{2(a+bx)}\right)}{b^2} - \frac{\operatorname{PolyLog}\left(3, e^{2(a+bx)}\right)}{2b^3} - \frac{x \coth(a+bx)}{b^2} + \frac{\log(\sinh(a+bx))}{b^3} + \frac{x^2 \log\left(1 - e^{2(a+bx)}\right)}{b} - \frac{x^2 \coth^2(a+bx)}{2b}$$

[Out] $x^2/(2*b) - x^3/3 - (x*\operatorname{Coth}[a + b*x])/b^2 - (x^2*\operatorname{Coth}[a + b*x]^2)/(2*b) + (x^2*\operatorname{Log}[1 - E^{(2*(a + b*x))}])/b + \operatorname{Log}[\operatorname{Sinh}[a + b*x]]/b^3 + (x*\operatorname{PolyLog}[2, E^{(2*(a + b*x))}])/b^2 - \operatorname{PolyLog}[3, E^{(2*(a + b*x))}]/(2*b^3)$

Rubi [A] time = 0.20707, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3720, 3475, 30, 3716, 2190, 2531, 2282, 6589}

$$\frac{x \operatorname{PolyLog}\left(2, e^{2(a+bx)}\right)}{b^2} - \frac{\operatorname{PolyLog}\left(3, e^{2(a+bx)}\right)}{2b^3} - \frac{x \coth(a+bx)}{b^2} + \frac{\log(\sinh(a+bx))}{b^3} + \frac{x^2 \log\left(1 - e^{2(a+bx)}\right)}{b} - \frac{x^2 \coth^2(a+bx)}{2b}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[x^2*\operatorname{Coth}[a + b*x]^3, x]$

[Out] $x^2/(2*b) - x^3/3 - (x*\operatorname{Coth}[a + b*x])/b^2 - (x^2*\operatorname{Coth}[a + b*x]^2)/(2*b) + (x^2*\operatorname{Log}[1 - E^{(2*(a + b*x))}])/b + \operatorname{Log}[\operatorname{Sinh}[a + b*x]]/b^3 + (x*\operatorname{PolyLog}[2, E^{(2*(a + b*x))}])/b^2 - \operatorname{PolyLog}[3, E^{(2*(a + b*x))}]/(2*b^3)$

Rule 3720

$\operatorname{Int}[\left((c_.) + (d_.)*(x_.)\right)^{(m_.)} * \left((b_.) * \tan[(e_.) + (f_.)*(x_.)]\right)^{(n_.)}, x_Symbol] \rightarrow \operatorname{Simp}[(b*(c + d*x)^m*(b*\operatorname{Tan}[e + f*x])^{n-1})/(f*(n-1)), x] + (-\operatorname{Dist}[(b*d*m)/(f*(n-1)), \operatorname{Int}[(c + d*x)^{m-1}*(b*\operatorname{Tan}[e + f*x])^{n-1}, x], x] - \operatorname{Dist}[b^2, \operatorname{Int}[(c + d*x)^m*(b*\operatorname{Tan}[e + f*x])^{n-2}, x], x]) /; \operatorname{FreeQ}\{b, c, d, e, f\}, x] \&\& \operatorname{GtQ}[n, 1] \&\& \operatorname{GtQ}[m, 0]$

Rule 3475

$\operatorname{Int}[\tan[(c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow -\operatorname{Simp}[\operatorname{Log}[\operatorname{RemoveContent}[\operatorname{Cos}[c + d*x], x]]/d, x] /; \operatorname{FreeQ}\{c, d\}, x]$

Rule 30

```
Int[(x_)^(m_.), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_]
)*(f_.)*(x_)], x_Symbol] := -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-I*
e) + f*fz*x))/E^(2*I*k*Pi))], x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_.) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a]]/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int x^2 \coth^3(a + bx) dx &= -\frac{x^2 \coth^2(a + bx)}{2b} + \frac{\int x \coth^2(a + bx) dx}{b} + \int x^2 \coth(a + bx) dx \\
&= -\frac{x^3}{3} - \frac{x \coth(a + bx)}{b^2} - \frac{x^2 \coth^2(a + bx)}{2b} - 2 \int \frac{e^{2(a+bx)} x^2}{1 - e^{2(a+bx)}} dx + \frac{\int \coth(a + bx) dx}{b^2} + \frac{\int x dx}{b} \\
&= \frac{x^2}{2b} - \frac{x^3}{3} - \frac{x \coth(a + bx)}{b^2} - \frac{x^2 \coth^2(a + bx)}{2b} + \frac{x^2 \log(1 - e^{2(a+bx)})}{b} + \frac{\log(\sinh(a + bx))}{b^3} - \frac{2}{b} \\
&= \frac{x^2}{2b} - \frac{x^3}{3} - \frac{x \coth(a + bx)}{b^2} - \frac{x^2 \coth^2(a + bx)}{2b} + \frac{x^2 \log(1 - e^{2(a+bx)})}{b} + \frac{\log(\sinh(a + bx))}{b^3} + \frac{x}{b} \\
&= \frac{x^2}{2b} - \frac{x^3}{3} - \frac{x \coth(a + bx)}{b^2} - \frac{x^2 \coth^2(a + bx)}{2b} + \frac{x^2 \log(1 - e^{2(a+bx)})}{b} + \frac{\log(\sinh(a + bx))}{b^3} + \frac{x}{b} \\
&= \frac{x^2}{2b} - \frac{x^3}{3} - \frac{x \coth(a + bx)}{b^2} - \frac{x^2 \coth^2(a + bx)}{2b} + \frac{x^2 \log(1 - e^{2(a+bx)})}{b} + \frac{\log(\sinh(a + bx))}{b^3} + \frac{x}{b}
\end{aligned}$$

Mathematica [B] time = 3.25824, size = 295, normalized size = 2.59

$$\frac{e^{2a} \left(6 \left(1 - e^{-2a} \right) \left(bx \operatorname{PolyLog} \left(2, -e^{-a-bx} \right) + \operatorname{PolyLog} \left(3, -e^{-a-bx} \right) \right) + 6 \left(1 - e^{-2a} \right) \left(bx \operatorname{PolyLog} \left(2, e^{-a-bx} \right) + \operatorname{PolyLog} \left(3, e^{-a-bx} \right) \right) \right)}{b^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^2*Coth[a + b*x]^3,x]

[Out] (x^3*Coth[a])/3 - (x^2*Csch[a + b*x]^2)/(2*b) - (E^(2*a))*((6*b*x)/E^(2*a) + (2*b^3*x^3)/E^(2*a) - (3*b^2*(-1 + E^(2*a))*x^2*Log[1 - E^(-a - b*x)])/E^(2*a) - (3*b^2*(-1 + E^(2*a))*x^2*Log[1 + E^(-a - b*x)])/E^(2*a) + 3*(1 - E^(-2*a))*(b*x - Log[1 - E^(a + b*x)]) + 3*(1 - E^(-2*a))*(b*x - Log[1 + E^(a + b*x)]) + 6*(1 - E^(-2*a))*(b*x*PolyLog[2, -E^(-a - b*x)] + PolyLog[3, -E^(-a - b*x)]) + 6*(1 - E^(-2*a))*(b*x*PolyLog[2, E^(-a - b*x)] + PolyLog[3, E^(-a - b*x)])))/(3*b^3*(-1 + E^(2*a))) + (x*Csch[a]*Csch[a + b*x]*Sinh[b*x])/b^2

Maple [B] time = 0.043, size = 246, normalized size = 2.2

$$-\frac{x^3}{3} - 2 \frac{x \left(b x e^{2bx+2a} + e^{2bx+2a} - 1 \right)}{b^2 \left(e^{2bx+2a} - 1 \right)^2} + \frac{4a^3}{3b^3} + \frac{a^2 \ln \left(e^{bx+a} - 1 \right)}{b^3} - 2 \frac{a^2 \ln \left(e^{bx+a} \right)}{b^3} - \frac{\ln \left(1 - e^{bx+a} \right) a^2}{b^3} - 2 \frac{\operatorname{polylog} \left(3, e^{bx+a} \right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*coth(b*x+a)^3,x)`

[Out]
$$-1/3*x^3-2*x*(b*x*\exp(2*b*x+2*a)+\exp(2*b*x+2*a)-1)/b^2/(\exp(2*b*x+2*a)-1)^2+4/3/b^3*a^3+1/b^3*a^2*\ln(\exp(b*x+a)-1)-2/b^3*a^2*\ln(\exp(b*x+a))-1/b^3*\ln(1-\exp(b*x+a))*a^2-2/b^3*\text{polylog}(3,\exp(b*x+a))-2/b^3*\ln(\exp(b*x+a))+1/b^3*\ln(1+\exp(b*x+a))-2/b^3*\text{polylog}(3,-\exp(b*x+a))+1/b*\ln(1+\exp(b*x+a))*x^2+2/b^2*\text{polylog}(2,-\exp(b*x+a))*x+1/b*\ln(1-\exp(b*x+a))*x^2+2/b^2*\text{polylog}(2,\exp(b*x+a))*x+1/b^3*\ln(\exp(b*x+a)-1)+2/b^2*a^2*x$$

Maxima [B] time = 1.26328, size = 305, normalized size = 2.68

$$-\frac{2}{3}x^3 + \frac{b^2x^3e^{(4bx+4a)} + b^2x^3 - 2(b^2x^3e^{(2a)} + 3bx^2e^{(2a)} + 3xe^{(2a)})e^{(2bx)} + 6x}{3(b^2e^{(4bx+4a)} - 2b^2e^{(2bx+2a)} + b^2)} - \frac{2x}{b^2} + \frac{b^2x^2 \log(e^{(bx+a)} + 1) + 2bx\text{Li}_2(-e^{(bx+a)})}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*coth(b*x+a)^3,x, algorithm="maxima")`

[Out]
$$-2/3*x^3 + 1/3*(b^2*x^3*e^{(4*b*x + 4*a)} + b^2*x^3 - 2*(b^2*x^3*e^{(2*a)} + 3*b*x^2*e^{(2*a)} + 3*x*e^{(2*a)})*e^{(2*b*x)} + 6*x)/(b^2*e^{(4*b*x + 4*a)} - 2*b^2*e^{(2*b*x + 2*a)} + b^2) - 2*x/b^2 + (b^2*x^2*\log(e^{(b*x + a)} + 1) + 2*b*x*\text{dilog}(-e^{(b*x + a)}) - 2*\text{polylog}(3, -e^{(b*x + a)}))/b^3 + (b^2*x^2*\log(-e^{(b*x + a)} + 1) + 2*b*x*\text{dilog}(e^{(b*x + a)}) - 2*\text{polylog}(3, e^{(b*x + a)}))/b^3 + \log(e^{(b*x + a)} + 1)/b^3 + \log(e^{(b*x + a)} - 1)/b^3$$

Fricas [C] time = 2.2779, size = 3729, normalized size = 32.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*coth(b*x+a)^3,x, algorithm="fricas")`

[Out]
$$-1/3*(b^3*x^3 + (b^3*x^3 + 2*a^3 + 6*b*x + 6*a)*\cosh(b*x + a)^4 + 4*(b^3*x^3 + 2*a^3 + 6*b*x + 6*a)*\cosh(b*x + a)*\sinh(b*x + a)^3 + (b^3*x^3 + 2*a^3 + 6*b*x + 6*a)*\sinh(b*x + a)^4 + 2*a^3 - 2*(b^3*x^3 - 3*b^2*x^2 + 2*a^3 + 3*b*x + 6*a)*\cosh(b*x + a)^2 - 2*(b^3*x^3 - 3*b^2*x^2 + 2*a^3 - 3*(b^3*x^3 + 2*a^3 + 6*b*x + 6*a)*\cosh(b*x + a)^2 + 3*b*x + 6*a)*\sinh(b*x + a)^2 - 6*(b*x*\cosh(b*x + a)^4 + 4*b*x*\cosh(b*x + a)*\sinh(b*x + a)^3 + b*x*\sinh(b*x + a)$$

```

^4 - 2*b*x*cosh(b*x + a)^2 + 2*(3*b*x*cosh(b*x + a)^2 - b*x)*sinh(b*x + a)^
2 + b*x + 4*(b*x*cosh(b*x + a)^3 - b*x*cosh(b*x + a))*sinh(b*x + a)*dilog(
cosh(b*x + a) + sinh(b*x + a)) - 6*(b*x*cosh(b*x + a)^4 + 4*b*x*cosh(b*x +
a)*sinh(b*x + a)^3 + b*x*sinh(b*x + a)^4 - 2*b*x*cosh(b*x + a)^2 + 2*(3*b*x
*cosh(b*x + a)^2 - b*x)*sinh(b*x + a)^2 + b*x + 4*(b*x*cosh(b*x + a)^3 - b
*x*cosh(b*x + a))*sinh(b*x + a)*dilog(-cosh(b*x + a) - sinh(b*x + a)) - 3*(
(b^2*x^2 + 1)*cosh(b*x + a)^4 + 4*(b^2*x^2 + 1)*cosh(b*x + a)*sinh(b*x + a)
^3 + (b^2*x^2 + 1)*sinh(b*x + a)^4 + b^2*x^2 - 2*(b^2*x^2 + 1)*cosh(b*x + a
)^2 - 2*(b^2*x^2 - 3*(b^2*x^2 + 1)*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 4
*((b^2*x^2 + 1)*cosh(b*x + a)^3 - (b^2*x^2 + 1)*cosh(b*x + a))*sinh(b*x + a
) + 1)*log(cosh(b*x + a) + sinh(b*x + a) + 1) - 3*((a^2 + 1)*cosh(b*x + a)^
4 + 4*(a^2 + 1)*cosh(b*x + a)*sinh(b*x + a)^3 + (a^2 + 1)*sinh(b*x + a)^4 -
2*(a^2 + 1)*cosh(b*x + a)^2 + 2*(3*(a^2 + 1)*cosh(b*x + a)^2 - a^2 - 1)*si
nh(b*x + a)^2 + a^2 + 4*((a^2 + 1)*cosh(b*x + a)^3 - (a^2 + 1)*cosh(b*x + a
))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh(b*x + a) - 1) - 3*((b^2*x^2
- a^2)*cosh(b*x + a)^4 + 4*(b^2*x^2 - a^2)*cosh(b*x + a)*sinh(b*x + a)^3 +
(b^2*x^2 - a^2)*sinh(b*x + a)^4 + b^2*x^2 - 2*(b^2*x^2 - a^2)*cosh(b*x + a)
^2 - 2*(b^2*x^2 - 3*(b^2*x^2 - a^2)*cosh(b*x + a)^2 - a^2)*sinh(b*x + a)^2
- a^2 + 4*((b^2*x^2 - a^2)*cosh(b*x + a)^3 - (b^2*x^2 - a^2)*cosh(b*x + a))
*sinh(b*x + a))*log(-cosh(b*x + a) - sinh(b*x + a) + 1) + 6*(cosh(b*x + a)^
4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^
2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x
+ a))*sinh(b*x + a) + 1)*polylog(3, cosh(b*x + a) + sinh(b*x + a)) + 6*(cos
h(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cos
h(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3
- cosh(b*x + a))*sinh(b*x + a) + 1)*polylog(3, -cosh(b*x + a) - sinh(b*x +
a)) + 4*((b^3*x^3 + 2*a^3 + 6*b*x + 6*a)*cosh(b*x + a)^3 - (b^3*x^3 - 3*b^2
*x^2 + 2*a^3 + 3*b*x + 6*a)*cosh(b*x + a))*sinh(b*x + a) + 6*a)/(b^3*cosh(b
*x + a)^4 + 4*b^3*cosh(b*x + a)*sinh(b*x + a)^3 + b^3*sinh(b*x + a)^4 - 2*b
^3*cosh(b*x + a)^2 + b^3 + 2*(3*b^3*cosh(b*x + a)^2 - b^3)*sinh(b*x + a)^2
+ 4*(b^3*cosh(b*x + a)^3 - b^3*cosh(b*x + a))*sinh(b*x + a))

```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \coth^3(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*coth(b*x+a)**3,x)

[Out] Integral(x**2*coth(a + b*x)**3, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \coth(bx + a)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*coth(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x^2*coth(b*x + a)^3, x)
```

3.13 $\int x \coth^3(a + bx) dx$

Optimal. Leaf size=82

$$\frac{\text{PolyLog}\left(2, e^{2(a+bx)}\right)}{2b^2} - \frac{\coth(a+bx)}{2b^2} + \frac{x \log\left(1 - e^{2(a+bx)}\right)}{b} - \frac{x \coth^2(a+bx)}{2b} + \frac{x}{2b} - \frac{x^2}{2}$$

[Out] $x/(2*b) - x^2/2 - \text{Coth}[a + b*x]/(2*b^2) - (x*\text{Coth}[a + b*x]^2)/(2*b) + (x*\text{Log}[1 - E^{2*(a + b*x)}])/b + \text{PolyLog}[2, E^{2*(a + b*x)}]/(2*b^2)$

Rubi [A] time = 0.121605, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.7$, Rules used = {3720, 3473, 8, 3716, 2190, 2279, 2391}

$$\frac{\text{PolyLog}\left(2, e^{2(a+bx)}\right)}{2b^2} - \frac{\coth(a+bx)}{2b^2} + \frac{x \log\left(1 - e^{2(a+bx)}\right)}{b} - \frac{x \coth^2(a+bx)}{2b} + \frac{x}{2b} - \frac{x^2}{2}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x*\text{Coth}[a + b*x]^3, x]$

[Out] $x/(2*b) - x^2/2 - \text{Coth}[a + b*x]/(2*b^2) - (x*\text{Coth}[a + b*x]^2)/(2*b) + (x*\text{Log}[1 - E^{2*(a + b*x)}])/b + \text{PolyLog}[2, E^{2*(a + b*x)}]/(2*b^2)$

Rule 3720

$\text{Int}[\left((c_.) + (d_.)*(x_.)\right)^{(m_.)} * \left((b_.) * \tan[(e_.) + (f_.)*(x_.)]\right)^{(n_.)}, x_Symbol] \rightarrow \text{Simp}[(b*(c + d*x)^m*(b*\text{Tan}[e + f*x])^{(n-1)})/(f*(n-1)), x] + (-\text{Dist}[(b*d*m)/(f*(n-1)), \text{Int}[(c + d*x)^{(m-1)}*(b*\text{Tan}[e + f*x])^{(n-1)}, x], x] - \text{Dist}[b^2, \text{Int}[(c + d*x)^m*(b*\text{Tan}[e + f*x])^{(n-2)}, x], x]) /; \text{FreeQ}[\{b, c, d, e, f\}, x] \&\& \text{GtQ}[n, 1] \&\& \text{GtQ}[m, 0]$

Rule 3473

$\text{Int}[\left((b_.) * \tan[(c_.) + (d_.)*(x_.)]\right)^{(n_.)}, x_Symbol] \rightarrow \text{Simp}[(b*(b*\text{Tan}[c + d*x])^{(n-1)})/(d*(n-1)), x] - \text{Dist}[b^2, \text{Int}[(b*\text{Tan}[c + d*x])^{(n-2)}, x], x] /; \text{FreeQ}[\{b, c, d\}, x] \&\& \text{GtQ}[n, 1]$

Rule 8

$\text{Int}[a_, x_Symbol] \rightarrow \text{Simp}[a*x, x] /; \text{FreeQ}[a, x]$

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] := -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-I*
e) + f*fz*x))/E^(2*I*k*Pi))], x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a]]/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:= Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int x \coth^3(a + bx) dx &= -\frac{x \coth^2(a + bx)}{2b} + \frac{\int \coth^2(a + bx) dx}{2b} + \int x \coth(a + bx) dx \\
&= -\frac{x^2}{2} - \frac{\coth(a + bx)}{2b^2} - \frac{x \coth^2(a + bx)}{2b} - 2 \int \frac{e^{2(a+bx)} x}{1 - e^{2(a+bx)}} dx + \frac{\int 1 dx}{2b} \\
&= \frac{x}{2b} - \frac{x^2}{2} - \frac{\coth(a + bx)}{2b^2} - \frac{x \coth^2(a + bx)}{2b} + \frac{x \log(1 - e^{2(a+bx)})}{b} - \frac{\int \log(1 - e^{2(a+bx)}) dx}{b} \\
&= \frac{x}{2b} - \frac{x^2}{2} - \frac{\coth(a + bx)}{2b^2} - \frac{x \coth^2(a + bx)}{2b} + \frac{x \log(1 - e^{2(a+bx)})}{b} - \frac{\text{Subst}\left(\int \frac{\log(1-x)}{x} dx, x, e^{2(a+bx)}\right)}{2b^2} \\
&= \frac{x}{2b} - \frac{x^2}{2} - \frac{\coth(a + bx)}{2b^2} - \frac{x \coth^2(a + bx)}{2b} + \frac{x \log(1 - e^{2(a+bx)})}{b} + \frac{\text{Li}_2(e^{2(a+bx)})}{2b^2}
\end{aligned}$$

Mathematica [C] time = 6.13174, size = 232, normalized size = 2.83

$$\text{csch}(a)\text{sech}(a) \left(-b^2 x^2 e^{-\tanh^{-1}(\tanh(a))} + \frac{i \tanh(a) \left(i \text{PolyLog} \left(2, e^{2i \left(\tanh^{-1}(\tanh(a)) + ibx \right)} \right) - bx \left(-\pi + 2i \tanh^{-1}(\tanh(a)) \right) - 2 \left(i \tanh^{-1}(\tanh(a)) + ibx \right) \right)} \right)}{2b^2 \sqrt{\text{sech}^2(a) \left(\cosh^2(a) - \sinh^2(a) \right)}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x*Coth[a + b*x]^3, x]

[Out] (x^2*Coth[a])/2 - (x*Csch[a + b*x]^2)/(2*b) + (Csch[a]*Csch[a + b*x]*Sinh[b*x])/(2*b^2) + (Csch[a]*Sech[a]*(-(b^2*x^2)/E^ArcTanh[Tanh[a]]) + (I*(-(b*x*(-Pi + (2*I)*ArcTanh[Tanh[a]])) - Pi*Log[1 + E^(2*b*x)] - 2*(I*b*x + I*ArcTanh[Tanh[a]])*Log[1 - E^((2*I)*(I*b*x + I*ArcTanh[Tanh[a]])])]) + Pi*Log[Cosh[b*x] + (2*I)*ArcTanh[Tanh[a]]*Log[I*Sinh[b*x + ArcTanh[Tanh[a]]]]) + I*PolyLog[2, E^((2*I)*(I*b*x + I*ArcTanh[Tanh[a]])])]*Tanh[a])/Sqrt[1 - Tanh[a]^2])/(2*b^2*Sqrt[Sech[a]^2*(Cosh[a]^2 - Sinh[a]^2)])

Maple [B] time = 0.034, size = 164, normalized size = 2.

$$-\frac{x^2}{2} - \frac{2bx e^{2bx+2a} + e^{2bx+2a} - 1}{b^2 (e^{2bx+2a} - 1)^2} - 2 \frac{ax}{b} - \frac{a^2}{b^2} + \frac{\ln(1 + e^{bx+a})x}{b} + \frac{\text{polylog}(2, -e^{bx+a})}{b^2} + \frac{\ln(1 - e^{bx+a})x}{b} + \frac{\ln(1 - e^{bx+a})}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*coth(b*x+a)^3, x)

[Out] -1/2*x^2-(2*b*x*exp(2*b*x+2*a)+exp(2*b*x+2*a)-1)/b^2/(exp(2*b*x+2*a)-1)^2-2/b*a*x-a^2/b^2+1/b*ln(1+exp(b*x+a))*x+1/b^2*polylog(2,-exp(b*x+a))+1/b*ln(1-exp(b*x+a))*x+1/b^2*ln(1-exp(b*x+a))*a+1/b^2*polylog(2,exp(b*x+a))-1/b^2*a*ln(exp(b*x+a)-1)+2/b^2*a*ln(exp(b*x+a))

Maxima [B] time = 1.22287, size = 201, normalized size = 2.45

$$-x^2 + \frac{b^2 x^2 e^{(4bx+4a)} + b^2 x^2 - 2(b^2 x^2 e^{(2a)} + 2bx e^{(2a)} + e^{(2a)})e^{(2bx)} + 2}{2(b^2 e^{(4bx+4a)} - 2b^2 e^{(2bx+2a)} + b^2)} + \frac{bx \log(e^{(bx+a)} + 1) + \text{Li}_2(-e^{(bx+a)})}{b^2} + \frac{bx \log(-e^{(bx+a)})}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*coth(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -x^2 + 1/2*(b^2*x^2*e^(4*b*x + 4*a) + b^2*x^2 - 2*(b^2*x^2*e^(2*a) + 2*b*x*
e^(2*a) + e^(2*a))*e^(2*b*x) + 2)/(b^2*e^(4*b*x + 4*a) - 2*b^2*e^(2*b*x + 2
*a) + b^2) + (b*x*log(e^(b*x + a) + 1) + dilog(-e^(b*x + a)))/b^2 + (b*x*lo
g(-e^(b*x + a) + 1) + dilog(e^(b*x + a)))/b^2
```

Fricas [B] time = 2.2154, size = 2569, normalized size = 31.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*coth(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/2*((b^2*x^2 - 2*a^2)*cosh(b*x + a)^4 + 4*(b^2*x^2 - 2*a^2)*cosh(b*x + a)
*sinh(b*x + a)^3 + (b^2*x^2 - 2*a^2)*sinh(b*x + a)^4 + b^2*x^2 - 2*(b^2*x^2
- 2*a^2 - 2*b*x - 1)*cosh(b*x + a)^2 - 2*(b^2*x^2 - 3*(b^2*x^2 - 2*a^2)*co
sh(b*x + a)^2 - 2*a^2 - 2*b*x - 1)*sinh(b*x + a)^2 - 2*a^2 - 2*(cosh(b*x +
a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x +
a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 - cosh(b
*x + a))*sinh(b*x + a) + 1)*dilog(cosh(b*x + a) + sinh(b*x + a)) - 2*(cosh(
b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(
b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 -
cosh(b*x + a))*sinh(b*x + a) + 1)*dilog(-cosh(b*x + a) - sinh(b*x + a)) - 2
*(b*x*cosh(b*x + a)^4 + 4*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + b*x*sinh(b*x
+ a)^4 - 2*b*x*cosh(b*x + a)^2 + 2*(3*b*x*cosh(b*x + a)^2 - b*x)*sinh(b*x +
a)^2 + b*x + 4*(b*x*cosh(b*x + a)^3 - b*x*cosh(b*x + a))*sinh(b*x + a))*lo
g(cosh(b*x + a) + sinh(b*x + a) + 1) + 2*(a*cosh(b*x + a)^4 + 4*a*cosh(b*x
+ a)*sinh(b*x + a)^3 + a*sinh(b*x + a)^4 - 2*a*cosh(b*x + a)^2 + 2*(3*a*cos
h(b*x + a)^2 - a)*sinh(b*x + a)^2 + 4*(a*cosh(b*x + a)^3 - a*cosh(b*x + a)
)*sinh(b*x + a) + a)*log(cosh(b*x + a) + sinh(b*x + a) - 1) - 2*((b*x + a)*c
osh(b*x + a)^4 + 4*(b*x + a)*cosh(b*x + a)*sinh(b*x + a)^3 + (b*x + a)*sinh
(b*x + a)^4 - 2*(b*x + a)*cosh(b*x + a)^2 + 2*(3*(b*x + a)*cosh(b*x + a)^2
- b*x - a)*sinh(b*x + a)^2 + b*x + 4*((b*x + a)*cosh(b*x + a)^3 - (b*x + a)
*cosh(b*x + a))*sinh(b*x + a) + a)*log(-cosh(b*x + a) - sinh(b*x + a) + 1)
+ 4*((b^2*x^2 - 2*a^2)*cosh(b*x + a)^3 - (b^2*x^2 - 2*a^2 - 2*b*x - 1)*cosh
(b*x + a))*sinh(b*x + a) - 2)/(b^2*cosh(b*x + a)^4 + 4*b^2*cosh(b*x + a)*si
nh(b*x + a)^3 + b^2*sinh(b*x + a)^4 - 2*b^2*cosh(b*x + a)^2 + 2*(3*b^2*cosh
(b*x + a)^2 - b^2)*sinh(b*x + a)^2 + b^2 + 4*(b^2*cosh(b*x + a)^3 - b^2*cos
```


`h(b*x + a))*sinh(b*x + a))`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x \coth^3(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*coth(b*x+a)**3,x)`

[Out] `Integral(x*coth(a + b*x)**3, x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \coth (bx + a)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*coth(b*x+a)^3,x, algorithm="giac")`

[Out] `integrate(x*coth(b*x + a)^3, x)`

$$3.14 \quad \int \frac{\coth^3(a+bx)}{x} dx$$

Optimal. Leaf size=14

$$\text{Unintegrable}\left(\frac{\coth^3(a+bx)}{x}, x\right)$$

[Out] Unintegrable[Coth[a + b*x]^3/x, x]

Rubi [A] time = 0.0283809, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\coth^3(a+bx)}{x} dx$$

Verification is Not applicable to the result.

[In] Int[Coth[a + b*x]^3/x, x]

[Out] Defer[Int][Coth[a + b*x]^3/x, x]

Rubi steps

$$\int \frac{\coth^3(a+bx)}{x} dx = \int \frac{\coth^3(a+bx)}{x} dx$$

Mathematica [A] time = 14.8989, size = 0, normalized size = 0.

$$\int \frac{\coth^3(a+bx)}{x} dx$$

Verification is Not applicable to the result.

[In] Integrate[Coth[a + b*x]^3/x, x]

[Out] Integrate[Coth[a + b*x]^3/x, x]

Maple [A] time = 0.29, size = 0, normalized size = 0.

$$\int \frac{(\coth(bx + a))^3}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(b*x+a)^3/x,x)

[Out] int(coth(b*x+a)^3/x,x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{(2bx e^{(2a)} - e^{(2a)})e^{(2bx)} + 1}{b^2 x^2 e^{(4bx+4a)} - 2b^2 x^2 e^{(2bx+2a)} + b^2 x^2} - \int \frac{b^2 x^2 + 1}{b^2 x^3 e^{(bx+a)} + b^2 x^3} dx + \int \frac{b^2 x^2 + 1}{b^2 x^3 e^{(bx+a)} - b^2 x^3} dx + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(b*x+a)^3/x,x, algorithm="maxima")

[Out] -((2*b*x*e^(2*a) - e^(2*a))*e^(2*b*x) + 1)/(b^2*x^2*e^(4*b*x + 4*a) - 2*b^2*x^2*e^(2*b*x + 2*a) + b^2*x^2) - integrate((b^2*x^2 + 1)/(b^2*x^3*e^(b*x + a) + b^2*x^3), x) + integrate((b^2*x^2 + 1)/(b^2*x^3*e^(b*x + a) - b^2*x^3), x) + log(x)

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\coth(bx + a)^3}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(b*x+a)^3/x,x, algorithm="fricas")

[Out] integral(coth(b*x + a)^3/x, x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth^3(a + bx)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(b*x+a)**3/x,x)

[Out] Integral(coth(a + b*x)**3/x, x)

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth^3(bx + a)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(b*x+a)^3/x,x, algorithm="giac")

[Out] integrate(coth(b*x + a)^3/x, x)

$$3.15 \quad \int \frac{\coth^3(a+bx)}{x^2} dx$$

Optimal. Leaf size=14

$$\text{Unintegrable}\left(\frac{\coth^3(a+bx)}{x^2}, x\right)$$

[Out] Unintegrable[Coth[a + b*x]^3/x^2, x]

Rubi [A] time = 0.0286649, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\coth^3(a+bx)}{x^2} dx$$

Verification is Not applicable to the result.

[In] Int[Coth[a + b*x]^3/x^2,x]

[Out] Defer[Int][Coth[a + b*x]^3/x^2, x]

Rubi steps

$$\int \frac{\coth^3(a+bx)}{x^2} dx = \int \frac{\coth^3(a+bx)}{x^2} dx$$

Mathematica [A] time = 11.1396, size = 0, normalized size = 0.

$$\int \frac{\coth^3(a+bx)}{x^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[Coth[a + b*x]^3/x^2,x]

[Out] Integrate[Coth[a + b*x]^3/x^2, x]

Maple [A] time = 0.227, size = 0, normalized size = 0.

$$\int \frac{(\coth(bx + a))^3}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(b*x+a)^3/x^2,x)

[Out] int(coth(b*x+a)^3/x^2,x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{b^2x^2e^{(4bx+4a)} + b^2x^2 - 2(b^2x^2e^{(2a)} - bxe^{(2a)} + e^{(2a)})e^{(2bx)} + 2}{b^2x^3e^{(4bx+4a)} - 2b^2x^3e^{(2bx+2a)} + b^2x^3} - \int \frac{b^2x^2 + 3}{b^2x^4e^{(bx+a)} + b^2x^4} dx + \int \frac{b^2x^2 + 3}{b^2x^4e^{(bx+a)} - b^2x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(b*x+a)^3/x^2,x, algorithm="maxima")

[Out] $-(b^2x^2e^{(4bx+4a)} + b^2x^2 - 2(b^2x^2e^{(2a)} - bxe^{(2a)} + e^{(2a)})e^{(2bx)} + 2)/(b^2x^3e^{(4bx+4a)} - 2b^2x^3e^{(2bx+2a)} + b^2x^3) - \text{integrate}((b^2x^2 + 3)/(b^2x^4e^{(bx+a)} + b^2x^4), x) + \text{integrate}((b^2x^2 + 3)/(b^2x^4e^{(bx+a)} - b^2x^4), x)$

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\coth(bx + a)^3}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(b*x+a)^3/x^2,x, algorithm="fricas")

[Out] integral(coth(b*x + a)^3/x^2, x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth^3(a + bx)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(b*x+a)**3/x**2,x)

[Out] Integral(coth(a + b*x)**3/x**2, x)

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth^3(bx + a)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(b*x+a)^3/x^2,x, algorithm="giac")

[Out] integrate(coth(b*x + a)^3/x^2, x)

$$3.16 \quad \int \frac{(c+dx)^3}{a+a \coth(e+fx)} dx$$

Optimal. Leaf size=169

$$\frac{3d^2(c+dx)}{4f^3(a \coth(e+fx)+a)} - \frac{3d(c+dx)^2}{4f^2(a \coth(e+fx)+a)} - \frac{(c+dx)^3}{2f(a \coth(e+fx)+a)} + \frac{3d(c+dx)^2}{8af^2} + \frac{(c+dx)^3}{4af} + \frac{(c+dx)^4}{8ad} -$$

[Out] (3*d^3*x)/(8*a*f^3) + (3*d*(c + d*x)^2)/(8*a*f^2) + (c + d*x)^3/(4*a*f) + (c + d*x)^4/(8*a*d) - (3*d^3)/(8*f^4*(a + a*Coth[e + f*x])) - (3*d^2*(c + d*x))/(4*f^3*(a + a*Coth[e + f*x])) - (3*d*(c + d*x)^2)/(4*f^2*(a + a*Coth[e + f*x])) - (c + d*x)^3/(2*f*(a + a*Coth[e + f*x]))

Rubi [A] time = 0.185485, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.15$, Rules used = {3723, 3479, 8}

$$\frac{3d^2(c+dx)}{4f^3(a \coth(e+fx)+a)} - \frac{3d(c+dx)^2}{4f^2(a \coth(e+fx)+a)} - \frac{(c+dx)^3}{2f(a \coth(e+fx)+a)} + \frac{3d(c+dx)^2}{8af^2} + \frac{(c+dx)^3}{4af} + \frac{(c+dx)^4}{8ad} -$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)^3/(a + a*Coth[e + f*x]),x]

[Out] (3*d^3*x)/(8*a*f^3) + (3*d*(c + d*x)^2)/(8*a*f^2) + (c + d*x)^3/(4*a*f) + (c + d*x)^4/(8*a*d) - (3*d^3)/(8*f^4*(a + a*Coth[e + f*x])) - (3*d^2*(c + d*x))/(4*f^3*(a + a*Coth[e + f*x])) - (3*d*(c + d*x)^2)/(4*f^2*(a + a*Coth[e + f*x])) - (c + d*x)^3/(2*f*(a + a*Coth[e + f*x]))

Rule 3723

Int[((c_.) + (d_.)*(x_))^(m_.)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] := Simp[(c + d*x)^(m + 1)/(2*a*d*(m + 1)), x] + (Dist[(a*d*m)/(2*b*f), Int[(c + d*x)^(m - 1)/(a + b*Tan[e + f*x]), x], x] - Simp[(a*(c + d*x)^m)/(2*b*f*(a + b*Tan[e + f*x])), x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2 + b^2, 0] && GtQ[m, 0]

Rule 3479

Int[((a_.) + (b_.)*tan[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Simp[(a*(a + b*Tan[c + d*x])^n)/(2*b*d*n), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(n

$n + 1), x], x] /; \text{FreeQ}\{a, b, c, d\}, x\} \ \&\& \ \text{EqQ}[a^2 + b^2, 0] \ \&\& \ \text{LtQ}[n, 0]$

Rule 8

$\text{Int}[a_, x_Symbol] \ :> \ \text{Simp}[a*x, x] /; \text{FreeQ}[a, x]$

Rubi steps

$$\begin{aligned} \int \frac{(c+dx)^3}{a+a \coth(e+fx)} dx &= \frac{(c+dx)^4}{8ad} - \frac{(c+dx)^3}{2f(a+a \coth(e+fx))} + \frac{(3d) \int \frac{(c+dx)^2}{a+a \coth(e+fx)} dx}{2f} \\ &= \frac{(c+dx)^3}{4af} + \frac{(c+dx)^4}{8ad} - \frac{3d(c+dx)^2}{4f^2(a+a \coth(e+fx))} - \frac{(c+dx)^3}{2f(a+a \coth(e+fx))} + \frac{(3d^2) \int \frac{c}{a+a \coth(e+fx)} dx}{2f^2} \\ &= \frac{3d(c+dx)^2}{8af^2} + \frac{(c+dx)^3}{4af} + \frac{(c+dx)^4}{8ad} - \frac{3d^2(c+dx)}{4f^3(a+a \coth(e+fx))} - \frac{3d(c+dx)^2}{4f^2(a+a \coth(e+fx))} \\ &= \frac{3d(c+dx)^2}{8af^2} + \frac{(c+dx)^3}{4af} + \frac{(c+dx)^4}{8ad} - \frac{3d^3}{8f^4(a+a \coth(e+fx))} - \frac{3d^2(c+dx)}{4f^3(a+a \coth(e+fx))} \\ &= \frac{3d^3x}{8af^3} + \frac{3d(c+dx)^2}{8af^2} + \frac{(c+dx)^3}{4af} + \frac{(c+dx)^4}{8ad} - \frac{3d^3}{8f^4(a+a \coth(e+fx))} - \frac{3d^2(c+dx)}{4f^3(a+a \coth(e+fx))} \end{aligned}$$

Mathematica [A] time = 0.400777, size = 244, normalized size = 1.44

$\frac{\text{csch}(e+fx)(\sinh(fx) + \cosh(fx))(2f^4x(6c^2dx + 4c^3 + 4cd^2x^2 + d^3x^3)(\sinh(e) + \cosh(e)) + (\cosh(e) - \sinh(e)) \cos(e+fx))}{(16af^4(1 + \coth(e+fx)))}$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)^3/(a + a*Coth[e + f*x]),x]

[Out] (Csch[e + f*x]*(Cosh[f*x] + Sinh[f*x])*((4*c^3*f^3 + 6*c^2*d*f^2*(1 + 2*f*x) + 6*c*d^2*f*(1 + 2*f*x + 2*f^2*x^2) + d^3*(3 + 6*f*x + 6*f^2*x^2 + 4*f^3*x^3))*Cosh[2*f*x]*(Cosh[e] - Sinh[e]) + 2*f^4*x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3)*(Cosh[e] + Sinh[e]) + (4*c^3*f^3 + 6*c^2*d*f^2*(1 + 2*f*x) + 6*c*d^2*f*(1 + 2*f*x + 2*f^2*x^2) + d^3*(3 + 6*f*x + 6*f^2*x^2 + 4*f^3*x^3))*(-Cosh[e] + Sinh[e])*Sinh[2*f*x]))/(16*a*f^4*(1 + Coth[e + f*x]))

Maple [B] time = 0.077, size = 959, normalized size = 5.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x+c)^3/(a+a*coth(f*x+e)),x)`

[Out]
$$\frac{1}{a} \frac{1}{f} \left(-\frac{1}{f^3 d^3} \left(\frac{1}{2} (f*x+e)^3 \cosh(f*x+e) \sinh(f*x+e) - \frac{1}{8} (f*x+e)^4 - \frac{3}{4} (f*x+e)^2 \cosh(f*x+e)^2 + \frac{3}{4} (f*x+e) \cosh(f*x+e) \sinh(f*x+e) + \frac{3}{8} (f*x+e)^2 - \frac{3}{8} \cosh(f*x+e)^2 \right) + \frac{1}{f^3 d^3} \left(\frac{1}{2} (f*x+e)^3 \cosh(f*x+e)^2 - \frac{3}{4} (f*x+e)^2 \cosh(f*x+e) \sinh(f*x+e) - \frac{1}{4} (f*x+e)^3 + \frac{3}{4} (f*x+e) \cosh(f*x+e)^2 - \frac{3}{8} \cosh(f*x+e) \sinh(f*x+e) - \frac{3}{8} f*x - \frac{3}{8} e \right) + \frac{3}{f^3 d^3} e \left(\frac{1}{2} (f*x+e)^2 \cosh(f*x+e) \sinh(f*x+e) - \frac{1}{6} (f*x+e)^3 - \frac{1}{2} (f*x+e) \cosh(f*x+e)^2 + \frac{1}{4} \cosh(f*x+e) \sinh(f*x+e) + \frac{1}{4} f*x + \frac{1}{4} e \right) - \frac{3}{f^3 d^3} e \left(\frac{1}{2} (f*x+e)^2 \cosh(f*x+e)^2 - \frac{1}{2} (f*x+e) \cosh(f*x+e) \sinh(f*x+e) - \frac{1}{4} (f*x+e)^2 + \frac{1}{4} \cosh(f*x+e)^2 \right) - \frac{3}{f^2 d^2} c \left(\frac{1}{2} (f*x+e)^2 \cosh(f*x+e) \sinh(f*x+e) - \frac{1}{6} (f*x+e)^3 - \frac{1}{2} (f*x+e) \cosh(f*x+e)^2 + \frac{1}{4} \cosh(f*x+e) \sinh(f*x+e) + \frac{1}{4} f*x + \frac{1}{4} e \right) + \frac{3}{f^2 d^2} c \left(\frac{1}{2} (f*x+e)^2 \cosh(f*x+e)^2 - \frac{1}{2} (f*x+e) \cosh(f*x+e) \sinh(f*x+e) - \frac{1}{4} (f*x+e)^2 + \frac{1}{4} \cosh(f*x+e)^2 \right) - \frac{3}{f^3 d^3} e^2 \left(\frac{1}{2} (f*x+e) \cosh(f*x+e) \sinh(f*x+e) - \frac{1}{4} (f*x+e)^2 - \frac{1}{4} \cosh(f*x+e)^2 \right) + \frac{3}{f^3 d^3} e^2 \left(\frac{1}{2} (f*x+e) \cosh(f*x+e)^2 - \frac{1}{4} \cosh(f*x+e) \sinh(f*x+e) - \frac{1}{4} f*x - \frac{1}{4} e \right) + \frac{6}{f^2 d^2} e^2 c \left(\frac{1}{2} (f*x+e) \cosh(f*x+e) \sinh(f*x+e) - \frac{1}{4} (f*x+e)^2 - \frac{1}{4} \cosh(f*x+e)^2 \right) - \frac{6}{f^2 d^2} e^2 c \left(\frac{1}{2} (f*x+e) \cosh(f*x+e)^2 - \frac{1}{4} \cosh(f*x+e) \sinh(f*x+e) - \frac{1}{4} f*x - \frac{1}{4} e \right) - \frac{3}{f d^2} c^2 \left(\frac{1}{2} (f*x+e) \cosh(f*x+e) \sinh(f*x+e) - \frac{1}{4} (f*x+e)^2 - \frac{1}{4} \cosh(f*x+e)^2 \right) + \frac{3}{f d^2} c^2 \left(\frac{1}{2} (f*x+e) \cosh(f*x+e)^2 - \frac{1}{4} \cosh(f*x+e) \sinh(f*x+e) - \frac{1}{4} f*x - \frac{1}{4} e \right) + d^3 e^3 / f^3 \left(\frac{1}{2} \cosh(f*x+e) \sinh(f*x+e) - \frac{1}{2} f*x - \frac{1}{2} e \right) - \frac{1}{2} d^3 e^3 / f^3 \cosh(f*x+e)^2 - \frac{3}{2} d^2 e^2 / f^2 c \left(\frac{1}{2} \cosh(f*x+e) \sinh(f*x+e) - \frac{1}{2} f*x - \frac{1}{2} e \right) + \frac{3}{2} d^2 e^2 / f^2 c \cosh(f*x+e)^2 + \frac{3}{2} d e / f c^2 \left(\frac{1}{2} \cosh(f*x+e) \sinh(f*x+e) - \frac{1}{2} f*x - \frac{1}{2} e \right) - \frac{3}{2} d e / f c^2 \cosh(f*x+e)^2 - \frac{3}{2} \left(\frac{1}{2} \cosh(f*x+e) \sinh(f*x+e) - \frac{1}{2} f*x - \frac{1}{2} e \right) + \frac{1}{2} c^3 \cosh(f*x+e)^2 \right)$$

Maxima [A] time = 1.37541, size = 247, normalized size = 1.46

$$\frac{1}{4} c^3 \left(\frac{2(fx+e)}{af} + \frac{e^{(-2fx-2e)}}{af} \right) + \frac{3 \left(2f^2 x^2 e^{(2e)} + (2fx+1)e^{(-2fx)} \right) c^2 d e^{(-2e)}}{8af^2} + \frac{\left(4f^3 x^3 e^{(2e)} + 3(2f^2 x^2 + 2fx+1)e^{(-2fx)} \right)}{8af^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)^3/(a+a*coth(f*x+e)),x, algorithm="maxima")`

```
[Out] 1/4*c^3*(2*(f*x + e)/(a*f) + e^(-2*f*x - 2*e)/(a*f)) + 3/8*(2*f^2*x^2*e^(2*
e) + (2*f*x + 1)*e^(-2*f*x))*c^2*d*e^(-2*e)/(a*f^2) + 1/8*(4*f^3*x^3*e^(2*e
) + 3*(2*f^2*x^2 + 2*f*x + 1)*e^(-2*f*x))*c*d^2*e^(-2*e)/(a*f^3) + 1/16*(2*
f^4*x^4*e^(2*e) + (4*f^3*x^3 + 6*f^2*x^2 + 6*f*x + 3)*e^(-2*f*x))*d^3*e^(-2
*e)/(a*f^4)
```

Fricas [A] time = 2.0833, size = 640, normalized size = 3.79

$$(2d^3f^4x^4 + 4c^3f^3 + 6c^2df^2 + 6cd^2f + 4(2cd^2f^4 + d^3f^3)x^3 + 3d^3 + 6(2c^2df^4 + 2cd^2f^3 + d^3f^2)x^2 + 2(4c^3f^4 + 6c^2df^3 + 6cd^2f^2 + 6f^2d^2 + 6fd^2 + 3d^2)x + 2d^2)e^{\frac{3}{a+a\coth(fx+e)}} + \text{constant}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+a*coth(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/16*((2*d^3*f^4*x^4 + 4*c^3*f^3 + 6*c^2*d*f^2 + 6*c*d^2*f + 4*(2*c*d^2*f^4
+ d^3*f^3))*x^3 + 3*d^3 + 6*(2*c^2*d*f^4 + 2*c*d^2*f^3 + d^3*f^2))*x^2 + 2*(
4*c^3*f^4 + 6*c^2*d*f^3 + 6*c*d^2*f^2 + 3*d^3*f)*x)*cosh(f*x + e) + (2*d^3*
f^4*x^4 - 4*c^3*f^3 - 6*c^2*d*f^2 - 6*c*d^2*f + 4*(2*c*d^2*f^4 - d^3*f^3))*x
^3 - 3*d^3 + 6*(2*c^2*d*f^4 - 2*c*d^2*f^3 - d^3*f^2))*x^2 + 2*(4*c^3*f^4 - 6
*c^2*d*f^3 - 6*c*d^2*f^2 - 3*d^3*f)*x)*sinh(f*x + e))/(a*f^4*cosh(f*x + e)
+ a*f^4*sinh(f*x + e))
```

Sympy [A] time = 5.25127, size = 864, normalized size = 5.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3/(a+a*coth(f*x+e)),x)
```

```
[Out] Piecewise((4*c**3*f**4*x*tanh(e + f*x)/(8*a*f**4*tanh(e + f*x) + 8*a*f**4)
+ 4*c**3*f**4*x/(8*a*f**4*tanh(e + f*x) + 8*a*f**4) + 4*c**3*f**3/(8*a*f**4
*tanh(e + f*x) + 8*a*f**4) + 6*c**2*d*f**4*x**2*tanh(e + f*x)/(8*a*f**4*tan
h(e + f*x) + 8*a*f**4) + 6*c**2*d*f**4*x**2/(8*a*f**4*tanh(e + f*x) + 8*a*f
**4) - 6*c**2*d*f**3*x*tanh(e + f*x)/(8*a*f**4*tanh(e + f*x) + 8*a*f**4) +
6*c**2*d*f**3*x/(8*a*f**4*tanh(e + f*x) + 8*a*f**4) + 6*c**2*d*f**2/(8*a*f
**4*tanh(e + f*x) + 8*a*f**4) + 4*c*d**2*f**4*x**3*tanh(e + f*x)/(8*a*f**4*t
anh(e + f*x) + 8*a*f**4) + 4*c*d**2*f**4*x**3/(8*a*f**4*tanh(e + f*x) + 8*a
```

```

*f**4) - 6*c*d**2*f**3*x**2*tanh(e + f*x)/(8*a*f**4*tanh(e + f*x) + 8*a*f**
4) + 6*c*d**2*f**3*x**2/(8*a*f**4*tanh(e + f*x) + 8*a*f**4) - 6*c*d**2*f**2
*x*tanh(e + f*x)/(8*a*f**4*tanh(e + f*x) + 8*a*f**4) + 6*c*d**2*f**2*x/(8*a
*f**4*tanh(e + f*x) + 8*a*f**4) + 6*c*d**2*f/(8*a*f**4*tanh(e + f*x) + 8*a*
f**4) + d**3*f**4*x**4*tanh(e + f*x)/(8*a*f**4*tanh(e + f*x) + 8*a*f**4) +
d**3*f**4*x**4/(8*a*f**4*tanh(e + f*x) + 8*a*f**4) - 2*d**3*f**3*x**3*tanh(
e + f*x)/(8*a*f**4*tanh(e + f*x) + 8*a*f**4) + 2*d**3*f**3*x**3/(8*a*f**4*t
anh(e + f*x) + 8*a*f**4) - 3*d**3*f**2*x**2*tanh(e + f*x)/(8*a*f**4*tanh(e
+ f*x) + 8*a*f**4) + 3*d**3*f**2*x**2/(8*a*f**4*tanh(e + f*x) + 8*a*f**4) -
3*d**3*f*x*tanh(e + f*x)/(8*a*f**4*tanh(e + f*x) + 8*a*f**4) + 3*d**3*f*x/
(8*a*f**4*tanh(e + f*x) + 8*a*f**4) + 3*d**3/(8*a*f**4*tanh(e + f*x) + 8*a*
f**4), Ne(f, 0)), ((c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3 + d**3*x**4/4)/(
a*coth(e) + a), True))

```

Giac [A] time = 1.184, size = 261, normalized size = 1.54

$$\frac{(2d^3f^4x^4e^{(2fx+2e)} + 8cd^2f^4x^3e^{(2fx+2e)} + 12c^2df^4x^2e^{(2fx+2e)} + 4d^3f^3x^3 + 8c^3f^4xe^{(2fx+2e)} + 12cd^2f^3x^2 + 12c^2df^3x + d^3f^3 + 8c^3f^4x + 12cd^2f^3 + 12c^2df^3 + d^3f^3)e^{(-2fx-2e)}}{16af^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3/(a+a*coth(f*x+e)),x, algorithm="giac")

[Out] 1/16*(2*d^3*f^4*x^4*e^(2*f*x + 2*e) + 8*c*d^2*f^4*x^3*e^(2*f*x + 2*e) + 12*c^2*d*f^4*x^2*e^(2*f*x + 2*e) + 4*d^3*f^3*x^3 + 8*c^3*f^4*x*e^(2*f*x + 2*e) + 12*c*d^2*f^3*x^2 + 12*c^2*d*f^3*x + 6*d^3*f^2*x^2 + 4*c^3*f^3 + 12*c*d^2*f^2*x + 6*c^2*d*f^2 + 6*d^3*f*x + 6*c*d^2*f + 3*d^3)*e^(-2*f*x - 2*e)/(a*f^4)

$$3.17 \quad \int \frac{(c+dx)^2}{a+a \coth(e+fx)} dx$$

Optimal. Leaf size=122

$$-\frac{d(c+dx)}{2f^2(a \coth(e+fx)+a)} - \frac{(c+dx)^2}{2f(a \coth(e+fx)+a)} + \frac{(c+dx)^2}{4af} + \frac{(c+dx)^3}{6ad} - \frac{d^2}{4f^3(a \coth(e+fx)+a)} + \frac{d^2x}{4af^2}$$

[Out] (d^2*x)/(4*a*f^2) + (c + d*x)^2/(4*a*f) + (c + d*x)^3/(6*a*d) - d^2/(4*f^3*(a + a*Coth[e + f*x])) - (d*(c + d*x))/(2*f^2*(a + a*Coth[e + f*x])) - (c + d*x)^2/(2*f*(a + a*Coth[e + f*x]))

Rubi [A] time = 0.117448, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.15$, Rules used = {3723, 3479, 8}

$$-\frac{d(c+dx)}{2f^2(a \coth(e+fx)+a)} - \frac{(c+dx)^2}{2f(a \coth(e+fx)+a)} + \frac{(c+dx)^2}{4af} + \frac{(c+dx)^3}{6ad} - \frac{d^2}{4f^3(a \coth(e+fx)+a)} + \frac{d^2x}{4af^2}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)^2/(a + a*Coth[e + f*x]),x]

[Out] (d^2*x)/(4*a*f^2) + (c + d*x)^2/(4*a*f) + (c + d*x)^3/(6*a*d) - d^2/(4*f^3*(a + a*Coth[e + f*x])) - (d*(c + d*x))/(2*f^2*(a + a*Coth[e + f*x])) - (c + d*x)^2/(2*f*(a + a*Coth[e + f*x]))

Rule 3723

Int[((c_.) + (d_.)*(x_))^(m_.)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(2*a*d*(m + 1)), x] + (Dist[(a*d*m)/(2*b*f), Int[(c + d*x)^(m - 1)/(a + b*Tan[e + f*x]), x], x] - Simp[(a*(c + d*x)^m)/(2*b*f*(a + b*Tan[e + f*x])), x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2 + b^2, 0] && GtQ[m, 0]

Rule 3479

Int[((a_.) + (b_.)*tan[(c_.) + (d_.)*(x_)]^(n_.), x_Symbol] :> Simp[(a*(a + b*Tan[c + d*x])^n)/(2*b*d*n), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rubi steps

$$\begin{aligned}
 \int \frac{(c+dx)^2}{a+a \coth(e+fx)} dx &= \frac{(c+dx)^3}{6ad} - \frac{(c+dx)^2}{2f(a+a \coth(e+fx))} + \frac{d \int \frac{c+dx}{a+a \coth(e+fx)} dx}{f} \\
 &= \frac{(c+dx)^2}{4af} + \frac{(c+dx)^3}{6ad} - \frac{d(c+dx)}{2f^2(a+a \coth(e+fx))} - \frac{(c+dx)^2}{2f(a+a \coth(e+fx))} + \frac{d^2 \int \frac{1}{a+a \coth(e+fx)}}{2f^2} \\
 &= \frac{(c+dx)^2}{4af} + \frac{(c+dx)^3}{6ad} - \frac{d^2}{4f^3(a+a \coth(e+fx))} - \frac{d(c+dx)}{2f^2(a+a \coth(e+fx))} - \frac{(c+dx)}{2f(a+a \coth(e+fx))} \\
 &= \frac{d^2x}{4af^2} + \frac{(c+dx)^2}{4af} + \frac{(c+dx)^3}{6ad} - \frac{d^2}{4f^3(a+a \coth(e+fx))} - \frac{d(c+dx)}{2f^2(a+a \coth(e+fx))} - \frac{(c+dx)}{2f(a+a \coth(e+fx))}
 \end{aligned}$$

Mathematica [A] time = 0.252905, size = 169, normalized size = 1.39

$$\frac{\operatorname{csch}(e+fx)(\sinh(fx) + \cosh(fx)) \left(\frac{4}{3} f^3 x (3c^2 + 3cdx + d^2x^2) (\sinh(e) + \cosh(e)) + (\cosh(e) - \sinh(e)) \cosh(2fx) (2c^2 + 3cdx + d^2x^2) \right)}{8af^3(\coth(e+fx) + 1)}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)^2/(a + a*Coth[e + f*x]),x]

[Out] (Csch[e + f*x]*(Cosh[f*x] + Sinh[f*x])*((2*c^2*f^2 + 2*c*d*f*(1 + 2*f*x) + d^2*(1 + 2*f*x + 2*f^2*x^2))*Cosh[2*f*x]*(Cosh[e] - Sinh[e]) + (4*f^3*x*(3*c^2 + 3*c*d*x + d^2*x^2)*(Cosh[e] + Sinh[e]))/3 + (2*c^2*f^2 + 2*c*d*f*(1 + 2*f*x) + d^2*(1 + 2*f*x + 2*f^2*x^2))*(-Cosh[e] + Sinh[e])*Sinh[2*f*x]))/(8*a*f^3*(1 + Coth[e + f*x]))

Maple [B] time = 0.065, size = 469, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^2/(a+a*coth(f*x+e)),x)

```
[Out] 1/a/f*(-1/f^2*d^2*(1/2*(f*x+e)^2*cosh(f*x+e)*sinh(f*x+e)-1/6*(f*x+e)^3-1/2*(f*x+e)*cosh(f*x+e)^2+1/4*cosh(f*x+e)*sinh(f*x+e)+1/4*f*x+1/4*e)+1/f^2*d^2*(1/2*(f*x+e)^2*cosh(f*x+e)^2-1/2*(f*x+e)*cosh(f*x+e)*sinh(f*x+e)-1/4*(f*x+e)^2+1/4*cosh(f*x+e)^2)+2/f^2*d^2*e*(1/2*(f*x+e)*cosh(f*x+e)*sinh(f*x+e)-1/4*(f*x+e)^2-1/4*cosh(f*x+e)^2)-2/f^2*d^2*e*(1/2*(f*x+e)*cosh(f*x+e)^2-1/4*cosh(f*x+e)*sinh(f*x+e)-1/4*f*x-1/4*e)-2/f*d*c*(1/2*(f*x+e)*cosh(f*x+e)*sinh(f*x+e)-1/4*(f*x+e)^2-1/4*cosh(f*x+e)^2)+2/f*d*c*(1/2*(f*x+e)*cosh(f*x+e)^2-1/4*cosh(f*x+e)*sinh(f*x+e)-1/4*f*x-1/4*e)-d^2*e^2/f^2*(1/2*cosh(f*x+e)*sinh(f*x+e)-1/2*f*x-1/2*e)+1/2*d^2*e^2/f^2*cosh(f*x+e)^2+2*d*e/f*c*(1/2*cosh(f*x+e)*sinh(f*x+e)-1/2*f*x-1/2*e)-d*e/f*c*cosh(f*x+e)^2-c^2*(1/2*cosh(f*x+e)*sinh(f*x+e)-1/2*f*x-1/2*e)+1/2*c^2*cosh(f*x+e)^2)
```

Maxima [A] time = 1.25439, size = 167, normalized size = 1.37

$$\frac{1}{4}c^2\left(\frac{2(fx+e)}{af} + \frac{e^{(-2fx-2e)}}{af}\right) + \frac{(2f^2x^2e^{(2e)} + (2fx+1)e^{(-2fx)})cde^{(-2e)}}{4af^2} + \frac{(4f^3x^3e^{(2e)} + 3(2f^2x^2 + 2fx+1)e^{(-2fx)})}{24af^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+a*coth(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/4*c^2*(2*(f*x + e)/(a*f) + e^(-2*f*x - 2*e)/(a*f)) + 1/4*(2*f^2*x^2*e^(2*e) + (2*f*x + 1)*e^(-2*f*x))*c*d*e^(-2*e)/(a*f^2) + 1/24*(4*f^3*x^3*e^(2*e) + 3*(2*f^2*x^2 + 2*f*x + 1)*e^(-2*f*x))*d^2*e^(-2*e)/(a*f^3)
```

Fricas [A] time = 2.08446, size = 419, normalized size = 3.43

$$\frac{(4d^2f^3x^3 + 6c^2f^2 + 6cdf + 6(2cdf^3 + d^2f^2)x^2 + 3d^2 + 6(2c^2f^3 + 2cdf^2 + d^2f)x) \cosh(fx + e) + (4d^2f^3x^3 - 6c^2f^2 + 6(2c^2f^3 + 2cdf^2 + d^2f)x) \sinh(fx + e)}{24(af^3 \cosh(fx + e) + af^3 \sinh(fx + e))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+a*coth(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/24*((4*d^2*f^3*x^3 + 6*c^2*f^2 + 6*c*d*f + 6*(2*c*d*f^3 + d^2*f^2)*x^2 + 3*d^2 + 6*(2*c^2*f^3 + 2*c*d*f^2 + d^2*f)*x)*cosh(f*x + e) + (4*d^2*f^3*x^3 - 6*c^2*f^2 - 6*c*d*f + 6*(2*c*d*f^3 - d^2*f^2)*x^2 - 3*d^2 + 6*(2*c^2*f^3 - 2*c*d*f^2 - d^2*f)*x)*sinh(f*x + e))/(a*f^3*cosh(f*x + e) + a*f^3*sinh(f*x + e))
```

*x + e))

Sympy [A] time = 2.18102, size = 522, normalized size = 4.28

$$\left(\frac{6c^2 f^3 x \tanh(e+fx)}{12af^3 \tanh(e+fx)+12af^3} + \frac{6c^2 f^3 x}{12af^3 \tanh(e+fx)+12af^3} + \frac{6c^2 f^2}{12af^3 \tanh(e+fx)+12af^3} + \frac{6cdf^3 x^2 \tanh(e+fx)}{12af^3 \tanh(e+fx)+12af^3} + \frac{6cdf^3 x^2}{12af^3 \tanh(e+fx)+12af^3} - \frac{c^2 x + cd x^2 + \frac{d^2 x^3}{3}}{a \coth(e)+a} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**2/(a+a*coth(f*x+e)),x)

[Out] Piecewise(((6*c**2*f**3*x*tanh(e + f*x)/(12*a*f**3*tanh(e + f*x) + 12*a*f**3) + 6*c**2*f**3*x/(12*a*f**3*tanh(e + f*x) + 12*a*f**3) + 6*c**2*f**2/(12*a*f**3*tanh(e + f*x) + 12*a*f**3) + 6*c*d*f**3*x**2*tanh(e + f*x)/(12*a*f**3*tanh(e + f*x) + 12*a*f**3) + 6*c*d*f**3*x**2/(12*a*f**3*tanh(e + f*x) + 12*a*f**3) - 6*c*d*f**2*x*tanh(e + f*x)/(12*a*f**3*tanh(e + f*x) + 12*a*f**3) + 6*c*d*f**2*x/(12*a*f**3*tanh(e + f*x) + 12*a*f**3) + 6*c*d*f/(12*a*f**3*tanh(e + f*x) + 12*a*f**3) + 2*d**2*f**3*x**3*tanh(e + f*x)/(12*a*f**3*tanh(e + f*x) + 12*a*f**3) + 2*d**2*f**3*x**3/(12*a*f**3*tanh(e + f*x) + 12*a*f**3) - 3*d**2*f**2*x**2*tanh(e + f*x)/(12*a*f**3*tanh(e + f*x) + 12*a*f**3) + 3*d**2*f**2*x**2/(12*a*f**3*tanh(e + f*x) + 12*a*f**3) - 3*d**2*f*x*tanh(e + f*x)/(12*a*f**3*tanh(e + f*x) + 12*a*f**3) + 3*d**2*f*x/(12*a*f**3*tanh(e + f*x) + 12*a*f**3) + 3*d**2/(12*a*f**3*tanh(e + f*x) + 12*a*f**3), Ne(f, 0)), ((c**2*x + c*d*x**2 + d**2*x**3/3)/(a*coth(e) + a), True))

Giac [A] time = 1.15187, size = 166, normalized size = 1.36

$$\frac{\left(4d^2 f^3 x^3 e^{(2fx+2e)} + 12cdf^3 x^2 e^{(2fx+2e)} + 12c^2 f^3 x e^{(2fx+2e)} + 6d^2 f^2 x^2 + 12cdf^2 x + 6c^2 f^2 + 6d^2 f x + 6cdf + 3d^2\right) e^{(-2fx-2e)}}{24af^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2/(a+a*coth(f*x+e)),x, algorithm="giac")

[Out] 1/24*(4*d^2*f^3*x^3*e^(2*f*x + 2*e) + 12*c*d*f^3*x^2*e^(2*f*x + 2*e) + 12*c^2*f^3*x*e^(2*f*x + 2*e) + 6*d^2*f^2*x^2 + 12*c*d*f^2*x + 6*c^2*f^2 + 6*d^2*f*x + 6*c*d*f + 3*d^2)*e^(-2*f*x - 2*e)/(a*f^3)

$$3.18 \quad \int \frac{c+dx}{a+a \coth(e+fx)} dx$$

Optimal. Leaf size=74

$$-\frac{c+dx}{2f(a \coth(e+fx)+a)} + \frac{(c+dx)^2}{4ad} - \frac{d}{4f^2(a \coth(e+fx)+a)} + \frac{dx}{4af}$$

[Out] (d*x)/(4*a*f) + (c + d*x)^2/(4*a*d) - d/(4*f^2*(a + a*Coth[e + f*x])) - (c + d*x)/(2*f*(a + a*Coth[e + f*x]))

Rubi [A] time = 0.051007, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3723, 3479, 8}

$$-\frac{c+dx}{2f(a \coth(e+fx)+a)} + \frac{(c+dx)^2}{4ad} - \frac{d}{4f^2(a \coth(e+fx)+a)} + \frac{dx}{4af}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)/(a + a*Coth[e + f*x]),x]

[Out] (d*x)/(4*a*f) + (c + d*x)^2/(4*a*d) - d/(4*f^2*(a + a*Coth[e + f*x])) - (c + d*x)/(2*f*(a + a*Coth[e + f*x]))

Rule 3723

Int[((c_.) + (d_.)*(x_))^(m_.)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] := Simp[(c + d*x)^(m + 1)/(2*a*d*(m + 1)), x] + (Dist[(a*d*m)/(2*b*f), Int[(c + d*x)^(m - 1)/(a + b*Tan[e + f*x]), x], x] - Simp[(a*(c + d*x)^m)/(2*b*f*(a + b*Tan[e + f*x])), x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2 + b^2, 0] && GtQ[m, 0]

Rule 3479

Int[((a_.) + (b_.)*tan[(c_.) + (d_.)*(x_)]^(n_.), x_Symbol] := Simp[(a*(a + b*Tan[c + d*x])^n)/(2*b*d*n), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rubi steps

$$\begin{aligned}
\int \frac{c+dx}{a+a \coth(e+fx)} dx &= \frac{(c+dx)^2}{4ad} - \frac{c+dx}{2f(a+a \coth(e+fx))} + \frac{d \int \frac{1}{a+a \coth(e+fx)} dx}{2f} \\
&= \frac{(c+dx)^2}{4ad} - \frac{d}{4f^2(a+a \coth(e+fx))} - \frac{c+dx}{2f(a+a \coth(e+fx))} + \frac{d \int 1 dx}{4af} \\
&= \frac{dx}{4af} + \frac{(c+dx)^2}{4ad} - \frac{d}{4f^2(a+a \coth(e+fx))} - \frac{c+dx}{2f(a+a \coth(e+fx))}
\end{aligned}$$

Mathematica [A] time = 0.262145, size = 81, normalized size = 1.09

$$\frac{(2cf(2fx+1) + d(2f^2x^2 + 2fx + 1)) \coth(e+fx) + 2cf(2fx-1) + d(2f^2x^2 - 2fx - 1)}{8af^2(\coth(e+fx) + 1)}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)/(a + a*Coth[e + f*x]),x]

[Out] (2*c*f*(-1 + 2*f*x) + d*(-1 - 2*f*x + 2*f^2*x^2) + (2*c*f*(1 + 2*f*x) + d*(1 + 2*f*x + 2*f^2*x^2))*Coth[e + f*x])/(8*a*f^2*(1 + Coth[e + f*x]))

Maple [B] time = 0.058, size = 175, normalized size = 2.4

$$\frac{1}{af} \left(-\frac{d}{f} \left(\frac{(fx+e) \cosh(fx+e) \sinh(fx+e)}{2} - \frac{(fx+e)^2}{4} - \frac{(\cosh(fx+e))^2}{4} \right) + \frac{d}{f} \left(\frac{(fx+e) (\cosh(fx+e))^2}{2} - \frac{\cosh(fx+e)}{2} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)/(a+a*coth(f*x+e)),x)

[Out] 1/a/f*(-1/f*d*(1/2*(f*x+e)*cosh(f*x+e)*sinh(f*x+e)-1/4*(f*x+e)^2-1/4*cosh(f*x+e)^2)+1/f*d*(1/2*(f*x+e)*cosh(f*x+e)^2-1/4*cosh(f*x+e)*sinh(f*x+e)-1/4*f*x-1/4*e)+d*e/f*(1/2*cosh(f*x+e)*sinh(f*x+e)-1/2*f*x-1/2*e)-1/2*d*e/f*cosh(f*x+e)^2-c*(1/2*cosh(f*x+e)*sinh(f*x+e)-1/2*f*x-1/2*e)+1/2*c*cosh(f*x+e)^2)

Maxima [A] time = 1.16564, size = 97, normalized size = 1.31

$$\frac{1}{4} c \left(\frac{2(fx + e)}{af} + \frac{e^{(-2fx-2e)}}{af} \right) + \frac{(2f^2x^2e^{(2e)} + (2fx + 1)e^{(-2fx)})de^{(-2e)}}{8af^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)/(a+a*coth(f*x+e)),x, algorithm="maxima")

[Out] 1/4*c*(2*(f*x + e)/(a*f) + e^(-2*f*x - 2*e)/(a*f)) + 1/8*(2*f^2*x^2*e^(2*e) + (2*f*x + 1)*e^(-2*f*x))*d*e^(-2*e)/(a*f^2)

Fricas [A] time = 2.11715, size = 239, normalized size = 3.23

$$\frac{(2df^2x^2 + 2cf + 2(2cf^2 + df)x + d) \cosh(fx + e) + (2df^2x^2 - 2cf + 2(2cf^2 - df)x - d) \sinh(fx + e)}{8(a f^2 \cosh(fx + e) + a f^2 \sinh(fx + e))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)/(a+a*coth(f*x+e)),x, algorithm="fricas")

[Out] 1/8*((2*d*f^2*x^2 + 2*c*f + 2*(2*c*f^2 + d*f)*x + d)*cosh(f*x + e) + (2*d*f^2*x^2 - 2*c*f + 2*(2*c*f^2 - d*f)*x - d)*sinh(f*x + e))/(a*f^2*cosh(f*x + e) + a*f^2*sinh(f*x + e))

Sympy [A] time = 1.45016, size = 250, normalized size = 3.38

$$\left\{ \begin{array}{l} \frac{2cf^2x \tanh(e+fx)}{4af^2 \tanh(e+fx)+4af^2} + \frac{2cf^2x}{4af^2 \tanh(e+fx)+4af^2} + \frac{2cf}{4af^2 \tanh(e+fx)+4af^2} + \frac{df^2x^2 \tanh(e+fx)}{4af^2 \tanh(e+fx)+4af^2} + \frac{df^2x^2}{4af^2 \tanh(e+fx)+4af^2} - \frac{dfx \tanh(e+fx)}{4af^2 \tanh(e+fx)+4af^2} \\ \frac{cx + \frac{dx^2}{2}}{a \coth(e)+a} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)/(a+a*coth(f*x+e)),x)

[Out] Piecewise(((2*c*f**2*x*tanh(e + f*x))/(4*a*f**2*tanh(e + f*x) + 4*a*f**2) + 2*c*f**2*x/(4*a*f**2*tanh(e + f*x) + 4*a*f**2) + 2*c*f/(4*a*f**2*tanh(e + f*x) + 4*a*f**2) + (d*f**2*x**2*tanh(e + f*x))/(4*a*f**2*tanh(e + f*x) + 4*a*f**2) + (d*f**2*x**2)/(4*a*f**2*tanh(e + f*x) + 4*a*f**2) - (d*f*x*tanh(e + f*x))/(4*a*f**2*tanh(e + f*x) + 4*a*f**2) + (c*x + d*x**2/2)/(a*coth(e) + a))

```
x) + 4*a*f**2) + d*f**2*x**2*tanh(e + f*x)/(4*a*f**2*tanh(e + f*x) + 4*a*f*
*2) + d*f**2*x**2/(4*a*f**2*tanh(e + f*x) + 4*a*f**2) - d*f*x*tanh(e + f*x)
/(4*a*f**2*tanh(e + f*x) + 4*a*f**2) + d*f*x/(4*a*f**2*tanh(e + f*x) + 4*a*
f**2) + d/(4*a*f**2*tanh(e + f*x) + 4*a*f**2), Ne(f, 0)), ((c*x + d*x**2/2)
/(a*coth(e) + a), True))
```

Giac [A] time = 1.18953, size = 88, normalized size = 1.19

$$\frac{(2df^2x^2e^{(2fx+2e)} + 4cf^2xe^{(2fx+2e)} + 2dfx + 2cf + d)e^{(-2fx-2e)}}{8af^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+a*coth(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/8*(2*d*f^2*x^2*e^(2*f*x + 2*e) + 4*c*f^2*x*e^(2*f*x + 2*e) + 2*d*f*x + 2*
c*f + d)*e^(-2*f*x - 2*e)/(a*f^2)
```

$$3.19 \quad \int \frac{1}{(c+dx)(a+a \coth(e+fx))} dx$$

Optimal. Leaf size=157

$$\frac{\operatorname{Chi}\left(2xf + \frac{2cf}{d}\right) \sinh\left(2e - \frac{2cf}{d}\right)}{2ad} - \frac{\operatorname{Chi}\left(2xf + \frac{2cf}{d}\right) \cosh\left(2e - \frac{2cf}{d}\right)}{2ad} - \frac{\sinh\left(2e - \frac{2cf}{d}\right) \operatorname{Shi}\left(2xf + \frac{2cf}{d}\right)}{2ad} + \frac{\cosh\left(2e - \frac{2cf}{d}\right)}{2ad}$$

[Out] $-(\operatorname{Cosh}[2*e - (2*c*f)/d]*\operatorname{CoshIntegral}[(2*c*f)/d + 2*f*x])/(2*a*d) + \operatorname{Log}[c + d*x]/(2*a*d) + (\operatorname{CoshIntegral}[(2*c*f)/d + 2*f*x]*\operatorname{Sinh}[2*e - (2*c*f)/d])/(2*a*d) + (\operatorname{Cosh}[2*e - (2*c*f)/d]*\operatorname{SinhIntegral}[(2*c*f)/d + 2*f*x])/(2*a*d) - (\operatorname{Sinh}[2*e - (2*c*f)/d]*\operatorname{SinhIntegral}[(2*c*f)/d + 2*f*x])/(2*a*d)$

Rubi [A] time = 0.251087, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {3726, 3303, 3298, 3301}

$$\frac{\operatorname{Chi}\left(2xf + \frac{2cf}{d}\right) \sinh\left(2e - \frac{2cf}{d}\right)}{2ad} - \frac{\operatorname{Chi}\left(2xf + \frac{2cf}{d}\right) \cosh\left(2e - \frac{2cf}{d}\right)}{2ad} - \frac{\sinh\left(2e - \frac{2cf}{d}\right) \operatorname{Shi}\left(2xf + \frac{2cf}{d}\right)}{2ad} + \frac{\cosh\left(2e - \frac{2cf}{d}\right)}{2ad}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[1/((c + d*x)*(a + a*\operatorname{Coth}[e + f*x])), x]$

[Out] $-(\operatorname{Cosh}[2*e - (2*c*f)/d]*\operatorname{CoshIntegral}[(2*c*f)/d + 2*f*x])/(2*a*d) + \operatorname{Log}[c + d*x]/(2*a*d) + (\operatorname{CoshIntegral}[(2*c*f)/d + 2*f*x]*\operatorname{Sinh}[2*e - (2*c*f)/d])/(2*a*d) + (\operatorname{Cosh}[2*e - (2*c*f)/d]*\operatorname{SinhIntegral}[(2*c*f)/d + 2*f*x])/(2*a*d) - (\operatorname{Sinh}[2*e - (2*c*f)/d]*\operatorname{SinhIntegral}[(2*c*f)/d + 2*f*x])/(2*a*d)$

Rule 3726

$\operatorname{Int}[1/(((c_.) + (d_.)*(x_.))*((a_.) + (b_.)*\tan[(e_.) + (f_.)*(x_.)]))], x_Symbol] \rightarrow \operatorname{Simp}[\operatorname{Log}[c + d*x]/(2*a*d), x] + (\operatorname{Dist}[1/(2*a), \operatorname{Int}[\operatorname{Cos}[2*e + 2*f*x]/(c + d*x), x], x] + \operatorname{Dist}[1/(2*b), \operatorname{Int}[\operatorname{Sin}[2*e + 2*f*x]/(c + d*x), x], x]) / ; \operatorname{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \operatorname{EqQ}[a^2 + b^2, 0]$

Rule 3303

$\operatorname{Int}[\sin[(e_.) + (f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x_Symbol] \rightarrow \operatorname{Dist}[\operatorname{Cos}[(d*e - c*f)/d], \operatorname{Int}[\operatorname{Sin}[(c*f)/d + f*x]/(c + d*x), x], x] + \operatorname{Dist}[\operatorname{Sin}[(d*e - c*f)/d], \operatorname{Int}[\operatorname{Cos}[(c*f)/d + f*x]/(c + d*x), x], x] / ; \operatorname{FreeQ}\{c, d, e, f\}, x] \&\&$

NeQ[d*e - c*f, 0]

Rule 3298

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*e - c*f*fz*I, 0]

Rule 3301

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{(c+dx)(a+a \coth(e+fx))} dx &= \frac{\log(c+dx)}{2ad} + \frac{i \int \frac{\sin(2(i e + \frac{\pi}{2}) + 2ifx)}{c+dx} dx}{2a} + \frac{\int \frac{\cos(2(i e + \frac{\pi}{2}) + 2ifx)}{c+dx} dx}{2a} \\ &= \frac{\log(c+dx)}{2ad} - \frac{\cosh\left(2e - \frac{2cf}{d}\right) \int \frac{\cosh\left(\frac{2cf}{d} + 2fx\right)}{c+dx} dx}{2a} + \frac{\cosh\left(2e - \frac{2cf}{d}\right) \int \frac{\sinh\left(\frac{2cf}{d} + 2fx\right)}{c+dx} dx}{2a} \\ &= -\frac{\cosh\left(2e - \frac{2cf}{d}\right) \operatorname{Chi}\left(\frac{2cf}{d} + 2fx\right)}{2ad} + \frac{\log(c+dx)}{2ad} + \frac{\operatorname{Chi}\left(\frac{2cf}{d} + 2fx\right) \sinh\left(2e - \frac{2cf}{d}\right)}{2ad} \end{aligned}$$

Mathematica [A] time = 0.250206, size = 122, normalized size = 0.78

$$\frac{\operatorname{csch}(e+fx)(\sinh(fx) + \cosh(fx)) \left(\operatorname{Chi}\left(\frac{2f(c+dx)}{d}\right) \left(\sinh\left(e - \frac{2cf}{d}\right) - \cosh\left(e - \frac{2cf}{d}\right) \right) + \operatorname{Shi}\left(\frac{2f(c+dx)}{d}\right) \left(\cosh\left(e - \frac{2cf}{d}\right) - \sinh\left(e - \frac{2cf}{d}\right) \right) \right)}{2ad(\coth(e+fx) + 1)}$$

Antiderivative was successfully verified.

[In] Integrate[1/((c + d*x)*(a + a*Coth[e + f*x])),x]

[Out] (Csch[e + f*x]*(Cosh[f*x] + Sinh[f*x])*(Log[f*(c + d*x)]*(Cosh[e] + Sinh[e]) + CoshIntegral[(2*f*(c + d*x))/d]*(-Cosh[e - (2*c*f)/d] + Sinh[e - (2*c*f)/d]) + (Cosh[e - (2*c*f)/d] - Sinh[e - (2*c*f)/d])*SinhIntegral[(2*f*(c + d*x))/d]))/(2*a*d*(1 + Coth[e + f*x]))

Maple [A] time = 0.182, size = 61, normalized size = 0.4

$$\frac{\ln(dx+c)}{2da} + \frac{1}{2da} e^{2\frac{cf-de}{d}} \operatorname{Ei}\left(1, 2fx + 2e + 2\frac{cf-de}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(d*x+c)/(a+a*coth(f*x+e)),x)`

[Out] `1/2*ln(d*x+c)/d/a+1/2/a/d*exp(2*(c*f-d*e)/d)*Ei(1,2*f*x+2*e+2*(c*f-d*e)/d)`

Maxima [A] time = 1.75167, size = 65, normalized size = 0.41

$$\frac{e^{\left(-2e+\frac{2cf}{d}\right)} E_1\left(\frac{2(dx+c)f}{d}\right)}{2ad} + \frac{\log(dx+c)}{2ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(d*x+c)/(a+a*coth(f*x+e)),x, algorithm="maxima")`

[Out] `1/2*e^(-2*e + 2*c*f/d)*exp_integral_e(1, 2*(d*x + c)*f/d)/(a*d) + 1/2*log(d*x + c)/(a*d)`

Fricas [A] time = 2.18103, size = 166, normalized size = 1.06

$$\frac{\operatorname{Ei}\left(-\frac{2(dfxc+cf)}{d}\right) \cosh\left(-\frac{2(de-cf)}{d}\right) + \operatorname{Ei}\left(-\frac{2(dfxc+cf)}{d}\right) \sinh\left(-\frac{2(de-cf)}{d}\right) - \log(dx+c)}{2ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(d*x+c)/(a+a*coth(f*x+e)),x, algorithm="fricas")`

[Out] `-1/2*(Ei(-2*(d*f*x + c*f)/d)*cosh(-2*(d*e - c*f)/d) + Ei(-2*(d*f*x + c*f)/d)*sinh(-2*(d*e - c*f)/d) - log(d*x + c))/(a*d)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{c \coth(e+fx)+c+dx \coth(e+fx)+dx} dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d*x+c)/(a+a*coth(f*x+e)),x)

[Out] Integral(1/(c*coth(e + f*x) + c + d*x*coth(e + f*x) + d*x), x)/a

Giac [A] time = 1.14969, size = 68, normalized size = 0.43

$$-\frac{\left(\operatorname{Ei}\left(-\frac{2(dfxc+cf)}{d}\right)e^{\left(\frac{2cf}{d}\right)} - e^{(2e)} \log(dx+c)\right)e^{(-2e)}}{2ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d*x+c)/(a+a*coth(f*x+e)),x, algorithm="giac")

[Out] -1/2*(Ei(-2*(d*f*x + c*f)/d)*e^(2*c*f/d) - e^(2*e)*log(d*x + c))*e^(-2*e)/(a*d)

$$3.20 \quad \int \frac{1}{(c+dx)^2(a+a \coth(e+fx))} dx$$

Optimal. Leaf size=159

$$\frac{f \operatorname{Chi}\left(2xf + \frac{2cf}{d}\right) \sinh\left(2e - \frac{2cf}{d}\right)}{ad^2} + \frac{f \operatorname{Chi}\left(2xf + \frac{2cf}{d}\right) \cosh\left(2e - \frac{2cf}{d}\right)}{ad^2} + \frac{f \sinh\left(2e - \frac{2cf}{d}\right) \operatorname{Shi}\left(2xf + \frac{2cf}{d}\right)}{ad^2} - \frac{f \cosh\left(2e - \frac{2cf}{d}\right) \operatorname{Chi}\left(2xf + \frac{2cf}{d}\right)}{ad^2}$$

[Out] (f*Cosh[2*e - (2*c*f)/d]*CoshIntegral[(2*c*f)/d + 2*f*x])/(a*d^2) - 1/(d*(c + d*x)*(a + a*Coth[e + f*x])) - (f*CoshIntegral[(2*c*f)/d + 2*f*x]*Sinh[2*e - (2*c*f)/d])/(a*d^2) - (f*Cosh[2*e - (2*c*f)/d]*SinhIntegral[(2*c*f)/d + 2*f*x])/(a*d^2) + (f*Sinh[2*e - (2*c*f)/d]*SinhIntegral[(2*c*f)/d + 2*f*x])/(a*d^2)

Rubi [A] time = 0.213013, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {3724, 3303, 3298, 3301}

$$\frac{f \operatorname{Chi}\left(2xf + \frac{2cf}{d}\right) \sinh\left(2e - \frac{2cf}{d}\right)}{ad^2} + \frac{f \operatorname{Chi}\left(2xf + \frac{2cf}{d}\right) \cosh\left(2e - \frac{2cf}{d}\right)}{ad^2} + \frac{f \sinh\left(2e - \frac{2cf}{d}\right) \operatorname{Shi}\left(2xf + \frac{2cf}{d}\right)}{ad^2} - \frac{f \cosh\left(2e - \frac{2cf}{d}\right) \operatorname{Chi}\left(2xf + \frac{2cf}{d}\right)}{ad^2}$$

Antiderivative was successfully verified.

[In] Int[1/((c + d*x)^2*(a + a*Coth[e + f*x])),x]

[Out] (f*Cosh[2*e - (2*c*f)/d]*CoshIntegral[(2*c*f)/d + 2*f*x])/(a*d^2) - 1/(d*(c + d*x)*(a + a*Coth[e + f*x])) - (f*CoshIntegral[(2*c*f)/d + 2*f*x]*Sinh[2*e - (2*c*f)/d])/(a*d^2) - (f*Cosh[2*e - (2*c*f)/d]*SinhIntegral[(2*c*f)/d + 2*f*x])/(a*d^2) + (f*Sinh[2*e - (2*c*f)/d]*SinhIntegral[(2*c*f)/d + 2*f*x])/(a*d^2)

Rule 3724

Int[1/(((c_.) + (d_.)*(x_))^(2*((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]))), x_Symbol] :> -Simp[(d*(c + d*x)*(a + b*Tan[e + f*x]))^(-1), x] + (-Dist[f/(a*d), Int[Sin[2*e + 2*f*x]/(c + d*x), x], x] + Dist[f/(b*d), Int[Cos[2*e + 2*f*x]/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2 + b^2, 0]

Rule 3303

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x]

/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]

Rule 3298

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*e - c*f*fz*I, 0]

Rule 3301

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{(c+dx)^2(a+a\coth(e+fx))} dx &= -\frac{1}{d(c+dx)(a+a\coth(e+fx))} - \frac{(if) \int \frac{\sin(2(i e + \frac{\pi}{2}) + 2ifx)}{c+dx} dx}{ad} - \frac{f \int \frac{\cos(2(i e + \frac{\pi}{2}) + 2ifx)}{c+dx}}{ad} \\ &= -\frac{1}{d(c+dx)(a+a\coth(e+fx))} + \frac{\left(f \cosh\left(2e - \frac{2cf}{d}\right)\right) \int \frac{\cosh\left(\frac{2cf}{d} + 2fx\right)}{c+dx} dx}{ad} - \frac{\left(f \cosh\left(2e - \frac{2cf}{d}\right)\right) \int \frac{\sinh\left(\frac{2cf}{d} + 2fx\right)}{c+dx} dx}{ad} \\ &= \frac{f \cosh\left(2e - \frac{2cf}{d}\right) \text{Chi}\left(\frac{2cf}{d} + 2fx\right)}{ad^2} - \frac{1}{d(c+dx)(a+a\coth(e+fx))} - \frac{f \text{Chi}\left(\frac{2cf}{d} + 2fx\right)}{ad} \end{aligned}$$

Mathematica [A] time = 0.764114, size = 206, normalized size = 1.3

$$\frac{\text{csch}(e+fx) \left(\sinh\left(\frac{cf}{d}\right) + \cosh\left(\frac{cf}{d}\right) \right) \left(2f(c+dx) \text{Chi}\left(\frac{2f(c+dx)}{d}\right) \left(\sinh\left(e - \frac{f(c+dx)}{d}\right) - \cosh\left(e - \frac{f(c+dx)}{d}\right) \right) + 2f(c+dx) \text{Shi}\left(\frac{2f(c+dx)}{d}\right) \right)}{d^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/((c + d*x)^2*(a + a*Coth[e + f*x])),x]

[Out] -(Csch[e + f*x]*(Cosh[(c*f)/d] + Sinh[(c*f)/d])*(d*(Cosh[e + f*(-(c/d) + x)] - Cosh[e + f*(c/d + x)] + Sinh[e + f*(-(c/d) + x)] + Sinh[e + f*(c/d + x)]) + 2*f*(c + d*x)*CoshIntegral[(2*f*(c + d*x))/d]*(-Cosh[e - (f*(c + d*x))/d] + Sinh[e - (f*(c + d*x))/d]) + 2*f*(c + d*x)*(Cosh[e - (f*(c + d*x))/d]

- Sinh[e - (f*(c + d*x))/d])*SinhIntegral[(2*f*(c + d*x))/d]))/(2*a*d^2*(c + d*x)*(1 + Coth[e + f*x]))

Maple [A] time = 0.203, size = 91, normalized size = 0.6

$$-\frac{1}{2(dx+c)ad} + \frac{fe^{-2fx-2e}}{2da(dfx+cf)} - \frac{f}{ad^2}e^{2\frac{cf-de}{d}}\text{Ei}\left(1, 2fx+2e+2\frac{cf-de}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(d*x+c)^2/(a+a*coth(f*x+e)),x)

[Out] -1/2/d/a/(d*x+c)+1/2/a*f*exp(-2*f*x-2*e)/d/(d*f*x+c*f)-1/a*f/d^2*exp(2*(c*f-d*e)/d)*Ei(1,2*f*x+2*e+2*(c*f-d*e)/d)

Maxima [A] time = 2.25829, size = 76, normalized size = 0.48

$$-\frac{1}{2(ad^2x+acd)} + \frac{e^{\left(-2e+\frac{2cf}{d}\right)}E_2\left(\frac{2(dx+c)f}{d}\right)}{2(dx+c)ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d*x+c)^2/(a+a*coth(f*x+e)),x, algorithm="maxima")

[Out] -1/2/(a*d^2*x + a*c*d) + 1/2*e^(-2*e + 2*c*f/d)*exp_integral_e(2, 2*(d*x + c)*f/d)/((d*x + c)*a*d)

Fricas [A] time = 2.13375, size = 498, normalized size = 3.13

$$\frac{(dfx+cf)\text{Ei}\left(-\frac{2(dfx+cf)}{d}\right)\cosh(fx+e)\cosh\left(-\frac{2(de-cf)}{d}\right) + (dfx+cf)\text{Ei}\left(-\frac{2(dfx+cf)}{d}\right)\cosh(fx+e)\sinh\left(-\frac{2(de-cf)}{d}\right)}{(ad^3x+acd^2)\cosh(fx+e) + (a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d*x+c)^2/(a+a*coth(f*x+e)),x, algorithm="fricas")

[Out] $((d*f*x + c*f)*Ei(-2*(d*f*x + c*f)/d)*\cosh(f*x + e)*\cosh(-2*(d*e - c*f)/d) + (d*f*x + c*f)*Ei(-2*(d*f*x + c*f)/d)*\cosh(f*x + e)*\sinh(-2*(d*e - c*f)/d) + ((d*f*x + c*f)*Ei(-2*(d*f*x + c*f)/d)*\cosh(-2*(d*e - c*f)/d) + (d*f*x + c*f)*Ei(-2*(d*f*x + c*f)/d)*\sinh(-2*(d*e - c*f)/d) - d)*\sinh(f*x + e))/((a*d^3*x + a*c*d^2)*\cosh(f*x + e) + (a*d^3*x + a*c*d^2)*\sinh(f*x + e))$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{c^2 \coth(e+fx)+c^2+2cdx \coth(e+fx)+2cdx+d^2x^2 \coth(e+fx)+d^2x^2} dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d*x+c)**2/(a+a*coth(f*x+e)),x)

[Out] Integral(1/(c**2*coth(e + f*x) + c**2 + 2*c*d*x*coth(e + f*x) + 2*c*d*x + d**2*x**2*coth(e + f*x) + d**2*x**2), x)/a

Giac [A] time = 1.33347, size = 149, normalized size = 0.94

$$\frac{2dfxEi\left(-\frac{2(dfxc+f)}{d}\right)e^{\left(\frac{2cf}{d}-2e\right)} + 2cfEi\left(-\frac{2(dfxc+f)}{d}\right)e^{\left(\frac{2cf}{d}-2e\right)} + de^{(-2fx-2e)}}{2(d^3x + cd^2)a} - \frac{1}{2(dx + c)ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d*x+c)^2/(a+a*coth(f*x+e)),x, algorithm="giac")

[Out] $1/2*(2*d*f*x*Ei(-2*(d*f*x + c*f)/d)*e^{(2*c*f/d - 2*e)} + 2*c*f*Ei(-2*(d*f*x + c*f)/d)*e^{(2*c*f/d - 2*e)} + d*e^{(-2*f*x - 2*e)})/((d^3*x + c*d^2)*a) - 1/2/((d*x + c)*a*d)$

$$3.21 \quad \int \frac{1}{(c+dx)^3(a+a \coth(e+fx))} dx$$

Optimal. Leaf size=211

$$\frac{f^2 \operatorname{Chi}\left(2xf + \frac{2cf}{d}\right) \sinh\left(2e - \frac{2cf}{d}\right)}{ad^3} - \frac{f^2 \operatorname{Chi}\left(2xf + \frac{2cf}{d}\right) \cosh\left(2e - \frac{2cf}{d}\right)}{ad^3} - \frac{f^2 \sinh\left(2e - \frac{2cf}{d}\right) \operatorname{Shi}\left(2xf + \frac{2cf}{d}\right)}{ad^3} + \frac{f^2 \operatorname{coth}\left(2e - \frac{2cf}{d}\right) \operatorname{Chi}\left(2xf + \frac{2cf}{d}\right)}{ad^3}$$

[Out] $-f/(2*a*d^2*(c + d*x)) - (f^2*\operatorname{Cosh}[2*e - (2*c*f)/d]*\operatorname{CoshIntegral}[(2*c*f)/d + 2*f*x])/(a*d^3) - 1/(2*d*(c + d*x)^2*(a + a*\operatorname{Coth}[e + f*x])) + f/(d^2*(c + d*x)*(a + a*\operatorname{Coth}[e + f*x])) + (f^2*\operatorname{CoshIntegral}[(2*c*f)/d + 2*f*x]*\operatorname{Sinh}[2*e - (2*c*f)/d])/(a*d^3) + (f^2*\operatorname{Cosh}[2*e - (2*c*f)/d]*\operatorname{SinhIntegral}[(2*c*f)/d + 2*f*x])/(a*d^3) - (f^2*\operatorname{Sinh}[2*e - (2*c*f)/d]*\operatorname{SinhIntegral}[(2*c*f)/d + 2*f*x])/(a*d^3)$

Rubi [A] time = 0.300802, antiderivative size = 211, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {3725, 3724, 3303, 3298, 3301}

$$\frac{f^2 \operatorname{Chi}\left(2xf + \frac{2cf}{d}\right) \sinh\left(2e - \frac{2cf}{d}\right)}{ad^3} - \frac{f^2 \operatorname{Chi}\left(2xf + \frac{2cf}{d}\right) \cosh\left(2e - \frac{2cf}{d}\right)}{ad^3} - \frac{f^2 \sinh\left(2e - \frac{2cf}{d}\right) \operatorname{Shi}\left(2xf + \frac{2cf}{d}\right)}{ad^3} + \frac{f^2 \operatorname{coth}\left(2e - \frac{2cf}{d}\right) \operatorname{Chi}\left(2xf + \frac{2cf}{d}\right)}{ad^3}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[1/((c + d*x)^3*(a + a*\operatorname{Coth}[e + f*x])), x]$

[Out] $-f/(2*a*d^2*(c + d*x)) - (f^2*\operatorname{Cosh}[2*e - (2*c*f)/d]*\operatorname{CoshIntegral}[(2*c*f)/d + 2*f*x])/(a*d^3) - 1/(2*d*(c + d*x)^2*(a + a*\operatorname{Coth}[e + f*x])) + f/(d^2*(c + d*x)*(a + a*\operatorname{Coth}[e + f*x])) + (f^2*\operatorname{CoshIntegral}[(2*c*f)/d + 2*f*x]*\operatorname{Sinh}[2*e - (2*c*f)/d])/(a*d^3) + (f^2*\operatorname{Cosh}[2*e - (2*c*f)/d]*\operatorname{SinhIntegral}[(2*c*f)/d + 2*f*x])/(a*d^3) - (f^2*\operatorname{Sinh}[2*e - (2*c*f)/d]*\operatorname{SinhIntegral}[(2*c*f)/d + 2*f*x])/(a*d^3)$

Rule 3725

$\operatorname{Int}[(c_. + (d_.)*(x_.))^(m_.)/((a_.) + (b_.)*\tan[(e_.) + (f_.)*(x_.)]), x_Sym\ bol] \rightarrow \operatorname{Simp}[f*(c + d*x)^(m + 2)/(b*d^2*(m + 1)*(m + 2)), x] + (\operatorname{Dist}[(2*b*f)/(a*d*(m + 1)), \operatorname{Int}[(c + d*x)^(m + 1)/(a + b*\tan[e + f*x]), x], x] + \operatorname{Simp}[(c + d*x)^(m + 1)/(d*(m + 1)*(a + b*\tan[e + f*x])), x]) /; \operatorname{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \operatorname{EqQ}[a^2 + b^2, 0] \&\& \operatorname{LtQ}[m, -1] \&\& \operatorname{NeQ}[m, -2]$

Rule 3724

```
Int[1/(((c_.) + (d_.)*(x_))2*((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])), x_Symbol]
:> -Simp[(d*(c + d*x)*(a + b*Tan[e + f*x]))-1, x] + (-Dist[f/(a*d), Int[Sin[2*e + 2*f*x]/(c + d*x), x], x] + Dist[f/(b*d), Int[Cos[2*e + 2*f*x]/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a2 + b2, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{(c+dx)^3(a+a \coth(e+fx))} dx &= -\frac{f}{2ad^2(c+dx)} - \frac{1}{2d(c+dx)^2(a+a \coth(e+fx))} - \frac{f \int \frac{1}{(c+dx)^2(a+a \coth(e+fx))} dx}{d} \\ &= -\frac{f}{2ad^2(c+dx)} - \frac{1}{2d(c+dx)^2(a+a \coth(e+fx))} + \frac{f}{d^2(c+dx)(a+a \coth(e+fx))} \\ &= -\frac{f}{2ad^2(c+dx)} - \frac{1}{2d(c+dx)^2(a+a \coth(e+fx))} + \frac{f}{d^2(c+dx)(a+a \coth(e+fx))} \\ &= -\frac{f}{2ad^2(c+dx)} - \frac{f^2 \cosh\left(2e - \frac{2cf}{d}\right) \text{Chi}\left(\frac{2cf}{d} + 2fx\right)}{ad^3} - \frac{1}{2d(c+dx)^2(a+a \coth(e+fx))} \end{aligned}$$

Mathematica [A] time = 1.07348, size = 265, normalized size = 1.26

$$\operatorname{csch}(e + fx) \left(\sinh\left(\frac{cf}{d}\right) + \cosh\left(\frac{cf}{d}\right) \right) \left(4f^2(c + dx)^2 \operatorname{Chi}\left(\frac{2f(c+dx)}{d}\right) \left(\cosh\left(e - \frac{f(c+dx)}{d}\right) - \sinh\left(e - \frac{f(c+dx)}{d}\right) \right) + 4f^2(c + dx) \right)$$

Antiderivative was successfully verified.

[In] Integrate[1/((c + d*x)^3*(a + a*Coth[e + f*x])),x]

[Out] $-(\operatorname{Csch}[e + f*x] * (\operatorname{Cosh}[(c*f)/d] + \operatorname{Sinh}[(c*f)/d]) * (d * (d * \operatorname{Cosh}[e + f * (-c/d) + x]) + (-d + 2*c*f + 2*d*f*x) * \operatorname{Cosh}[e + f*(c/d + x)] + d * \operatorname{Sinh}[e + f * (-c/d) + x]) + d * \operatorname{Sinh}[e + f*(c/d + x)] - 2*c*f * \operatorname{Sinh}[e + f*(c/d + x)] - 2*d*f*x * \operatorname{Sinh}[e + f*(c/d + x)]) + 4*f^2*(c + d*x)^2 * \operatorname{CoshIntegral}[(2*f*(c + d*x))/d] * (\operatorname{Cosh}[e - (f*(c + d*x))/d] - \operatorname{Sinh}[e - (f*(c + d*x))/d]) + 4*f^2*(c + d*x)^2 * (-\operatorname{Cosh}[e - (f*(c + d*x))/d] + \operatorname{Sinh}[e - (f*(c + d*x))/d]) * \operatorname{SinhIntegral}[(2*f*(c + d*x))/d]) / (4*a*d^3*(c + d*x)^2*(1 + \operatorname{Coth}[e + f*x]))$

Maple [A] time = 0.219, size = 210, normalized size = 1.

$$-\frac{1}{4da(dx+c)^2} - \frac{f^3 e^{-2fx-2e} x}{2da(d^2 f^2 x^2 + 2cdf^2 x + c^2 f^2)} - \frac{f^3 e^{-2fx-2e} c}{2ad^2(d^2 f^2 x^2 + 2cdf^2 x + c^2 f^2)} + \frac{f^2 e^{-2fx-2e}}{4da(d^2 f^2 x^2 + 2cdf^2 x + c^2 f^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(d*x+c)^3/(a+a*coth(f*x+e)),x)

[Out] $-1/4/d/a/(d*x+c)^2 - 1/2/a*f^3*\exp(-2*f*x-2*e)/d/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2)*x - 1/2/a*f^3*\exp(-2*f*x-2*e)/d^2/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2)*c + 1/4/a*f^2*\exp(-2*f*x-2*e)/d/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2) + 1/a*f^2/d^3*\exp(2*(c*f-d*e)/d)*\operatorname{Ei}(1,2*f*x+2*e+2*(c*f-d*e)/d)$

Maxima [A] time = 3.05869, size = 92, normalized size = 0.44

$$-\frac{1}{4(ad^3x^2 + 2acd^2x + ac^2d)} + \frac{e^{\left(-2e + \frac{2cf}{d}\right)} E_3\left(\frac{2(dx+c)f}{d}\right)}{2(dx+c)^2 ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d*x+c)^3/(a+a*coth(f*x+e)),x, algorithm="maxima")

[Out] $-1/4/(a*d^3*x^2 + 2*a*c*d^2*x + a*c^2*d) + 1/2*e^{(-2*e + 2*c*f/d)*exp_integral_e(3, 2*(d*x + c)*f/d)/((d*x + c)^2*a*d)}$

Fricas [A] time = 2.16983, size = 748, normalized size = 3.55

$$2\left(d^2f^2x^2 + 2cdf^2x + c^2f^2\right)Ei\left(-\frac{2(dfxc+f)}{d}\right)\cosh(fx + e)\sinh\left(-\frac{2(de-cf)}{d}\right) + \left(d^2fx + cdf + 2\left(d^2f^2x^2 + 2cdf^2x + c^2f^2\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d*x+c)^3/(a+a*coth(f*x+e)),x, algorithm="fricas")

[Out] $-1/2*(2*(d^2*f^2*x^2 + 2*c*d*f^2*x + c^2*f^2)*Ei(-2*(d*f*x + c*f)/d)*\cosh(f*x + e)*\sinh(-2*(d*e - c*f)/d) + (d^2*f*x + c*d*f + 2*(d^2*f^2*x^2 + 2*c*d*f^2*x + c^2*f^2)*Ei(-2*(d*f*x + c*f)/d)*\cosh(-2*(d*e - c*f)/d))*\cosh(f*x + e) - (d^2*f*x + c*d*f - 2*(d^2*f^2*x^2 + 2*c*d*f^2*x + c^2*f^2)*Ei(-2*(d*f*x + c*f)/d)*\cosh(-2*(d*e - c*f)/d) - 2*(d^2*f^2*x^2 + 2*c*d*f^2*x + c^2*f^2)*Ei(-2*(d*f*x + c*f)/d)*\sinh(-2*(d*e - c*f)/d) - d^2*\sinh(f*x + e))/((a*d^5*x^2 + 2*a*c*d^4*x + a*c^2*d^3)*\cosh(f*x + e) + (a*d^5*x^2 + 2*a*c*d^4*x + a*c^2*d^3)*\sinh(f*x + e))$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{c^3 \coth(e+fx)+c^3+3c^2dx \coth(e+fx)+3c^2dx+3cd^2x^2 \coth(e+fx)+3cd^2x^2+d^3x^3 \coth(e+fx)+d^3x^3} dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d*x+c)**3/(a+a*coth(f*x+e)),x)

[Out] $Integral(1/(c**3*\coth(e + f*x) + c**3 + 3*c**2*d*x*\coth(e + f*x) + 3*c**2*d*x + 3*c*d**2*x**2*\coth(e + f*x) + 3*c*d**2*x**2 + d**3*x**3*\coth(e + f*x) + d**3*x**3), x)/a$

Giac [A] time = 1.14919, size = 242, normalized size = 1.15

$$\frac{4d^2f^2x^2\text{Ei}\left(-\frac{2(dfxc+f)}{d}\right)e^{\left(\frac{2cf}{d}\right)} + 8cdf^2x\text{Ei}\left(-\frac{2(dfxc+f)}{d}\right)e^{\left(\frac{2cf}{d}\right)} + 4c^2f^2\text{Ei}\left(-\frac{2(dfxc+f)}{d}\right)e^{\left(\frac{2cf}{d}\right)} + 2d^2fxe^{(-2fx)} + 2cdf}{4(ad^5x^2e^{(2e)} + 2acd^4xe^{(2e)} + ac^2d^3e^{(2e)})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d*x+c)^3/(a+a*coth(f*x+e)),x, algorithm="giac")

[Out] -1/4*(4*d^2*f^2*x^2*Ei(-2*(d*f*x + c*f)/d)*e^(2*c*f/d) + 8*c*d*f^2*x*Ei(-2*(d*f*x + c*f)/d)*e^(2*c*f/d) + 4*c^2*f^2*Ei(-2*(d*f*x + c*f)/d)*e^(2*c*f/d) + 2*d^2*f*x*e^(-2*f*x) + 2*c*d*f*e^(-2*f*x) - d^2*e^(-2*f*x) + d^2*e^(2*e))/(a*d^5*x^2*e^(2*e) + 2*a*c*d^4*x*e^(2*e) + a*c^2*d^3*e^(2*e))

$$3.22 \quad \int \frac{(c+dx)^3}{(a+a \coth(e+fx))^2} dx$$

Optimal. Leaf size=230

$$-\frac{3d^2(c+dx)e^{-4e-4fx}}{128a^2f^3} + \frac{3d^2(c+dx)e^{-2e-2fx}}{8a^2f^3} - \frac{3d(c+dx)^2e^{-4e-4fx}}{64a^2f^2} + \frac{3d(c+dx)^2e^{-2e-2fx}}{8a^2f^2} - \frac{(c+dx)^3e^{-4e-4fx}}{16a^2f} + \frac{(c+dx)^3e^{-2e-2fx}}{4a^2f}$$

[Out] $(-3*d^3*E^{(-4*e - 4*f*x)})/(512*a^2*f^4) + (3*d^3*E^{(-2*e - 2*f*x)})/(16*a^2*f^4) - (3*d^2*E^{(-4*e - 4*f*x)}*(c + d*x))/(128*a^2*f^3) + (3*d^2*E^{(-2*e - 2*f*x)}*(c + d*x))/(8*a^2*f^3) - (3*d*E^{(-4*e - 4*f*x)}*(c + d*x)^2)/(64*a^2*f^2) + (3*d*E^{(-2*e - 2*f*x)}*(c + d*x)^2)/(8*a^2*f^2) - (E^{(-4*e - 4*f*x)}*(c + d*x)^3)/(16*a^2*f) + (E^{(-2*e - 2*f*x)}*(c + d*x)^3)/(4*a^2*f) + (c + d*x)^4/(16*a^2*d)$

Rubi [A] time = 0.26818, antiderivative size = 230, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.15$, Rules used = {3729, 2176, 2194}

$$-\frac{3d^2(c+dx)e^{-4e-4fx}}{128a^2f^3} + \frac{3d^2(c+dx)e^{-2e-2fx}}{8a^2f^3} - \frac{3d(c+dx)^2e^{-4e-4fx}}{64a^2f^2} + \frac{3d(c+dx)^2e^{-2e-2fx}}{8a^2f^2} - \frac{(c+dx)^3e^{-4e-4fx}}{16a^2f} + \frac{(c+dx)^3e^{-2e-2fx}}{4a^2f}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)^3/(a + a*Coth[e + f*x])^2,x]

[Out] $(-3*d^3*E^{(-4*e - 4*f*x)})/(512*a^2*f^4) + (3*d^3*E^{(-2*e - 2*f*x)})/(16*a^2*f^4) - (3*d^2*E^{(-4*e - 4*f*x)}*(c + d*x))/(128*a^2*f^3) + (3*d^2*E^{(-2*e - 2*f*x)}*(c + d*x))/(8*a^2*f^3) - (3*d*E^{(-4*e - 4*f*x)}*(c + d*x)^2)/(64*a^2*f^2) + (3*d*E^{(-2*e - 2*f*x)}*(c + d*x)^2)/(8*a^2*f^2) - (E^{(-4*e - 4*f*x)}*(c + d*x)^3)/(16*a^2*f) + (E^{(-2*e - 2*f*x)}*(c + d*x)^3)/(4*a^2*f) + (c + d*x)^4/(16*a^2*d)$

Rule 3729

Int[((c_.) + (d_.)*(x_))^(m_)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (1/(2*a) + E^((2*a*(e + f*x))/b)/(2*a))^(n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && ILtQ[n, 0]

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] := Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] && !$UseGamma == True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] := Simp[(F^(c*(a +
b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(c + dx)^3}{(a + a \coth(e + fx))^2} dx &= \int \left(\frac{(c + dx)^3}{4a^2} + \frac{e^{-4e-4fx}(c + dx)^3}{4a^2} - \frac{e^{-2e-2fx}(c + dx)^3}{2a^2} \right) dx \\
&= \frac{(c + dx)^4}{16a^2d} + \frac{\int e^{-4e-4fx}(c + dx)^3 dx}{4a^2} - \frac{\int e^{-2e-2fx}(c + dx)^3 dx}{2a^2} \\
&= -\frac{e^{-4e-4fx}(c + dx)^3}{16a^2f} + \frac{e^{-2e-2fx}(c + dx)^3}{4a^2f} + \frac{(c + dx)^4}{16a^2d} + \frac{(3d) \int e^{-4e-4fx}(c + dx)^2 dx}{16a^2f} - \frac{(3d) \int e^{-2e-2fx}(c + dx)^2 dx}{16a^2f} \\
&= -\frac{3de^{-4e-4fx}(c + dx)^2}{64a^2f^2} + \frac{3de^{-2e-2fx}(c + dx)^2}{8a^2f^2} - \frac{e^{-4e-4fx}(c + dx)^3}{16a^2f} + \frac{e^{-2e-2fx}(c + dx)^3}{4a^2f} + \frac{(c + dx)^4}{16a^2d} \\
&= -\frac{3d^2e^{-4e-4fx}(c + dx)}{128a^2f^3} + \frac{3d^2e^{-2e-2fx}(c + dx)}{8a^2f^3} - \frac{3de^{-4e-4fx}(c + dx)^2}{64a^2f^2} + \frac{3de^{-2e-2fx}(c + dx)^2}{8a^2f^2} \\
&= -\frac{3d^3e^{-4e-4fx}}{512a^2f^4} + \frac{3d^3e^{-2e-2fx}}{16a^2f^4} - \frac{3d^2e^{-4e-4fx}(c + dx)}{128a^2f^3} + \frac{3d^2e^{-2e-2fx}(c + dx)}{8a^2f^3} - \frac{3de^{-4e-4fx}(c + dx)^2}{64a^2f^2}
\end{aligned}$$

Mathematica [A] time = 1.02336, size = 420, normalized size = 1.83

$$\operatorname{csch}^2(e + fx)(\sinh(fx) + \cosh(fx))^2 \left(f^4 x (6c^2 dx + 4c^3 + 4cd^2 x^2 + d^3 x^3) (\sinh(2e) + \cosh(2e)) + \frac{1}{32} (\sinh(2e) - \cosh(2e)) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3/(a + a*Coth[e + f*x])^2,x]
```

```
[Out] (Csch[e + f*x]^2*(Cosh[f*x] + Sinh[f*x])^2*((4*c^3*f^3 + 6*c^2*d*f^2*(1 + 2
*f*x) + 6*c*d^2*f*(1 + 2*f*x + 2*f^2*x^2) + d^3*(3 + 6*f*x + 6*f^2*x^2 + 4*
f^3*x^3))*Cosh[2*f*x] + ((32*c^3*f^3 + 24*c^2*d*f^2*(1 + 4*f*x) + 12*c*d^2*
f*(1 + 4*f*x + 8*f^2*x^2) + d^3*(3 + 12*f*x + 24*f^2*x^2 + 32*f^3*x^3))*Cos
```

$$\frac{h[4*f*x]*(-\text{Cosh}[2*e] + \text{Sinh}[2*e])}{32} + f^4*x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3)*(\text{Cosh}[2*e] + \text{Sinh}[2*e]) - (4*c^3*f^3 + 6*c^2*d*f^2*(1 + 2*f*x) + 6*c*d^2*f*(1 + 2*f*x + 2*f^2*x^2) + d^3*(3 + 6*f*x + 6*f^2*x^2 + 4*f^3*x^3))*\text{Sinh}[2*f*x] + ((32*c^3*f^3 + 24*c^2*d*f^2*(1 + 4*f*x) + 12*c*d^2*f*(1 + 4*f*x + 8*f^2*x^2) + d^3*(3 + 12*f*x + 24*f^2*x^2 + 32*f^3*x^3))*(\text{Cosh}[2*e] - \text{Sinh}[2*e]))*\text{Sinh}[4*f*x])/32)/(16*a^2*f^4*(1 + \text{Coth}[e + f*x])^2)$$

Maple [B] time = 0.108, size = 2200, normalized size = 9.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((d*x+c)^3/(a+a*\text{coth}(f*x+e))^2,x)$

[Out] $\frac{1}{a^2} \frac{1}{f} (\frac{6}{f^2} \frac{d^2 e}{c} * (\frac{1}{2} (f*x+e) * \cosh(f*x+e) * \sinh(f*x+e) - \frac{1}{4} (f*x+e)^2 - \frac{1}{4} \cosh(f*x+e)^2) - 3 \frac{d^2 e^2}{f^2} * c * (\frac{1}{2} \cosh(f*x+e) * \sinh(f*x+e) - \frac{1}{2} f*x - \frac{1}{2} e) + 3 \frac{d e}{f} * c^2 * (\frac{1}{2} \cosh(f*x+e) * \sinh(f*x+e) - \frac{1}{2} f*x - \frac{1}{2} e) + 6 \frac{c^2}{f} * d * e * (\frac{1}{4} * \sinh(f*x+e)^2 * \cosh(f*x+e)^2 - \frac{1}{4} \cosh(f*x+e)^2) - 6 \frac{c^2}{f} * d * e * (\frac{1}{4} * \cosh(f*x+e)^3 * \sinh(f*x+e) - \frac{1}{8} \cosh(f*x+e) * \sinh(f*x+e) - \frac{1}{8} f*x - \frac{1}{8} e) + 12 \frac{d^2 e}{f^2} * c * (\frac{1}{4} * (f*x+e) * \sinh(f*x+e)^2 * \cosh(f*x+e)^2 - \frac{1}{4} * (f*x+e) * \cosh(f*x+e)^2 - \frac{1}{16} \cosh(f*x+e)^3 * \sinh(f*x+e) + \frac{5}{32} \cosh(f*x+e) * \sinh(f*x+e) + \frac{5}{32} f*x + \frac{5}{32} e) - 12 \frac{d^2 e}{f^2} * c * (\frac{1}{4} * (f*x+e) * \sinh(f*x+e) * \cosh(f*x+e)^3 - \frac{1}{8} * (f*x+e) * \cosh(f*x+e) * \sinh(f*x+e) - \frac{1}{16} * (f*x+e)^2 - \frac{1}{16} * \sinh(f*x+e)^2 * \cosh(f*x+e)^2) + 6 \frac{d^2 e^2}{f^2} * c * (\frac{1}{4} * \cosh(f*x+e)^3 * \sinh(f*x+e) - \frac{1}{8} \cosh(f*x+e) * \sinh(f*x+e) - \frac{1}{8} f*x - \frac{1}{8} e) - 6 \frac{d^2 e^2}{f^2} * c * (\frac{1}{4} * \sinh(f*x+e)^2 * \cosh(f*x+e)^2 - \frac{1}{4} \cosh(f*x+e)^2) - 3 \frac{d^2 e^2}{f^2} * c * (\frac{1}{2} * (f*x+e)^2 * \cosh(f*x+e) * \sinh(f*x+e) - \frac{1}{6} * (f*x+e)^3 - \frac{1}{2} * (f*x+e) * \cosh(f*x+e)^2 + \frac{1}{4} * \cosh(f*x+e) * \sinh(f*x+e) + \frac{1}{4} f*x + \frac{1}{4} e) - 3 \frac{d^3 e^2}{f^3} * (\frac{1}{2} * (f*x+e) * \cosh(f*x+e) * \sinh(f*x+e) - \frac{1}{4} * (f*x+e)^2 - \frac{1}{4} \cosh(f*x+e)^2) - 3 \frac{d^3 e^2}{f^3} * c * (\frac{1}{2} * (f*x+e) * \cosh(f*x+e) * \sinh(f*x+e) - \frac{1}{4} * (f*x+e)^2 - \frac{1}{4} \cosh(f*x+e)^2) + d^3 \frac{e^3}{f^3} * (\frac{1}{2} * \cosh(f*x+e) * \sinh(f*x+e) - \frac{1}{2} f*x - \frac{1}{2} e) + 3 \frac{d^3 e^3}{f^3} * (\frac{1}{2} * (f*x+e)^2 * \cosh(f*x+e) * \sinh(f*x+e) - \frac{1}{6} * (f*x+e)^3 - \frac{1}{2} * (f*x+e) * \cosh(f*x+e)^2 + \frac{1}{4} * \cosh(f*x+e) * \sinh(f*x+e) + \frac{1}{4} f*x + \frac{1}{4} e) - \frac{1}{f^3} \frac{d^3 e^3}{c} * (\frac{1}{2} * (f*x+e)^3 * \cosh(f*x+e) * \sinh(f*x+e) - \frac{1}{8} * (f*x+e)^4 - \frac{3}{4} * (f*x+e)^2 * \cosh(f*x+e)^2 + \frac{3}{4} * (f*x+e) * \cosh(f*x+e) * \sinh(f*x+e) + \frac{3}{8} * (f*x+e)^2 - \frac{3}{8} \cosh(f*x+e)^2) + 2 \frac{d^3 e^3}{f^3} * (\frac{1}{4} * (f*x+e)^3 * \sinh(f*x+e) * \cosh(f*x+e)^3 - \frac{1}{8} * (f*x+e)^3 * \cosh(f*x+e) * \sinh(f*x+e) - \frac{1}{32} * (f*x+e)^4 - \frac{3}{16} * (f*x+e)^2 * \sinh(f*x+e)^2 * \cosh(f*x+e)^2 + \frac{3}{32} * (f*x+e) * \sinh(f*x+e) * \cosh(f*x+e)^3 - \frac{3}{64} * (f*x+e) * \cosh(f*x+e) * \sinh(f*x+e) - \frac{3}{128} * (f*x+e)^2 - \frac{3}{128} * \sinh(f*x+e)^2 * \cosh(f*x+e)^2) - 2 \frac{d^3 e^3}{f^3} * (\frac{1}{4} * (f*x+e)^3 * \sinh(f*x+e)^2 * \cosh(f*x+e)^2 - \frac{1}{4} * (f*x+e)^3 * \cosh(f*x+e)^2 - \frac{3}{16} * (f*x+e)^2 * \sinh(f*x+e) * \cosh(f*x+e)^3 + \frac{15}{32} * (f*x+e)^2 * \cosh(f*x+e) * \sinh(f*x+e) + \frac{5}{32} * (f*x+e)^3 + \frac{3}{32} * (f*x+e) * \sinh(f*x+e)^2 * \cosh(f*x+e)^2 - \frac{3}{128} * \cosh(f*x+e)^3 * \sinh(f*x+e) + \frac{51}{256} * \cosh(f*x+e) * \sinh(f*x+e) + \frac{51}{256} * \sinh(f*x+e)^2 * \cosh(f*x+e)^2)$

$f*x+51/256*e^{-3/8*(f*x+e)*\cosh(f*x+e)^2}+6/f^3*d^3*e*(1/4*(f*x+e)^2*\sinh(f*x+e)^2*\cosh(f*x+e)^2-1/4*(f*x+e)^2*\cosh(f*x+e)^2-1/8*(f*x+e)*\sinh(f*x+e)*\cosh(f*x+e)^3+5/16*(f*x+e)*\cosh(f*x+e)*\sinh(f*x+e)+5/32*(f*x+e)^2+1/32*\sinh(f*x+e)^2*\cosh(f*x+e)^2-1/8*\cosh(f*x+e)^2)-6/f^3*d^3*e*(1/4*(f*x+e)^2*\sinh(f*x+e)*\cosh(f*x+e)^3-1/8*(f*x+e)^2*\cosh(f*x+e)*\sinh(f*x+e)-1/24*(f*x+e)^3-1/8*(f*x+e)*\sinh(f*x+e)^2*\cosh(f*x+e)^2+1/32*\cosh(f*x+e)^3*\sinh(f*x+e)-1/64*\cosh(f*x+e)*\sinh(f*x+e)-1/64*f*x-1/64*e)+6/f^3*d^3*e^2*(1/4*(f*x+e)*\sinh(f*x+e)*\cosh(f*x+e)^3-1/8*(f*x+e)*\cosh(f*x+e)*\sinh(f*x+e)-1/16*(f*x+e)^2-1/16*\sinh(f*x+e)^2*\cosh(f*x+e)^2)+6/f^2*c*d^2*(1/4*(f*x+e)^2*\sinh(f*x+e)*\cosh(f*x+e)^3-1/8*(f*x+e)^2*\cosh(f*x+e)*\sinh(f*x+e)-1/24*(f*x+e)^3-1/8*(f*x+e)*\sinh(f*x+e)^2*\cosh(f*x+e)^2+1/32*\cosh(f*x+e)^3*\sinh(f*x+e)-1/64*\cosh(f*x+e)*\sinh(f*x+e)-1/64*f*x-1/64*e)+6*c^2/f*d*(1/4*(f*x+e)*\sinh(f*x+e)*\cosh(f*x+e)^3-1/8*(f*x+e)*\cosh(f*x+e)*\sinh(f*x+e)-1/16*(f*x+e)^2-1/16*\sinh(f*x+e)^2*\cosh(f*x+e)^2)-6/f^3*d^3*e^2*(1/4*(f*x+e)*\sinh(f*x+e)^2*\cosh(f*x+e)^2-1/4*(f*x+e)*\cosh(f*x+e)^2-1/16*\cosh(f*x+e)^3*\sinh(f*x+e)+5/32*\cosh(f*x+e)*\sinh(f*x+e)+5/32*f*x+5/32*e)-2/f^3*d^3*e^3*(1/4*\cosh(f*x+e)^3*\sinh(f*x+e)-1/8*\cosh(f*x+e)*\sinh(f*x+e)-1/8*f*x-1/8*e)+2/f^3*d^3*e^3*(1/4*\sinh(f*x+e)^2*\cosh(f*x+e)^2-1/4*\cosh(f*x+e)^2)-6/f^2*c*d^2*(1/4*(f*x+e)^2*\sinh(f*x+e)^2*\cosh(f*x+e)^2-1/4*(f*x+e)^2*\cosh(f*x+e)^2-1/8*(f*x+e)*\sinh(f*x+e)*\cosh(f*x+e)^3+5/16*(f*x+e)*\cosh(f*x+e)*\sinh(f*x+e)+5/32*(f*x+e)^2+1/32*\sinh(f*x+e)^2*\cosh(f*x+e)^2-1/8*\cosh(f*x+e)^2)-6*c^2/f*d*(1/4*(f*x+e)*\sinh(f*x+e)^2*\cosh(f*x+e)^2-1/4*(f*x+e)*\cosh(f*x+e)^2-1/16*\cosh(f*x+e)^3*\sinh(f*x+e)+5/32*\cosh(f*x+e)*\sinh(f*x+e)+5/32*f*x+5/32*e)+2*c^3*(1/4*\cosh(f*x+e)^3*\sinh(f*x+e)-1/8*\cosh(f*x+e)*\sinh(f*x+e)-1/8*f*x-1/8*e)-2*c^3*(1/4*\sinh(f*x+e)^2*\cosh(f*x+e)^2-1/4*\cosh(f*x+e)^2)-c^3*(1/2*\cosh(f*x+e)*\sinh(f*x+e)-1/2*f*x-1/2*e))$

Maxima [A] time = 2.49278, size = 401, normalized size = 1.74

$$\frac{1}{16}c^3\left(\frac{4(fx+e)}{a^2f} + \frac{4e^{(-2fx-2e)} - e^{(-4fx-4e)}}{a^2f}\right) + \frac{3\left(8f^2x^2e^{(4e)} + 8\left(2fxe^{(2e)} + e^{(2e)}\right)e^{(-2fx)} - (4fx+1)e^{(-4fx)}\right)c^2de^{(-4e)}}{64a^2f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3/(a+a*coth(f*x+e))^2,x, algorithm="maxima")

[Out] 1/16*c^3*(4*(f*x + e)/(a^2*f) + (4*e^(-2*f*x - 2*e) - e^(-4*f*x - 4*e)))/(a^2*f) + 3/64*(8*f^2*x^2*e^(4*e) + 8*(2*f*x*e^(2*e) + e^(2*e))*e^(-2*f*x) - (4*f*x + 1)*e^(-4*f*x))*c^2*d*e^(-4*e)/(a^2*f^2) + 1/128*(32*f^3*x^3*e^(4*e) + 48*(2*f^2*x^2*e^(2*e) + 2*f*x*e^(2*e) + e^(2*e))*e^(-2*f*x) - 3*(8*f^2*x^2 + 4*f*x + 1)*e^(-4*f*x))*c*d^2*e^(-4*e)/(a^2*f^3) + 1/512*(32*f^4*x^4*e^(4*e) + 32*(4*f^3*x^3*e^(2*e) + 6*f^2*x^2*e^(2*e) + 6*f*x*e^(2*e) + 3*e^(2

$e))e^{(-2fx)} - (32f^3x^3 + 24f^2x^2 + 12fx + 3)e^{(-4fx))d^3e^{(-4e)/(a^2f^4)}$

Fricas [B] time = 2.17092, size = 1257, normalized size = 5.47

$128d^3f^3x^3 + 128c^3f^3 + 192c^2df^2 + 192cd^2f + 96d^3 + 192(2cd^2f^3 + d^3f^2)x^2 + (32d^3f^4x^4 - 32c^3f^3 - 24c^2df^2 - 12$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3/(a+a*coth(f*x+e))^2,x, algorithm="fricas")

[Out] $\frac{1}{512} \cdot (128d^3f^3x^3 + 128c^3f^3 + 192c^2df^2 + 192cd^2f + 96d^3 + 192(2c^2d^2f^3 + d^3f^2)x^2 + (32d^3f^4x^4 - 32c^3f^3 - 24c^2d^2f^2 - 12c^2d^2f + 32(4c^2d^2f^4 - d^3f^3))x^3 - 3d^3 + 24(8c^2d^2f^4 - 4c^2d^2f^3 - d^3f^2)x^2 + 4(32c^3f^4 - 24c^2d^2f^3 - 12c^2d^2f^2 - 3d^3f)x) \cdot \cosh(fx + e)^2 + 2(32d^3f^4x^4 + 32c^3f^3 + 24c^2d^2f^2 + 12c^2d^2f + 32(4c^2d^2f^4 + d^3f^3))x^3 + 3d^3 + 24(8c^2d^2f^4 + 4c^2d^2f^3 + d^3f^2)x^2 + 4(32c^3f^4 + 24c^2d^2f^3 + 12c^2d^2f^2 + 3d^3f)x) \cdot \cosh(fx + e) \cdot \sinh(fx + e) + (32d^3f^4x^4 - 32c^3f^3 - 24c^2d^2f^2 - 12c^2d^2f + 32(4c^2d^2f^4 - d^3f^3))x^3 - 3d^3 + 24(8c^2d^2f^4 - 4c^2d^2f^3 - d^3f^2)x^2 + 4(32c^3f^4 - 24c^2d^2f^3 - 12c^2d^2f^2 - 3d^3f)x) \cdot \sinh(fx + e)^2 + 192(2c^2d^2f^3 + 2c^2d^2f^2 + d^3f)x) / (a^2f^4 \cosh(fx + e)^2 + 2a^2f^4 \cosh(fx + e) \sinh(fx + e) + a^2f^4 \sinh(fx + e)^2)$

Sympy [A] time = 10.8872, size = 2200, normalized size = 9.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**3/(a+a*coth(f*x+e))**2,x)

[Out] $\text{Piecewise}((64c^{**3}f^{**4}x \cdot \tanh(e + fx))^{**2} / (256a^{**2}f^{**4} \cdot \tanh(e + fx))^{**2} + 512a^{**2}f^{**4} \cdot \tanh(e + fx) + 256a^{**2}f^{**4}) + 128c^{**3}f^{**4}x \cdot \tanh(e + fx) / (256a^{**2}f^{**4} \cdot \tanh(e + fx))^{**2} + 512a^{**2}f^{**4} \cdot \tanh(e + fx) + 256a^{**2}f^{**4}) + 64c^{**3}f^{**4}x / (256a^{**2}f^{**4} \cdot \tanh(e + fx))^{**2} + 512a^{**2}f^{**4} \cdot \tanh(e + fx) + 256a^{**2}f^{**4})$

$$\begin{aligned}
& \text{nh}(e + f*x) + 256*a**2*f**4) - 96*c**3*f**3*\tanh(e + f*x)**2/(256*a**2*f**4 \\
& * \tanh(e + f*x)**2 + 512*a**2*f**4*\tanh(e + f*x) + 256*a**2*f**4) + 32*c**3* \\
& f**3/(256*a**2*f**4*\tanh(e + f*x)**2 + 512*a**2*f**4*\tanh(e + f*x) + 256*a \\
& **2*f**4) + 96*c**2*d*f**4*x**2*\tanh(e + f*x)**2/(256*a**2*f**4*\tanh(e + f*x) \\
&)**2 + 512*a**2*f**4*\tanh(e + f*x) + 256*a**2*f**4) + 192*c**2*d*f**4*x**2* \\
& \tanh(e + f*x)/(256*a**2*f**4*\tanh(e + f*x)**2 + 512*a**2*f**4*\tanh(e + f*x) \\
& + 256*a**2*f**4) + 96*c**2*d*f**4*x**2/(256*a**2*f**4*\tanh(e + f*x)**2 + 5 \\
& 12*a**2*f**4*\tanh(e + f*x) + 256*a**2*f**4) - 240*c**2*d*f**3*x*\tanh(e + f* \\
& x)**2/(256*a**2*f**4*\tanh(e + f*x)**2 + 512*a**2*f**4*\tanh(e + f*x) + 256*a \\
& **2*f**4) + 96*c**2*d*f**3*x*\tanh(e + f*x)/(256*a**2*f**4*\tanh(e + f*x)**2 \\
& + 512*a**2*f**4*\tanh(e + f*x) + 256*a**2*f**4) + 144*c**2*d*f**3*x/(256*a** \\
& 2*f**4*\tanh(e + f*x)**2 + 512*a**2*f**4*\tanh(e + f*x) + 256*a**2*f**4) - 12 \\
& 0*c**2*d*f**2*\tanh(e + f*x)**2/(256*a**2*f**4*\tanh(e + f*x)**2 + 512*a**2*f \\
& **4*\tanh(e + f*x) + 256*a**2*f**4) + 72*c**2*d*f**2/(256*a**2*f**4*\tanh(e + \\
& f*x)**2 + 512*a**2*f**4*\tanh(e + f*x) + 256*a**2*f**4) + 64*c*d**2*f**4*x* \\
& **3*\tanh(e + f*x)**2/(256*a**2*f**4*\tanh(e + f*x)**2 + 512*a**2*f**4*\tanh(e \\
& + f*x) + 256*a**2*f**4) + 128*c*d**2*f**4*x**3*\tanh(e + f*x)/(256*a**2*f**4 \\
& *\tanh(e + f*x)**2 + 512*a**2*f**4*\tanh(e + f*x) + 256*a**2*f**4) + 64*c*d** \\
& 2*f**4*x**3/(256*a**2*f**4*\tanh(e + f*x)**2 + 512*a**2*f**4*\tanh(e + f*x) + \\
& 256*a**2*f**4) - 240*c*d**2*f**3*x**2*\tanh(e + f*x)**2/(256*a**2*f**4*\tanh \\
& (e + f*x)**2 + 512*a**2*f**4*\tanh(e + f*x) + 256*a**2*f**4) + 96*c*d**2*f** \\
& 3*x**2*\tanh(e + f*x)/(256*a**2*f**4*\tanh(e + f*x)**2 + 512*a**2*f**4*\tanh(e \\
& + f*x) + 256*a**2*f**4) + 144*c*d**2*f**3*x**2/(256*a**2*f**4*\tanh(e + f*x) \\
&)**2 + 512*a**2*f**4*\tanh(e + f*x) + 256*a**2*f**4) - 216*c*d**2*f**2*x*\tan \\
& h(e + f*x)**2/(256*a**2*f**4*\tanh(e + f*x)**2 + 512*a**2*f**4*\tanh(e + f*x) \\
& + 256*a**2*f**4) + 48*c*d**2*f**2*x*\tanh(e + f*x)/(256*a**2*f**4*\tanh(e + \\
& f*x)**2 + 512*a**2*f**4*\tanh(e + f*x) + 256*a**2*f**4) + 168*c*d**2*f**2*x/ \\
& (256*a**2*f**4*\tanh(e + f*x)**2 + 512*a**2*f**4*\tanh(e + f*x) + 256*a**2*f* \\
& **4) - 108*c*d**2*f*\tanh(e + f*x)**2/(256*a**2*f**4*\tanh(e + f*x)**2 + 512*a \\
& **2*f**4*\tanh(e + f*x) + 256*a**2*f**4) + 84*c*d**2*f/(256*a**2*f**4*\tanh(e \\
& + f*x)**2 + 512*a**2*f**4*\tanh(e + f*x) + 256*a**2*f**4) + 16*d**3*f**4*x* \\
& **4*\tanh(e + f*x)**2/(256*a**2*f**4*\tanh(e + f*x)**2 + 512*a**2*f**4*\tanh(e \\
& + f*x) + 256*a**2*f**4) + 32*d**3*f**4*x**4*\tanh(e + f*x)/(256*a**2*f**4*ta \\
& nh(e + f*x)**2 + 512*a**2*f**4*\tanh(e + f*x) + 256*a**2*f**4) + 16*d**3*f** \\
& 4*x**4/(256*a**2*f**4*\tanh(e + f*x)**2 + 512*a**2*f**4*\tanh(e + f*x) + 256* \\
& a**2*f**4) - 80*d**3*f**3*x**3*\tanh(e + f*x)**2/(256*a**2*f**4*\tanh(e + f*x) \\
&)**2 + 512*a**2*f**4*\tanh(e + f*x) + 256*a**2*f**4) + 32*d**3*f**3*x**3*\tan \\
& h(e + f*x)/(256*a**2*f**4*\tanh(e + f*x)**2 + 512*a**2*f**4*\tanh(e + f*x) + \\
& 256*a**2*f**4) + 48*d**3*f**3*x**3/(256*a**2*f**4*\tanh(e + f*x)**2 + 512*a* \\
& **2*f**4*\tanh(e + f*x) + 256*a**2*f**4) - 108*d**3*f**2*x**2*\tanh(e + f*x)** \\
& 2/(256*a**2*f**4*\tanh(e + f*x)**2 + 512*a**2*f**4*\tanh(e + f*x) + 256*a**2* \\
& f**4) + 24*d**3*f**2*x**2*\tanh(e + f*x)/(256*a**2*f**4*\tanh(e + f*x)**2 + 5 \\
& 12*a**2*f**4*\tanh(e + f*x) + 256*a**2*f**4) + 84*d**3*f**2*x**2/(256*a**2*f \\
& **4*\tanh(e + f*x)**2 + 512*a**2*f**4*\tanh(e + f*x) + 256*a**2*f**4) - 102*d \\
& **3*f*x*\tanh(e + f*x)**2/(256*a**2*f**4*\tanh(e + f*x)**2 + 512*a**2*f**4*ta
\end{aligned}$$

```

nh(e + f*x) + 256*a**2*f**4) + 12*d**3*f*x*tanh(e + f*x)/(256*a**2*f**4*tan
h(e + f*x)**2 + 512*a**2*f**4*tanh(e + f*x) + 256*a**2*f**4) + 90*d**3*f*x/
(256*a**2*f**4*tanh(e + f*x)**2 + 512*a**2*f**4*tanh(e + f*x) + 256*a**2*f*
**4) - 51*d**3*tanh(e + f*x)**2/(256*a**2*f**4*tanh(e + f*x)**2 + 512*a**2*f
**4*tanh(e + f*x) + 256*a**2*f**4) + 45*d**3/(256*a**2*f**4*tanh(e + f*x)**
2 + 512*a**2*f**4*tanh(e + f*x) + 256*a**2*f**4), Ne(f, 0)), ((c**3*x + 3*c
**2*d*x**2/2 + c*d**2*x**3 + d**3*x**4/4)/(a*coth(e) + a)**2, True))

```

Giac [A] time = 1.16722, size = 517, normalized size = 2.25

$$\left(32d^3f^4x^4e^{(4fx+4e)} + 128cd^2f^4x^3e^{(4fx+4e)} + 192c^2df^4x^2e^{(4fx+4e)} + 128d^3f^3x^3e^{(2fx+2e)} - 32d^3f^3x^3 + 128c^3f^4xe^{(4fx+4e)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+a*coth(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/512*(32*d^3*f^4*x^4*e^(4*f*x + 4*e) + 128*c*d^2*f^4*x^3*e^(4*f*x + 4*e) +
192*c^2*d*f^4*x^2*e^(4*f*x + 4*e) + 128*d^3*f^3*x^3*e^(2*f*x + 2*e) - 32*d
^3*f^3*x^3 + 128*c^3*f^4*x*e^(4*f*x + 4*e) + 384*c*d^2*f^3*x^2*e^(2*f*x + 2
*e) - 96*c*d^2*f^3*x^2 + 384*c^2*d*f^3*x*e^(2*f*x + 2*e) + 192*d^3*f^2*x^2*
e^(2*f*x + 2*e) - 96*c^2*d*f^3*x - 24*d^3*f^2*x^2 + 128*c^3*f^3*e^(2*f*x +
2*e) + 384*c*d^2*f^2*x*e^(2*f*x + 2*e) - 32*c^3*f^3 - 48*c*d^2*f^2*x + 192*
c^2*d*f^2*e^(2*f*x + 2*e) + 192*d^3*f*x*e^(2*f*x + 2*e) - 24*c^2*d*f^2 - 12
*d^3*f*x + 192*c*d^2*f*e^(2*f*x + 2*e) - 12*c*d^2*f + 96*d^3*e^(2*f*x + 2*e
) - 3*d^3)*e^(-4*f*x - 4*e)/(a^2*f^4)
```


3.23

$$\int \frac{(c+dx)^2}{(a+a \coth(e+fx))^2} dx$$

Optimal. Leaf size=170

$$-\frac{d(c+dx)e^{-4e-4fx}}{32a^2f^2} + \frac{d(c+dx)e^{-2e-2fx}}{4a^2f^2} - \frac{(c+dx)^2e^{-4e-4fx}}{16a^2f} + \frac{(c+dx)^2e^{-2e-2fx}}{4a^2f} + \frac{(c+dx)^3}{12a^2d} - \frac{d^2e^{-4e-4fx}}{128a^2f^3} + \frac{d^2e^{-2e-2fx}}{8a^2f^3}$$

[Out] $-(d^2E^{(-4e - 4f*x)})/(128*a^2*f^3) + (d^2E^{(-2e - 2f*x)})/(8*a^2*f^3) - (dE^{(-4e - 4f*x)}*(c + d*x))/(32*a^2*f^2) + (dE^{(-2e - 2f*x)}*(c + d*x))/(4*a^2*f^2) - (E^{(-4e - 4f*x)}*(c + d*x)^2)/(16*a^2*f) + (E^{(-2e - 2f*x)}*(c + d*x)^2)/(4*a^2*f) + (c + d*x)^3/(12*a^2*d)$

Rubi [A] time = 0.191968, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.15$, Rules used = {3729, 2176, 2194}

$$-\frac{d(c+dx)e^{-4e-4fx}}{32a^2f^2} + \frac{d(c+dx)e^{-2e-2fx}}{4a^2f^2} - \frac{(c+dx)^2e^{-4e-4fx}}{16a^2f} + \frac{(c+dx)^2e^{-2e-2fx}}{4a^2f} + \frac{(c+dx)^3}{12a^2d} - \frac{d^2e^{-4e-4fx}}{128a^2f^3} + \frac{d^2e^{-2e-2fx}}{8a^2f^3}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)^2/(a + a*Coth[e + f*x])^2,x]

[Out] $-(d^2E^{(-4e - 4f*x)})/(128*a^2*f^3) + (d^2E^{(-2e - 2f*x)})/(8*a^2*f^3) - (dE^{(-4e - 4f*x)}*(c + d*x))/(32*a^2*f^2) + (dE^{(-2e - 2f*x)}*(c + d*x))/(4*a^2*f^2) - (E^{(-4e - 4f*x)}*(c + d*x)^2)/(16*a^2*f) + (E^{(-2e - 2f*x)}*(c + d*x)^2)/(4*a^2*f) + (c + d*x)^3/(12*a^2*d)$

Rule 3729

Int[((c_.) + (d_.)*(x_))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (1/(2*a) + E^((2*a*(e + f*x)))/b)/(2*a)]^(-n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && ILtQ[n, 0]

Rule 2176

Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]), x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m

] && !\$UseGamma === True

Rule 2194

Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a + b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]

Rubi steps

$$\begin{aligned} \int \frac{(c+dx)^2}{(a+a\coth(e+fx))^2} dx &= \int \left(\frac{(c+dx)^2}{4a^2} + \frac{e^{-4e-4fx}(c+dx)^2}{4a^2} - \frac{e^{-2e-2fx}(c+dx)^2}{2a^2} \right) dx \\ &= \frac{(c+dx)^3}{12a^2d} + \frac{\int e^{-4e-4fx}(c+dx)^2 dx}{4a^2} - \frac{\int e^{-2e-2fx}(c+dx)^2 dx}{2a^2} \\ &= -\frac{e^{-4e-4fx}(c+dx)^2}{16a^2f} + \frac{e^{-2e-2fx}(c+dx)^2}{4a^2f} + \frac{(c+dx)^3}{12a^2d} + \frac{d \int e^{-4e-4fx}(c+dx) dx}{8a^2f} - \frac{d \int e^{-2e-2fx}(c+dx) dx}{2a^2f} \\ &= -\frac{de^{-4e-4fx}(c+dx)}{32a^2f^2} + \frac{de^{-2e-2fx}(c+dx)}{4a^2f^2} - \frac{e^{-4e-4fx}(c+dx)^2}{16a^2f} + \frac{e^{-2e-2fx}(c+dx)^2}{4a^2f} + \frac{(c+dx)^3}{12a^2d} \\ &= -\frac{d^2e^{-4e-4fx}}{128a^2f^3} + \frac{d^2e^{-2e-2fx}}{8a^2f^3} - \frac{de^{-4e-4fx}(c+dx)}{32a^2f^2} + \frac{de^{-2e-2fx}(c+dx)}{4a^2f^2} - \frac{e^{-4e-4fx}(c+dx)^2}{16a^2f} + \frac{e^{-2e-2fx}(c+dx)^2}{4a^2f} + \frac{(c+dx)^3}{12a^2d} \end{aligned}$$

Mathematica [A] time = 0.946135, size = 207, normalized size = 1.22

$$\frac{\operatorname{csch}^2(e+fx) \left((24c^2f^2(4fx+1) + 12cdf(8f^2x^2+4fx+1) + d^2(32f^3x^3+24f^2x^2+12fx+3)) \sinh(2(e+fx)) + (24c^2f^2(4fx+1) + 12cdf(8f^2x^2+4fx+1) + d^2(32f^3x^3+24f^2x^2+12fx+3)) \right)}{(384a^2f^3(1+\operatorname{Coth}[e+fx]))^2}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)^2/(a + a*Coth[e + f*x])^2, x]

[Out] (Csch[e + f*x]^2*(48*(2*c^2*f^2 + 2*c*d*f*(1 + 2*f*x) + d^2*(1 + 2*f*x + 2*f^2*x^2)) + (24*c^2*f^2*(-1 + 4*f*x) + 12*c*d*f*(-1 - 4*f*x + 8*f^2*x^2) + d^2*(-3 - 12*f*x - 24*f^2*x^2 + 32*f^3*x^3))*Cosh[2*(e + f*x)] + (24*c^2*f^2*(1 + 4*f*x) + 12*c*d*f*(1 + 4*f*x + 8*f^2*x^2) + d^2*(3 + 12*f*x + 24*f^2*x^2 + 32*f^3*x^3))*Sinh[2*(e + f*x)])/(384*a^2*f^3*(1 + Coth[e + f*x])^2)

Maple [B] time = 0.097, size = 1087, normalized size = 6.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x+c)^2/(a+a*coth(f*x+e))^2,x)`

[Out] $1/a^2/f*(-2/f^2*d^2*(1/4*(f*x+e)^2*\sinh(f*x+e)^2*\cosh(f*x+e)^2-1/4*(f*x+e)^2*\cosh(f*x+e)^2-1/8*(f*x+e)*\sinh(f*x+e)*\cosh(f*x+e)^3+5/16*(f*x+e)*\cosh(f*x+e)*\sinh(f*x+e)+5/32*(f*x+e)^2+1/32*\sinh(f*x+e)^2*\cosh(f*x+e)^2-1/8*\cosh(f*x+e)^2)+2/f^2*d^2*(1/4*(f*x+e)^2*\sinh(f*x+e)*\cosh(f*x+e)^3-1/8*(f*x+e)^2*\cosh(f*x+e)*\sinh(f*x+e)-1/24*(f*x+e)^3-1/8*(f*x+e)*\sinh(f*x+e)^2*\cosh(f*x+e)^2+1/32*\cosh(f*x+e)^3*\sinh(f*x+e)-1/64*\cosh(f*x+e)*\sinh(f*x+e)-1/64*f*x-1/64*e)-1/f^2*d^2*(1/2*(f*x+e)^2*\cosh(f*x+e)*\sinh(f*x+e)-1/6*(f*x+e)^3-1/2*(f*x+e)*\cosh(f*x+e)^2+1/4*\cosh(f*x+e)*\sinh(f*x+e)+1/4*f*x+1/4*e)+4/f^2*d^2*e*(1/4*(f*x+e)*\sinh(f*x+e)^2*\cosh(f*x+e)^2-1/4*(f*x+e)*\cosh(f*x+e)^2-1/16*\cosh(f*x+e)^3*\sinh(f*x+e)+5/32*\cosh(f*x+e)*\sinh(f*x+e)+5/32*f*x+5/32*e)-4/f^2*d^2*e*(1/4*(f*x+e)*\sinh(f*x+e)*\cosh(f*x+e)^3-1/8*(f*x+e)*\cosh(f*x+e)*\sinh(f*x+e)-1/16*(f*x+e)^2-1/16*\sinh(f*x+e)^2*\cosh(f*x+e)^2)+2/f^2*d^2*e*(1/2*(f*x+e)*\cosh(f*x+e)*\sinh(f*x+e)-1/4*(f*x+e)^2-1/4*\cosh(f*x+e)^2)-2/f^2*d^2*e^2*(1/4*\sinh(f*x+e)^2*\cosh(f*x+e)^2-1/4*\cosh(f*x+e)^2)+2/f^2*d^2*e^2*(1/4*\cosh(f*x+e)^3*\sinh(f*x+e)-1/8*\cosh(f*x+e)*\sinh(f*x+e)-1/8*f*x-1/8*e)-d^2*e^2/f^2*(1/2*\cosh(f*x+e)*\sinh(f*x+e)-1/2*f*x-1/2*e)-4*c/f*d*(1/4*(f*x+e)*\sinh(f*x+e)^2*\cosh(f*x+e)^2-1/4*(f*x+e)*\cosh(f*x+e)^2-1/16*\cosh(f*x+e)^3*\sinh(f*x+e)+5/32*\cosh(f*x+e)*\sinh(f*x+e)+5/32*f*x+5/32*e)+4*c/f*d*(1/4*(f*x+e)*\sinh(f*x+e)*\cosh(f*x+e)^3-1/8*(f*x+e)*\cosh(f*x+e)*\sinh(f*x+e)-1/16*(f*x+e)^2-1/16*\sinh(f*x+e)^2*\cosh(f*x+e)^2)-2/f*d*c*(1/2*(f*x+e)*\cosh(f*x+e)*\sinh(f*x+e)-1/4*(f*x+e)^2-1/4*\cosh(f*x+e)^2)+4*c/f*d*e*(1/4*\sinh(f*x+e)^2*\cosh(f*x+e)^2-1/4*\cosh(f*x+e)^2)-4*c/f*d*e*(1/4*\cosh(f*x+e)^3*\sinh(f*x+e)-1/8*\cosh(f*x+e)*\sinh(f*x+e)-1/8*f*x-1/8*e)+2*d*e/f*c*(1/2*\cosh(f*x+e)*\sinh(f*x+e)-1/2*f*x-1/2*e)-2*c^2*(1/4*\sinh(f*x+e)^2*\cosh(f*x+e)^2-1/4*\cosh(f*x+e)^2)+2*c^2*(1/4*\cosh(f*x+e)^3*\sinh(f*x+e)-1/8*\cosh(f*x+e)*\sinh(f*x+e)-1/8*f*x-1/8*e)-c^2*(1/2*\cosh(f*x+e)*\sinh(f*x+e)-1/2*f*x-1/2*e))$

Maxima [A] time = 2.02766, size = 258, normalized size = 1.52

$$\frac{1}{16}c^2\left(\frac{4(fx+e)}{a^2f} + \frac{4e^{(-2fx-2e)} - e^{(-4fx-4e)}}{a^2f}\right) + \frac{(8f^2x^2e^{(4e)} + 8(2fxe^{(2e)} + e^{(2e)})e^{(-2fx)} - (4fx+1)e^{(-4fx)})cde^{(-4e)}}{32a^2f^2} +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)^2/(a+a*coth(f*x+e))^2,x, algorithm="maxima")`

[Out] $1/16*c^2*(4*(f*x + e)/(a^2*f) + (4*e^{(-2*f*x - 2*e)} - e^{(-4*f*x - 4*e)})/(a^2*f))$

$$2*f)) + 1/32*(8*f^2*x^2*e^{(4*e)} + 8*(2*f*x*e^{(2*e)} + e^{(2*e)})*e^{(-2*f*x)} - (4*f*x + 1)*e^{(-4*f*x)})*c*d*e^{(-4*e)}/(a^2*f^2) + 1/384*(32*f^3*x^3*e^{(4*e)} + 48*(2*f^2*x^2*e^{(2*e)} + 2*f*x*e^{(2*e)} + e^{(2*e)})*e^{(-2*f*x)} - 3*(8*f^2*x^2 + 4*f*x + 1)*e^{(-4*f*x)})*d^2*e^{(-4*e)}/(a^2*f^3)$$

Fricas [B] time = 2.26552, size = 811, normalized size = 4.77

$$96d^2f^2x^2 + 96c^2f^2 + 96cdf + (32d^2f^3x^3 - 24c^2f^2 - 12cdf + 24(4cdf^3 - d^2f^2)x^2 - 3d^2 + 12(8c^2f^3 - 4cdf^2 - d^2f^2))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2/(a+a*coth(f*x+e))^2,x, algorithm="fricas")

[Out] $1/384*(96*d^2*f^2*x^2 + 96*c^2*f^2 + 96*c*d*f + (32*d^2*f^3*x^3 - 24*c^2*f^2 - 12*c*d*f + 24*(4*c*d*f^3 - d^2*f^2)*x^2 - 3*d^2 + 12*(8*c^2*f^3 - 4*c*d*f^2 - d^2*f)*x)*\cosh(f*x + e)^2 + 2*(32*d^2*f^3*x^3 + 24*c^2*f^2 + 12*c*d*f + 24*(4*c*d*f^3 + d^2*f^2)*x^2 + 3*d^2 + 12*(8*c^2*f^3 + 4*c*d*f^2 + d^2*f)*x)*\cosh(f*x + e)*\sinh(f*x + e) + (32*d^2*f^3*x^3 - 24*c^2*f^2 - 12*c*d*f + 24*(4*c*d*f^3 - d^2*f^2)*x^2 - 3*d^2 + 12*(8*c^2*f^3 - 4*c*d*f^2 - d^2*f)*x)*\sinh(f*x + e)^2 + 48*d^2 + 96*(2*c*d*f^2 + d^2*f)*x)/(a^2*f^3*\cosh(f*x + e)^2 + 2*a^2*f^3*\cosh(f*x + e)*\sinh(f*x + e) + a^2*f^3*\sinh(f*x + e)^2)$

Sympy [A] time = 3.46332, size = 1358, normalized size = 7.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**2/(a+a*coth(f*x+e))**2,x)

[Out] Piecewise(((48*c**2*f**3*x*tanh(e + f*x)**2/(192*a**2*f**3*tanh(e + f*x)**2 + 384*a**2*f**3*tanh(e + f*x) + 192*a**2*f**3) + 96*c**2*f**3*x*tanh(e + f*x)/(192*a**2*f**3*tanh(e + f*x)**2 + 384*a**2*f**3*tanh(e + f*x) + 192*a**2*f**3) + 48*c**2*f**3*x/(192*a**2*f**3*tanh(e + f*x)**2 + 384*a**2*f**3*tanh(e + f*x) + 192*a**2*f**3) - 72*c**2*f**2*tanh(e + f*x)**2/(192*a**2*f**3*tanh(e + f*x)**2 + 384*a**2*f**3*tanh(e + f*x) + 192*a**2*f**3) + 24*c**2*f**2/(192*a**2*f**3*tanh(e + f*x)**2 + 384*a**2*f**3*tanh(e + f*x) + 192*a**2*f**3))

```

2*f**3) + 48*c*d*f**3*x**2*tanh(e + f*x)**2/(192*a**2*f**3*tanh(e + f*x)**2
+ 384*a**2*f**3*tanh(e + f*x) + 192*a**2*f**3) + 96*c*d*f**3*x**2*tanh(e +
f*x)/(192*a**2*f**3*tanh(e + f*x)**2 + 384*a**2*f**3*tanh(e + f*x) + 192*a
**2*f**3) + 48*c*d*f**3*x**2/(192*a**2*f**3*tanh(e + f*x)**2 + 384*a**2*f**
3*tanh(e + f*x) + 192*a**2*f**3) - 120*c*d*f**2*x*tanh(e + f*x)**2/(192*a**
2*f**3*tanh(e + f*x)**2 + 384*a**2*f**3*tanh(e + f*x) + 192*a**2*f**3) + 48
*c*d*f**2*x*tanh(e + f*x)/(192*a**2*f**3*tanh(e + f*x)**2 + 384*a**2*f**3*t
anh(e + f*x) + 192*a**2*f**3) + 72*c*d*f**2*x/(192*a**2*f**3*tanh(e + f*x)*
**2 + 384*a**2*f**3*tanh(e + f*x) + 192*a**2*f**3) - 60*c*d*f*tanh(e + f*x)*
**2/(192*a**2*f**3*tanh(e + f*x)**2 + 384*a**2*f**3*tanh(e + f*x) + 192*a**2
*f**3) + 36*c*d*f/(192*a**2*f**3*tanh(e + f*x)**2 + 384*a**2*f**3*tanh(e +
f*x) + 192*a**2*f**3) + 16*d**2*f**3*x**3*tanh(e + f*x)**2/(192*a**2*f**3*t
anh(e + f*x)**2 + 384*a**2*f**3*tanh(e + f*x) + 192*a**2*f**3) + 32*d**2*f*
**3*x**3*tanh(e + f*x)/(192*a**2*f**3*tanh(e + f*x)**2 + 384*a**2*f**3*tanh(
e + f*x) + 192*a**2*f**3) + 16*d**2*f**3*x**3/(192*a**2*f**3*tanh(e + f*x)*
**2 + 384*a**2*f**3*tanh(e + f*x) + 192*a**2*f**3) - 60*d**2*f**2*x**2*tanh(
e + f*x)**2/(192*a**2*f**3*tanh(e + f*x)**2 + 384*a**2*f**3*tanh(e + f*x) +
192*a**2*f**3) + 24*d**2*f**2*x**2*tanh(e + f*x)/(192*a**2*f**3*tanh(e + f
*x)**2 + 384*a**2*f**3*tanh(e + f*x) + 192*a**2*f**3) + 36*d**2*f**2*x**2/(
192*a**2*f**3*tanh(e + f*x)**2 + 384*a**2*f**3*tanh(e + f*x) + 192*a**2*f**
3) - 54*d**2*f*x*tanh(e + f*x)**2/(192*a**2*f**3*tanh(e + f*x)**2 + 384*a**
2*f**3*tanh(e + f*x) + 192*a**2*f**3) + 12*d**2*f*x*tanh(e + f*x)/(192*a**2
*f**3*tanh(e + f*x)**2 + 384*a**2*f**3*tanh(e + f*x) + 192*a**2*f**3) + 42*
d**2*f*x/(192*a**2*f**3*tanh(e + f*x)**2 + 384*a**2*f**3*tanh(e + f*x) + 19
2*a**2*f**3) - 27*d**2*tanh(e + f*x)**2/(192*a**2*f**3*tanh(e + f*x)**2 + 3
84*a**2*f**3*tanh(e + f*x) + 192*a**2*f**3) + 21*d**2/(192*a**2*f**3*tanh(e
+ f*x)**2 + 384*a**2*f**3*tanh(e + f*x) + 192*a**2*f**3), Ne(f, 0)), ((c**
2*x + c*d*x**2 + d**2*x**3/3)/(a*coth(e) + a)**2, True))

```

Giac [A] time = 1.1428, size = 306, normalized size = 1.8

$$\left(32d^2f^3x^3e^{(4fx+4e)} + 96cdf^3x^2e^{(4fx+4e)} + 96c^2f^3xe^{(4fx+4e)} + 96d^2f^2x^2e^{(2fx+2e)} - 24d^2f^2x^2 + 192cdf^2xe^{(2fx+2e)} - \dots\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2/(a+a*coth(f*x+e))^2,x, algorithm="giac")

[Out] 1/384*(32*d^2*f^3*x^3*e^(4*f*x + 4*e) + 96*c*d*f^3*x^2*e^(4*f*x + 4*e) + 96*c^2*f^3*x*e^(4*f*x + 4*e) + 96*d^2*f^2*x^2*e^(2*f*x + 2*e) - 24*d^2*f^2*x^2 + 192*c*d*f^2*x*e^(2*f*x + 2*e) - 48*c*d*f^2*x + 96*c^2*f^2*e^(2*f*x + 2*e) + 96*d^2*f*x*e^(2*f*x + 2*e) - 24*c^2*f^2 - 12*d^2*f*x + 96*c*d*f*e^(2*f

$$\frac{(x + 2e) - 12cd + 48d^2e^{2fx + 2e} - 3d^2e^{-4fx - 4e}}{a^{2f^3}}$$

$$3.24 \quad \int \frac{c+dx}{(a+a \coth(e+fx))^2} dx$$

Optimal. Leaf size=133

$$-\frac{c+dx}{4f(a^2 \coth(e+fx)+a^2)} + \frac{x(c+dx)}{4a^2} - \frac{3d}{16f^2(a^2 \coth(e+fx)+a^2)} + \frac{3dx}{16a^2f} - \frac{dx^2}{8a^2} - \frac{c+dx}{4f(a \coth(e+fx)+a)^2} - \frac{1}{16}$$

[Out] (3*d*x)/(16*a^2*f) - (d*x^2)/(8*a^2) + (x*(c + d*x))/(4*a^2) - d/(16*f^2*(a + a*Coth[e + f*x])^2) - (c + d*x)/(4*f*(a + a*Coth[e + f*x])^2) - (3*d)/(16*f^2*(a^2 + a^2*Coth[e + f*x])) - (c + d*x)/(4*f*(a^2 + a^2*Coth[e + f*x]))

Rubi [A] time = 0.129456, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3479, 8, 3730}

$$-\frac{c+dx}{4f(a^2 \coth(e+fx)+a^2)} + \frac{x(c+dx)}{4a^2} - \frac{3d}{16f^2(a^2 \coth(e+fx)+a^2)} + \frac{3dx}{16a^2f} - \frac{dx^2}{8a^2} - \frac{c+dx}{4f(a \coth(e+fx)+a)^2} - \frac{1}{16}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)/(a + a*Coth[e + f*x])^2,x]

[Out] (3*d*x)/(16*a^2*f) - (d*x^2)/(8*a^2) + (x*(c + d*x))/(4*a^2) - d/(16*f^2*(a + a*Coth[e + f*x])^2) - (c + d*x)/(4*f*(a + a*Coth[e + f*x])^2) - (3*d)/(16*f^2*(a^2 + a^2*Coth[e + f*x])) - (c + d*x)/(4*f*(a^2 + a^2*Coth[e + f*x]))

Rule 3479

Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(a*(a + b*Tan[c + d*x])^n)/(2*b*d*n), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 3730

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_),
  x_Symbol] :> With[{u = IntHide[(a + b*Tan[e + f*x])^n, x]}, Dist[(c + d*x)
^m, u, x] - Dist[d*m, Int[Dist[(c + d*x)^(m - 1), u, x], x], x]] /; FreeQ[{
a, b, c, d, e, f}, x] && EqQ[a^2 + b^2, 0] && ILtQ[n, -1] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{c + dx}{(a + a \coth(e + fx))^2} dx &= \frac{x(c + dx)}{4a^2} - \frac{c + dx}{4f(a + a \coth(e + fx))^2} - \frac{c + dx}{4f(a^2 + a^2 \coth(e + fx))} - d \int \left(\frac{x}{4a^2} - \frac{1}{4f(a + a \coth(e + fx))} \right) dx \\ &= -\frac{dx^2}{8a^2} + \frac{x(c + dx)}{4a^2} - \frac{c + dx}{4f(a + a \coth(e + fx))^2} - \frac{c + dx}{4f(a^2 + a^2 \coth(e + fx))} + \frac{d \int \frac{1}{(a + a \coth(e + fx))} dx}{4f} \\ &= -\frac{dx^2}{8a^2} + \frac{x(c + dx)}{4a^2} - \frac{d}{16f^2(a + a \coth(e + fx))^2} - \frac{c + dx}{4f(a + a \coth(e + fx))^2} - \frac{c + dx}{8f^2(a^2 + a^2 \coth(e + fx))} \\ &= \frac{dx}{8a^2 f} - \frac{dx^2}{8a^2} + \frac{x(c + dx)}{4a^2} - \frac{d}{16f^2(a + a \coth(e + fx))^2} - \frac{c + dx}{4f(a + a \coth(e + fx))^2} - \frac{c + dx}{16f^2(a^2 + a^2 \coth(e + fx))} \\ &= \frac{3dx}{16a^2 f} - \frac{dx^2}{8a^2} + \frac{x(c + dx)}{4a^2} - \frac{d}{16f^2(a + a \coth(e + fx))^2} - \frac{c + dx}{4f(a + a \coth(e + fx))^2} - \frac{c + dx}{16f^2(a^2 + a^2 \coth(e + fx))} \end{aligned}$$

Mathematica [A] time = 0.521503, size = 114, normalized size = 0.86

$$\frac{\operatorname{csch}^2(e + fx) \left((4cf(4fx + 1) + d(8f^2x^2 + 4fx + 1)) \sinh(2(e + fx)) + (4cf(4fx - 1) + d(8f^2x^2 - 4fx - 1)) \cosh(2(e + fx)) \right)}{64a^2f^2(\coth(e + fx) + 1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a + a*Coth[e + f*x])^2, x]
```

```
[Out] (Csch[e + f*x]^2*(8*(d + 2*c*f + 2*d*f*x) + (4*c*f*(-1 + 4*f*x) + d*(-1 - 4*f*x + 8*f^2*x^2))*Cosh[2*(e + f*x)] + (4*c*f*(1 + 4*f*x) + d*(1 + 4*f*x + 8*f^2*x^2))*Sinh[2*(e + f*x)])/(64*a^2*f^2*(1 + Coth[e + f*x])^2)
```

Maple [A] time = 0.122, size = 74, normalized size = 0.6

$$\frac{dx^2}{8a^2} + \frac{cx}{4a^2} + \frac{(2dfx + 2cf + d)e^{-2fx-2e}}{8a^2f^2} - \frac{(4dfx + 4cf + d)e^{-4fx-4e}}{64a^2f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x+c)/(a+a*coth(f*x+e))^2,x)`

[Out] $\frac{1}{8}d*x^2/a^2 + \frac{1}{4}a^2*c*x + \frac{1}{8}(2*d*f*x + 2*c*f + d)/a^2/f^2*\exp(-2*f*x - 2*e) - \frac{1}{6}4*(4*d*f*x + 4*c*f + d)/a^2/f^2*\exp(-4*f*x - 4*e)$

Maxima [A] time = 1.50981, size = 144, normalized size = 1.08

$$\frac{1}{16}c \left(\frac{4(fx + e)}{a^2f} + \frac{4e^{(-2fx-2e)} - e^{(-4fx-4e)}}{a^2f} \right) + \frac{\left(8f^2x^2e^{(4e)} + 8(2fxe^{(2e)} + e^{(2e)})e^{(-2fx)} - (4fx + 1)e^{(-4fx)} \right) de^{(-4e)}}{64a^2f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)/(a+a*coth(f*x+e))^2,x, algorithm="maxima")`

[Out] $\frac{1}{16}c*(4*(fx + e)/(a^2*f) + (4*e^{(-2*f*x - 2*e)} - e^{(-4*f*x - 4*e)})/(a^2*f)) + \frac{1}{64}*(8*f^2*x^2*e^{(4*e)} + 8*(2*f*x*e^{(2*e)} + e^{(2*e)})*e^{(-2*f*x)} - (4*f*x + 1)*e^{(-4*f*x)})*d*e^{(-4*e)}/(a^2*f^2)$

Fricas [A] time = 2.03748, size = 454, normalized size = 3.41

$$\frac{16dfx + (8df^2x^2 - 4cf + 4(4cf^2 - df)x - d) \cosh(fx + e)^2 + 2(8df^2x^2 + 4cf + 4(4cf^2 + df)x + d) \cosh(fx + e)}{64(a^2f^2 \cosh(fx + e))^2 + 2a^2f^2 \cosh(fx + e) \sinh(fx + e)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)/(a+a*coth(f*x+e))^2,x, algorithm="fricas")`

[Out] $\frac{1}{64}*(16*d*f*x + (8*d*f^2*x^2 - 4*c*f + 4*(4*c*f^2 - d*f)*x - d)*\cosh(f*x + e)^2 + 2*(8*d*f^2*x^2 + 4*c*f + 4*(4*c*f^2 + d*f)*x + d)*\cosh(f*x + e)*\sinh(f*x + e) + (8*d*f^2*x^2 - 4*c*f + 4*(4*c*f^2 - d*f)*x - d)*\sinh(f*x + e)^2 + 16*c*f + 8*d)/(a^2*f^2*\cosh(f*x + e)^2 + 2*a^2*f^2*\cosh(f*x + e)*\sinh(f*x + e) + a^2*f^2*\sinh(f*x + e)^2)$

Sympy [A] time = 2.84553, size = 700, normalized size = 5.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)/(a+a*coth(f*x+e))**2,x)

[Out] Piecewise(((4*c*f**2*x*tanh(e + f*x)**2/(16*a**2*f**2*tanh(e + f*x)**2 + 32*a**2*f**2*tanh(e + f*x) + 16*a**2*f**2) + 8*c*f**2*x*tanh(e + f*x)/(16*a**2*f**2*tanh(e + f*x)**2 + 32*a**2*f**2*tanh(e + f*x) + 16*a**2*f**2) + 4*c*f**2*x/(16*a**2*f**2*tanh(e + f*x)**2 + 32*a**2*f**2*tanh(e + f*x) + 16*a**2*f**2) + 12*c*f*tanh(e + f*x)/(16*a**2*f**2*tanh(e + f*x)**2 + 32*a**2*f**2*tanh(e + f*x) + 16*a**2*f**2) + 8*c*f/(16*a**2*f**2*tanh(e + f*x)**2 + 32*a**2*f**2*tanh(e + f*x) + 16*a**2*f**2) + 2*d*f**2*x**2*tanh(e + f*x)**2/(16*a**2*f**2*tanh(e + f*x)**2 + 32*a**2*f**2*tanh(e + f*x) + 16*a**2*f**2) + 4*d*f**2*x**2*tanh(e + f*x)/(16*a**2*f**2*tanh(e + f*x)**2 + 32*a**2*f**2*tanh(e + f*x) + 16*a**2*f**2) + 2*d*f**2*x**2/(16*a**2*f**2*tanh(e + f*x)**2 + 32*a**2*f**2*tanh(e + f*x) + 16*a**2*f**2) - 5*d*f*x*tanh(e + f*x)**2/(16*a**2*f**2*tanh(e + f*x)**2 + 32*a**2*f**2*tanh(e + f*x) + 16*a**2*f**2) + 2*d*f*x*tanh(e + f*x)/(16*a**2*f**2*tanh(e + f*x)**2 + 32*a**2*f**2*tanh(e + f*x) + 16*a**2*f**2) + 3*d*f*x/(16*a**2*f**2*tanh(e + f*x)**2 + 32*a**2*f**2*tanh(e + f*x) + 16*a**2*f**2) + 5*d*tanh(e + f*x)/(16*a**2*f**2*tanh(e + f*x)**2 + 32*a**2*f**2*tanh(e + f*x) + 16*a**2*f**2) + 4*d/(16*a**2*f**2*tanh(e + f*x)**2 + 32*a**2*f**2*tanh(e + f*x) + 16*a**2*f**2), Ne(f, 0)), ((c*x + d*x**2/2)/(a*coth(e) + a)**2, True))

Giac [A] time = 1.145, size = 147, normalized size = 1.11

$$\frac{(8df^2x^2e^{4fx+4e} + 16cf^2xe^{4fx+4e} + 16dfxe^{2fx+2e} - 4dfx + 16cfe^{2fx+2e} - 4cf + 8de^{2fx+2e} - d)e^{-4fx-4e}}{64a^2f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)/(a+a*coth(f*x+e))^2,x, algorithm="giac")

[Out] 1/64*(8*d*f^2*x^2*e^(4*f*x + 4*e) + 16*c*f^2*x*e^(4*f*x + 4*e) + 16*d*f*x*e^(2*f*x + 2*e) - 4*d*f*x + 16*c*f*e^(2*f*x + 2*e) - 4*c*f + 8*d*e^(2*f*x + 2*e) - d)*e^(-4*f*x - 4*e)/(a^2*f^2)

$$3.25 \quad \int \frac{1}{(c+dx)(a+a \coth(e+fx))^2} dx$$

Optimal. Leaf size=297

$$\frac{\text{Chi}\left(2xf + \frac{2cf}{d}\right) \sinh\left(2e - \frac{2cf}{d}\right)}{2a^2d} - \frac{\text{Chi}\left(4xf + \frac{4cf}{d}\right) \sinh\left(4e - \frac{4cf}{d}\right)}{4a^2d} - \frac{\text{Chi}\left(2xf + \frac{2cf}{d}\right) \cosh\left(2e - \frac{2cf}{d}\right)}{2a^2d} + \frac{\text{Chi}\left(4xf + \frac{4cf}{d}\right) \cosh\left(4e - \frac{4cf}{d}\right)}{4a^2d}$$

[Out] $-(\text{Cosh}[2*e - (2*c*f)/d]*\text{CoshIntegral}[(2*c*f)/d + 2*f*x])/(2*a^2*d) + (\text{Cosh}[4*e - (4*c*f)/d]*\text{CoshIntegral}[(4*c*f)/d + 4*f*x])/(4*a^2*d) + \text{Log}[c + d*x]/(4*a^2*d) - (\text{CoshIntegral}[(4*c*f)/d + 4*f*x]*\text{Sinh}[4*e - (4*c*f)/d])/(4*a^2*d) + (\text{CoshIntegral}[(2*c*f)/d + 2*f*x]*\text{Sinh}[2*e - (2*c*f)/d])/(2*a^2*d) + (\text{Cosh}[2*e - (2*c*f)/d]*\text{SinhIntegral}[(2*c*f)/d + 2*f*x])/(2*a^2*d) - (\text{Sinh}[2*e - (2*c*f)/d]*\text{SinhIntegral}[(2*c*f)/d + 2*f*x])/(2*a^2*d) - (\text{Cosh}[4*e - (4*c*f)/d]*\text{SinhIntegral}[(4*c*f)/d + 4*f*x])/(4*a^2*d) + (\text{Sinh}[4*e - (4*c*f)/d]*\text{SinhIntegral}[(4*c*f)/d + 4*f*x])/(4*a^2*d)$

Rubi [A] time = 0.703588, antiderivative size = 297, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {3728, 3303, 3298, 3301, 3312}

$$\frac{\text{Chi}\left(2xf + \frac{2cf}{d}\right) \sinh\left(2e - \frac{2cf}{d}\right)}{2a^2d} - \frac{\text{Chi}\left(4xf + \frac{4cf}{d}\right) \sinh\left(4e - \frac{4cf}{d}\right)}{4a^2d} - \frac{\text{Chi}\left(2xf + \frac{2cf}{d}\right) \cosh\left(2e - \frac{2cf}{d}\right)}{2a^2d} + \frac{\text{Chi}\left(4xf + \frac{4cf}{d}\right) \cosh\left(4e - \frac{4cf}{d}\right)}{4a^2d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[1/((c + d*x)*(a + a*\text{Coth}[e + f*x]))^2), x]$

[Out] $-(\text{Cosh}[2*e - (2*c*f)/d]*\text{CoshIntegral}[(2*c*f)/d + 2*f*x])/(2*a^2*d) + (\text{Cosh}[4*e - (4*c*f)/d]*\text{CoshIntegral}[(4*c*f)/d + 4*f*x])/(4*a^2*d) + \text{Log}[c + d*x]/(4*a^2*d) - (\text{CoshIntegral}[(4*c*f)/d + 4*f*x]*\text{Sinh}[4*e - (4*c*f)/d])/(4*a^2*d) + (\text{CoshIntegral}[(2*c*f)/d + 2*f*x]*\text{Sinh}[2*e - (2*c*f)/d])/(2*a^2*d) + (\text{Cosh}[2*e - (2*c*f)/d]*\text{SinhIntegral}[(2*c*f)/d + 2*f*x])/(2*a^2*d) - (\text{Sinh}[2*e - (2*c*f)/d]*\text{SinhIntegral}[(2*c*f)/d + 2*f*x])/(2*a^2*d) - (\text{Cosh}[4*e - (4*c*f)/d]*\text{SinhIntegral}[(4*c*f)/d + 4*f*x])/(4*a^2*d) + (\text{Sinh}[4*e - (4*c*f)/d]*\text{SinhIntegral}[(4*c*f)/d + 4*f*x])/(4*a^2*d)$

Rule 3728

$\text{Int}[(c + d*x)^m * ((a + b*\tan[(e + f*x)])^n), x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(c + d*x)^m, (1/(2*a) + \text{Cos}[2*e + 2*f*x])/($

```
2*a) + Sin[2*e + 2*f*x]/(2*b))^(-n), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
  && EqQ[a^2 + b^2, 0] && ILtQ[m, 0] && ILtQ[n, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
  NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] := Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] := Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] := In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(c+dx)(a+a\coth(e+fx))^2} dx &= \int \left(\frac{1}{4a^2(c+dx)} - \frac{\cosh(2e+2fx)}{2a^2(c+dx)} + \frac{\cosh^2(2e+2fx)}{4a^2(c+dx)} + \frac{\sinh(2e+2fx)}{2a^2(c+dx)} + \frac{\sinh^2(2e+2fx)}{4a^2(c+dx)} \right) dx \\
&= \frac{\log(c+dx)}{4a^2d} + \frac{\int \frac{\cosh^2(2e+2fx)}{c+dx} dx}{4a^2} + \frac{\int \frac{\sinh^2(2e+2fx)}{c+dx} dx}{4a^2} - \frac{\int \frac{\sinh(4e+4fx)}{c+dx} dx}{4a^2} - \frac{\int \frac{\cosh(4e+4fx)}{c+dx} dx}{4a^2} \\
&= \frac{\log(c+dx)}{4a^2d} - \frac{\int \left(\frac{1}{2(c+dx)} - \frac{\cosh(4e+4fx)}{2(c+dx)} \right) dx}{4a^2} + \frac{\int \left(\frac{1}{2(c+dx)} + \frac{\cosh(4e+4fx)}{2(c+dx)} \right) dx}{4a^2} - \frac{\int \frac{\cosh(4e+4fx)}{c+dx} dx}{4a^2} \\
&= -\frac{\cosh\left(2e - \frac{2cf}{d}\right) \operatorname{Chi}\left(\frac{2cf}{d} + 2fx\right)}{2a^2d} + \frac{\log(c+dx)}{4a^2d} - \frac{\operatorname{Chi}\left(\frac{4cf}{d} + 4fx\right) \sinh\left(4e - \frac{4cf}{d}\right)}{4a^2d} \\
&= -\frac{\cosh\left(2e - \frac{2cf}{d}\right) \operatorname{Chi}\left(\frac{2cf}{d} + 2fx\right)}{2a^2d} + \frac{\log(c+dx)}{4a^2d} - \frac{\operatorname{Chi}\left(\frac{4cf}{d} + 4fx\right) \sinh\left(4e - \frac{4cf}{d}\right)}{4a^2d} \\
&= -\frac{\cosh\left(2e - \frac{2cf}{d}\right) \operatorname{Chi}\left(\frac{2cf}{d} + 2fx\right)}{2a^2d} + \frac{\log(c+dx)}{4a^2d} - \frac{\operatorname{Chi}\left(\frac{4cf}{d} + 4fx\right) \sinh\left(4e - \frac{4cf}{d}\right)}{4a^2d}
\end{aligned}$$

Mathematica [A] time = 0.439323, size = 199, normalized size = 0.67

$$\frac{\left(\cosh\left(2e - \frac{2cf}{d}\right) - \sinh\left(2e - \frac{2cf}{d}\right)\right) \left(\operatorname{Chi}\left(\frac{4f(c+dx)}{d}\right) \left(\cosh\left(2e - \frac{2cf}{d}\right) - \sinh\left(2e - \frac{2cf}{d}\right)\right) - 2\operatorname{Chi}\left(\frac{2f(c+dx)}{d}\right) + \sinh\left(2e - \frac{2cf}{d}\right)\right)}{4a^2d}$$

Antiderivative was successfully verified.

[In] Integrate[1/((c + d*x)*(a + a*Coth[e + f*x])^2),x]

[Out] ((Cosh[2*e - (2*c*f)/d] - Sinh[2*e - (2*c*f)/d])*(-2*CoshIntegral[(2*f*(c + d*x))/d] + Cosh[2*e - (2*c*f)/d]*Log[f*(c + d*x)] + CoshIntegral[(4*f*(c + d*x))/d]*(Cosh[2*e - (2*c*f)/d] - Sinh[2*e - (2*c*f)/d]) + Log[f*(c + d*x)]*Sinh[2*e - (2*c*f)/d] + 2*SinhIntegral[(2*f*(c + d*x))/d] - Cosh[2*e - (2*c*f)/d]*SinhIntegral[(4*f*(c + d*x))/d] + Sinh[2*e - (2*c*f)/d]*SinhIntegral[(4*f*(c + d*x))/d]))/(4*a^2*d)

Maple [A] time = 0.27, size = 106, normalized size = 0.4

$$\frac{\ln(dx+c)}{4a^2d} - \frac{1}{4a^2d} e^{4\frac{cf-de}{d}} \operatorname{Ei}\left(1, 4fx + 4e + 4\frac{cf-de}{d}\right) + \frac{1}{2a^2d} e^{2\frac{cf-de}{d}} \operatorname{Ei}\left(1, 2fx + 2e + 2\frac{cf-de}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(d*x+c)/(a+a*coth(f*x+e))^2,x)`

[Out] $\frac{1}{4} \ln(d*x+c)/d/a^2 - \frac{1}{4} \frac{\exp(4*(c*f-d*e)/d) * Ei(1, 4*f*x+4*e+4*(c*f-d*e)/d) + 1/2/a^2/d * \exp(2*(c*f-d*e)/d) * Ei(1, 2*f*x+2*e+2*(c*f-d*e)/d)}{a^2}$

Maxima [A] time = 3.38104, size = 109, normalized size = 0.37

$$-\frac{e^{\left(-4e+\frac{4cf}{d}\right)} E_1\left(\frac{4(dx+c)f}{d}\right)}{4a^2d} + \frac{e^{\left(-2e+\frac{2cf}{d}\right)} E_1\left(\frac{2(dx+c)f}{d}\right)}{2a^2d} + \frac{\log(dx+c)}{4a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(d*x+c)/(a+a*coth(f*x+e))^2,x, algorithm="maxima")`

[Out] $-\frac{1}{4} e^{(-4e + 4*c*f/d)} \exp_integral_e(1, 4*(d*x + c)*f/d)/(a^2*d) + \frac{1}{2} e^{(-2e + 2*c*f/d)} \exp_integral_e(1, 2*(d*x + c)*f/d)/(a^2*d) + \frac{1}{4} \log(d*x + c)/(a^2*d)$

Fricas [A] time = 2.19348, size = 304, normalized size = 1.02

$$\frac{2 Ei\left(-\frac{2(dfxc+cf)}{d}\right) \cosh\left(-\frac{2(de-cf)}{d}\right) - Ei\left(-\frac{4(dfxc+cf)}{d}\right) \cosh\left(-\frac{4(de-cf)}{d}\right) + 2 Ei\left(-\frac{2(dfxc+cf)}{d}\right) \sinh\left(-\frac{2(de-cf)}{d}\right) - Ei\left(-\frac{4(dfxc+cf)}{d}\right) \sinh\left(-\frac{2(de-cf)}{d}\right) - \log(d*x + c)}{4a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(d*x+c)/(a+a*coth(f*x+e))^2,x, algorithm="fricas")`

[Out] $-\frac{1}{4} (2 * Ei(-2*(d*f*x + c*f)/d) * \cosh(-2*(d*e - c*f)/d) - Ei(-4*(d*f*x + c*f)/d) * \cosh(-4*(d*e - c*f)/d) + 2 * Ei(-2*(d*f*x + c*f)/d) * \sinh(-2*(d*e - c*f)/d) - Ei(-4*(d*f*x + c*f)/d) * \sinh(-4*(d*e - c*f)/d) - \log(d*x + c))/(a^2*d)$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{c \coth^2(e+fx)+2c \coth(e+fx)+c+dx \coth^2(e+fx)+2dx \coth(e+fx)+dx} dx$$

$$a^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d*x+c)/(a+a*coth(f*x+e))**2,x)

[Out] Integral(1/(c*coth(e + f*x)**2 + 2*c*coth(e + f*x) + c + d*x*coth(e + f*x)*
*2 + 2*d*x*coth(e + f*x) + d*x), x)/a**2

Giac [A] time = 1.19066, size = 105, normalized size = 0.35

$$\frac{\left(\operatorname{Ei}\left(-\frac{4(df_x+cf)}{d}\right) e^{\left(\frac{4cf}{d}\right)} - 2 \operatorname{Ei}\left(-\frac{2(df_x+cf)}{d}\right) e^{\left(\frac{2cf}{d}+2e\right)} + e^{(4e)} \log(dx+c) \right) e^{(-4e)}}{4a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d*x+c)/(a+a*coth(f*x+e))^2,x, algorithm="giac")

[Out] 1/4*(Ei(-4*(d*f*x + c*f)/d)*e^(4*c*f/d) - 2*Ei(-2*(d*f*x + c*f)/d)*e^(2*c*f/d + 2*e) + e^(4*e)*log(d*x + c))*e^(-4*e)/(a^2*d)

$$3.26 \quad \int \frac{1}{(c+dx)^2(a+a \coth(e+fx))^2} dx$$

Optimal. Leaf size=420

$$\frac{f \operatorname{Chi}\left(4xf + \frac{4cf}{d}\right) \sinh\left(4e - \frac{4cf}{d}\right)}{a^2 d^2} - \frac{f \operatorname{Chi}\left(2xf + \frac{2cf}{d}\right) \sinh\left(2e - \frac{2cf}{d}\right)}{a^2 d^2} + \frac{f \operatorname{Chi}\left(2xf + \frac{2cf}{d}\right) \cosh\left(2e - \frac{2cf}{d}\right)}{a^2 d^2} - \frac{f \operatorname{Chi}\left(4xf + \frac{4cf}{d}\right) \cosh\left(4e - \frac{4cf}{d}\right)}{a^2 d^2}$$

[Out] $-1/(4*a^2*d*(c + d*x)) + \operatorname{Cosh}[2*e + 2*f*x]/(2*a^2*d*(c + d*x)) - \operatorname{Cosh}[2*e + 2*f*x]^2/(4*a^2*d*(c + d*x)) + (f*\operatorname{Cosh}[2*e - (2*c*f)/d]*\operatorname{CoshIntegral}[(2*c*f)/d + 2*f*x])/(a^2*d^2) - (f*\operatorname{Cosh}[4*e - (4*c*f)/d]*\operatorname{CoshIntegral}[(4*c*f)/d + 4*f*x])/(a^2*d^2) + (f*\operatorname{CoshIntegral}[(4*c*f)/d + 4*f*x]*\operatorname{Sinh}[4*e - (4*c*f)/d])/(a^2*d^2) - (f*\operatorname{CoshIntegral}[(2*c*f)/d + 2*f*x]*\operatorname{Sinh}[2*e - (2*c*f)/d])/(a^2*d^2) - \operatorname{Sinh}[2*e + 2*f*x]/(2*a^2*d*(c + d*x)) - \operatorname{Sinh}[2*e + 2*f*x]^2/(4*a^2*d*(c + d*x)) + \operatorname{Sinh}[4*e + 4*f*x]/(4*a^2*d*(c + d*x)) - (f*\operatorname{Cosh}[2*e - (2*c*f)/d]*\operatorname{SinhIntegral}[(2*c*f)/d + 2*f*x])/(a^2*d^2) + (f*\operatorname{Sinh}[2*e - (2*c*f)/d]*\operatorname{SinhIntegral}[(2*c*f)/d + 2*f*x])/(a^2*d^2) + (f*\operatorname{Cosh}[4*e - (4*c*f)/d]*\operatorname{SinhIntegral}[(4*c*f)/d + 4*f*x])/(a^2*d^2) - (f*\operatorname{Sinh}[4*e - (4*c*f)/d]*\operatorname{SinhIntegral}[(4*c*f)/d + 4*f*x])/(a^2*d^2)$

Rubi [A] time = 0.732656, antiderivative size = 420, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.35$, Rules used = {3728, 3297, 3303, 3298, 3301, 3313, 12}

$$\frac{f \operatorname{Chi}\left(4xf + \frac{4cf}{d}\right) \sinh\left(4e - \frac{4cf}{d}\right)}{a^2 d^2} - \frac{f \operatorname{Chi}\left(2xf + \frac{2cf}{d}\right) \sinh\left(2e - \frac{2cf}{d}\right)}{a^2 d^2} + \frac{f \operatorname{Chi}\left(2xf + \frac{2cf}{d}\right) \cosh\left(2e - \frac{2cf}{d}\right)}{a^2 d^2} - \frac{f \operatorname{Chi}\left(4xf + \frac{4cf}{d}\right) \cosh\left(4e - \frac{4cf}{d}\right)}{a^2 d^2}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[1/((c + d*x)^2*(a + a*\operatorname{Coth}[e + f*x])^2), x]$

[Out] $-1/(4*a^2*d*(c + d*x)) + \operatorname{Cosh}[2*e + 2*f*x]/(2*a^2*d*(c + d*x)) - \operatorname{Cosh}[2*e + 2*f*x]^2/(4*a^2*d*(c + d*x)) + (f*\operatorname{Cosh}[2*e - (2*c*f)/d]*\operatorname{CoshIntegral}[(2*c*f)/d + 2*f*x])/(a^2*d^2) - (f*\operatorname{Cosh}[4*e - (4*c*f)/d]*\operatorname{CoshIntegral}[(4*c*f)/d + 4*f*x])/(a^2*d^2) + (f*\operatorname{CoshIntegral}[(4*c*f)/d + 4*f*x]*\operatorname{Sinh}[4*e - (4*c*f)/d])/(a^2*d^2) - (f*\operatorname{CoshIntegral}[(2*c*f)/d + 2*f*x]*\operatorname{Sinh}[2*e - (2*c*f)/d])/(a^2*d^2) - \operatorname{Sinh}[2*e + 2*f*x]/(2*a^2*d*(c + d*x)) - \operatorname{Sinh}[2*e + 2*f*x]^2/(4*a^2*d*(c + d*x)) + \operatorname{Sinh}[4*e + 4*f*x]/(4*a^2*d*(c + d*x)) - (f*\operatorname{Cosh}[2*e - (2*c*f)/d]*\operatorname{SinhIntegral}[(2*c*f)/d + 2*f*x])/(a^2*d^2) + (f*\operatorname{Sinh}[2*e - (2*c*f)/d]*\operatorname{SinhIntegral}[(2*c*f)/d + 2*f*x])/(a^2*d^2) + (f*\operatorname{Cosh}[4*e - (4*c*f)/d]*\operatorname{SinhIntegral}[(4*c*f)/d + 4*f*x])/(a^2*d^2) - (f*\operatorname{Sinh}[4*e - (4*c*f)/d]*\operatorname{SinhIntegral}[(4*c*f)/d + 4*f*x])/(a^2*d^2)$

tegral[(4*c*f)/d + 4*f*x]/(a^2*d^2)

Rule 3728

Int[((c_.) + (d_.)*(x_))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_),
x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (1/(2*a) + Cos[2*e + 2*f*x]/(2*a) + Sin[2*e + 2*f*x]/(2*b))^(-n), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& EqQ[a^2 + b^2, 0] && ILtQ[m, 0] && ILtQ[n, 0]

Rule 3297

Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]

Rule 3303

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]

Rule 3298

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*e - c*f*fz*I, 0]

Rule 3301

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]

Rule 3313

Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Simp[((c + d*x)^(m + 1)*Sin[e + f*x]^n)/(d*(m + 1)), x] - Dist[(f*n)/(d*(m + 1)), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n - 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] && LtQ[m, -1]

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rubi steps

$$\begin{aligned}
 \int \frac{1}{(c+dx)^2(a+a \coth(e+fx))^2} dx &= \int \left(\frac{1}{4a^2(c+dx)^2} - \frac{\cosh(2e+2fx)}{2a^2(c+dx)^2} + \frac{\cosh^2(2e+2fx)}{4a^2(c+dx)^2} + \frac{\sinh(2e+2fx)}{2a^2(c+dx)^2} + \frac{\sinh^2(2e+2fx)}{4a^2(c+dx)^2} \right) dx \\
 &= -\frac{1}{4a^2d(c+dx)} + \frac{\int \frac{\cosh^2(2e+2fx)}{(c+dx)^2} dx}{4a^2} + \frac{\int \frac{\sinh^2(2e+2fx)}{(c+dx)^2} dx}{4a^2} - \frac{\int \frac{\sinh(4e+4fx)}{(c+dx)^2} dx}{4a^2} - \frac{\int \frac{\cosh(4e+4fx)}{(c+dx)^2} dx}{4a^2} \\
 &= -\frac{1}{4a^2d(c+dx)} + \frac{\cosh(2e+2fx)}{2a^2d(c+dx)} - \frac{\cosh^2(2e+2fx)}{4a^2d(c+dx)} - \frac{\sinh(2e+2fx)}{2a^2d(c+dx)} - \frac{\sinh^2(2e+2fx)}{4a^2d(c+dx)} \\
 &= -\frac{1}{4a^2d(c+dx)} + \frac{\cosh(2e+2fx)}{2a^2d(c+dx)} - \frac{\cosh^2(2e+2fx)}{4a^2d(c+dx)} - \frac{\sinh(2e+2fx)}{2a^2d(c+dx)} - \frac{\sinh^2(2e+2fx)}{4a^2d(c+dx)} \\
 &= -\frac{1}{4a^2d(c+dx)} + \frac{\cosh(2e+2fx)}{2a^2d(c+dx)} - \frac{\cosh^2(2e+2fx)}{4a^2d(c+dx)} + \frac{f \cosh\left(2e - \frac{2cf}{d}\right) \text{Chi}\left(\frac{2c}{a}\right)}{a^2d^2} \\
 &= -\frac{1}{4a^2d(c+dx)} + \frac{\cosh(2e+2fx)}{2a^2d(c+dx)} - \frac{\cosh^2(2e+2fx)}{4a^2d(c+dx)} + \frac{f \cosh\left(2e - \frac{2cf}{d}\right) \text{Chi}\left(\frac{2c}{a}\right)}{a^2d^2}
 \end{aligned}$$

Mathematica [A] time = 1.41906, size = 442, normalized size = 1.05

$$\frac{(\sinh(2(f(x - \frac{c}{d}) + e)) - \cosh(2(f(x - \frac{c}{d}) + e))) \left(4f(c+dx) \text{Chi}\left(\frac{4f(c+dx)}{d}\right) \left(\cosh\left(2e - \frac{2f(c+dx)}{d}\right) - \sinh\left(2e - \frac{2f(c+dx)}{d}\right)\right)\right)}{a^2d^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/((c + d*x)^2*(a + a*Coth[e + f*x])^2), x]

[Out] ((-Cosh[2*(e + f*(-(c/d) + x))] + Sinh[2*(e + f*(-(c/d) + x))])*(-2*d*Cosh[(2*c*f)/d] + d*Cosh[2*(e + f*(-(c/d) + x))] + d*Cosh[2*(e + f*(c/d + x))] + 2*d*Sinh[(2*c*f)/d] - 4*f*(c + d*x)*CoshIntegral[(2*f*(c + d*x))/d]*(Cosh[2*f*x] + Sinh[2*f*x]) + d*Sinh[2*(e + f*(-(c/d) + x))] - d*Sinh[2*(e + f*(c/d + x))] + 4*f*(c + d*x)*CoshIntegral[(4*f*(c + d*x))/d]*(Cosh[2*e - (2*f*(c + d*x))/d] - Sinh[2*e - (2*f*(c + d*x))/d]) + 4*c*f*Cosh[2*f*x]*SinhInte

```

gral[(2*f*(c + d*x))/d] + 4*d*f*x*Cosh[2*f*x]*SinhIntegral[(2*f*(c + d*x))/
d] + 4*c*f*Sinh[2*f*x]*SinhIntegral[(2*f*(c + d*x))/d] + 4*d*f*x*Sinh[2*f*x
]*SinhIntegral[(2*f*(c + d*x))/d] - 4*c*f*Cosh[2*e - (2*f*(c + d*x))/d]*Sin
hIntegral[(4*f*(c + d*x))/d] - 4*d*f*x*Cosh[2*e - (2*f*(c + d*x))/d]*SinhIn
tegral[(4*f*(c + d*x))/d] + 4*c*f*Sinh[2*e - (2*f*(c + d*x))/d]*SinhIntegra
l[(4*f*(c + d*x))/d] + 4*d*f*x*Sinh[2*e - (2*f*(c + d*x))/d]*SinhIntegral[(
4*f*(c + d*x))/d]))/(4*a^2*d^2*(c + d*x))

```

Maple [A] time = 0.282, size = 164, normalized size = 0.4

$$-\frac{1}{4a^2d(dx+c)} - \frac{fe^{-4fx-4e}}{4a^2d(dfxc+cf)} + \frac{f}{a^2d^2}e^{4\frac{cf-de}{d}}\text{Ei}\left(1,4fx+4e+4\frac{cf-de}{d}\right) + \frac{fe^{-2fx-2e}}{2a^2d(dfxc+cf)} - \frac{f}{a^2d^2}e^{2\frac{cf-de}{d}}\text{Ei}\left(1,$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(d*x+c)^2/(a+a*coth(f*x+e))^2,x)

[Out] -1/4/a^2/d/(d*x+c)-1/4/a^2*f*exp(-4*f*x-4*e)/d/(d*f*x+c*f)+1/a^2*f/d^2*exp(4*(c*f-d*e)/d)*Ei(1,4*f*x+4*e+4*(c*f-d*e)/d)+1/2/a^2*f*exp(-2*f*x-2*e)/d/(d*f*x+c*f)-1/a^2*f/d^2*exp(2*(c*f-d*e)/d)*Ei(1,2*f*x+2*e+2*(c*f-d*e)/d)

Maxima [A] time = 6.0564, size = 135, normalized size = 0.32

$$-\frac{1}{4(a^2d^2x+a^2cd)} - \frac{e^{(-4e+\frac{4cf}{d})}E_2\left(\frac{4(dx+c)f}{d}\right)}{4(dx+c)a^2d} + \frac{e^{(-2e+\frac{2cf}{d})}E_2\left(\frac{2(dx+c)f}{d}\right)}{2(dx+c)a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d*x+c)^2/(a+a*coth(f*x+e))^2,x, algorithm="maxima")

[Out] -1/4/(a^2*d^2*x + a^2*c*d) - 1/4*e^(-4*e + 4*c*f/d)*exp_integral_e(2, 4*(d*x + c)*f/d)/((d*x + c)*a^2*d) + 1/2*e^(-2*e + 2*c*f/d)*exp_integral_e(2, 2*(d*x + c)*f/d)/((d*x + c)*a^2*d)

Fricas [A] time = 2.24221, size = 1413, normalized size = 3.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d*x+c)^2/(a+a*coth(f*x+e))^2,x, algorithm="fricas")

[Out] $\frac{1}{2} * (2 * (d * f * x + c * f) * \text{Ei}(-2 * (d * f * x + c * f) / d) * \cosh(f * x + e) ^ 2 * \sinh(-2 * (d * e - c * f) / d) - 2 * (d * f * x + c * f) * \text{Ei}(-4 * (d * f * x + c * f) / d) * \cosh(f * x + e) ^ 2 * \sinh(-4 * (d * e - c * f) / d) + (2 * (d * f * x + c * f) * \text{Ei}(-2 * (d * f * x + c * f) / d) * \cosh(-2 * (d * e - c * f) / d) - 2 * (d * f * x + c * f) * \text{Ei}(-4 * (d * f * x + c * f) / d) * \cosh(-4 * (d * e - c * f) / d) - d) * \cosh(f * x + e) ^ 2 + (2 * (d * f * x + c * f) * \text{Ei}(-2 * (d * f * x + c * f) / d) * \cosh(-2 * (d * e - c * f) / d) - 2 * (d * f * x + c * f) * \text{Ei}(-4 * (d * f * x + c * f) / d) * \cosh(-4 * (d * e - c * f) / d) + 2 * (d * f * x + c * f) * \text{Ei}(-2 * (d * f * x + c * f) / d) * \sinh(-2 * (d * e - c * f) / d) - 2 * (d * f * x + c * f) * \text{Ei}(-4 * (d * f * x + c * f) / d) * \sinh(-4 * (d * e - c * f) / d) - d) * \sinh(f * x + e) ^ 2 + 4 * ((d * f * x + c * f) * \text{Ei}(-2 * (d * f * x + c * f) / d) * \cosh(f * x + e) * \sinh(-2 * (d * e - c * f) / d) - (d * f * x + c * f) * \text{Ei}(-4 * (d * f * x + c * f) / d) * \cosh(f * x + e) * \sinh(-4 * (d * e - c * f) / d) + ((d * f * x + c * f) * \text{Ei}(-2 * (d * f * x + c * f) / d) * \cosh(-2 * (d * e - c * f) / d) - (d * f * x + c * f) * \text{Ei}(-4 * (d * f * x + c * f) / d) * \cosh(-4 * (d * e - c * f) / d)) * \cosh(f * x + e)) * \sinh(f * x + e) + d) / ((a ^ 2 * d ^ 3 * x + a ^ 2 * c * d ^ 2) * \cosh(f * x + e) ^ 2 + 2 * (a ^ 2 * d ^ 3 * x + a ^ 2 * c * d ^ 2) * \cosh(f * x + e) * \sinh(f * x + e) + (a ^ 2 * d ^ 3 * x + a ^ 2 * c * d ^ 2) * \sinh(f * x + e) ^ 2)$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{c^2 \coth^2(e+fx) + 2c^2 \coth(e+fx) + c^2 + 2cdx \coth^2(e+fx) + 4cdx \coth(e+fx) + 2cdx + d^2x^2 \coth^2(e+fx) + 2d^2x^2 \coth(e+fx) + d^2x^2} dx$$

$$\frac{1}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d*x+c)**2/(a+a*coth(f*x+e))**2,x)

[Out] Integral(1/(c**2*coth(e + f*x)**2 + 2*c**2*coth(e + f*x) + c**2 + 2*c*d*x*c*o*th(e + f*x)**2 + 4*c*d*x*coth(e + f*x) + 2*c*d*x + d**2*x**2*c*o*th(e + f*x)**2 + 2*d**2*x**2*c*o*th(e + f*x) + d**2*x**2), x)/a**2

Giac [A] time = 1.92818, size = 255, normalized size = 0.61

$$\frac{4dfxEi\left(-\frac{4(dfxc+f)}{d}\right)e^{\left(\frac{4cf}{d}-4e\right)} - 4dfxEi\left(-\frac{2(dfxc+f)}{d}\right)e^{\left(\frac{2cf}{d}-2e\right)} + 4cfEi\left(-\frac{4(dfxc+f)}{d}\right)e^{\left(\frac{4cf}{d}-4e\right)} - 4cfEi\left(-\frac{2(dfxc+f)}{d}\right)e^{\left(\frac{2cf}{d}-2e\right)}}{4(a^2d^3x + a^2cd^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+a*coth(f*x+e))^2,x, algorithm="giac")
```

```
[Out] -1/4*(4*d*f*x*Ei(-4*(d*f*x + c*f)/d)*e^(4*c*f/d - 4*e) - 4*d*f*x*Ei(-2*(d*f*x + c*f)/d)*e^(2*c*f/d - 2*e) + 4*c*f*Ei(-4*(d*f*x + c*f)/d)*e^(4*c*f/d - 4*e) - 4*c*f*Ei(-2*(d*f*x + c*f)/d)*e^(2*c*f/d - 2*e) - 2*d*e^(-2*f*x - 2*e) + d*e^(-4*f*x - 4*e))/(a^2*d^3*x + a^2*c*d^2) - 1/4/((d*x + c)*a^2*d)
```

$$3.27 \quad \int \frac{(c+dx)^3}{(a+a \coth(e+fx))^3} dx$$

Optimal. Leaf size=336

$$\frac{d^2(c+dx)e^{-6e-6fx}}{288a^3f^3} - \frac{9d^2(c+dx)e^{-4e-4fx}}{256a^3f^3} + \frac{9d^2(c+dx)e^{-2e-2fx}}{32a^3f^3} + \frac{d(c+dx)^2e^{-6e-6fx}}{96a^3f^2} - \frac{9d(c+dx)^2e^{-4e-4fx}}{128a^3f^2} + \frac{9d(c+dx)^2}{32a^3}$$

[Out] $(d^3E^{-6e-6fx})/(1728a^3f^4) - (9d^3E^{-4e-4fx})/(1024a^3f^4) + (9d^3E^{-2e-2fx})/(64a^3f^4) + (d^2E^{-6e-6fx})(c+dx)/(288a^3f^3) - (9d^2E^{-4e-4fx})(c+dx)/(256a^3f^3) + (9d^2E^{-2e-2fx})(c+dx)/(32a^3f^3) + (dE^{-6e-6fx})(c+dx)^2/(96a^3f^2) - (9dE^{-4e-4fx})(c+dx)^2/(128a^3f^2) + (9dE^{-2e-2fx})(c+dx)^2/(32a^3f^2) + (E^{-6e-6fx})(c+dx)^3/(48a^3f) - (3E^{-4e-4fx})(c+dx)^3/(32a^3f) + (3E^{-2e-2fx})(c+dx)^3/(16a^3f) + (c+dx)^4/(32a^3d)$

Rubi [A] time = 0.369896, antiderivative size = 336, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.15$, Rules used = {3729, 2176, 2194}

$$\frac{d^2(c+dx)e^{-6e-6fx}}{288a^3f^3} - \frac{9d^2(c+dx)e^{-4e-4fx}}{256a^3f^3} + \frac{9d^2(c+dx)e^{-2e-2fx}}{32a^3f^3} + \frac{d(c+dx)^2e^{-6e-6fx}}{96a^3f^2} - \frac{9d(c+dx)^2e^{-4e-4fx}}{128a^3f^2} + \frac{9d(c+dx)^2}{32a^3}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)^3/(a + a*Coth[e + f*x])^3,x]

[Out] $(d^3E^{-6e-6fx})/(1728a^3f^4) - (9d^3E^{-4e-4fx})/(1024a^3f^4) + (9d^3E^{-2e-2fx})/(64a^3f^4) + (d^2E^{-6e-6fx})(c+dx)/(288a^3f^3) - (9d^2E^{-4e-4fx})(c+dx)/(256a^3f^3) + (9d^2E^{-2e-2fx})(c+dx)/(32a^3f^3) + (dE^{-6e-6fx})(c+dx)^2/(96a^3f^2) - (9dE^{-4e-4fx})(c+dx)^2/(128a^3f^2) + (9dE^{-2e-2fx})(c+dx)^2/(32a^3f^2) + (E^{-6e-6fx})(c+dx)^3/(48a^3f) - (3E^{-4e-4fx})(c+dx)^3/(32a^3f) + (3E^{-2e-2fx})(c+dx)^3/(16a^3f) + (c+dx)^4/(32a^3d)$

Rule 3729

Int[((c_.) + (d_.)*(x_))^(m_)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, (1/(2*a) + E^((2*a*(e + f*x))/b)/(2*a))^(n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 + b^2

, 0] && ILtQ[n, 0]

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_)))^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]), x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m] && !$UseGamma == True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_)))^(n_.), x_Symbol] := Simp[(F^(c*(a + b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}
 \int \frac{(c + dx)^3}{(a + a \coth(e + fx))^3} dx &= \int \left(\frac{(c + dx)^3}{8a^3} - \frac{e^{-6e-6fx}(c + dx)^3}{8a^3} + \frac{3e^{-4e-4fx}(c + dx)^3}{8a^3} - \frac{3e^{-2e-2fx}(c + dx)^3}{8a^3} \right) dx \\
 &= \frac{(c + dx)^4}{32a^3d} - \frac{\int e^{-6e-6fx}(c + dx)^3 dx}{8a^3} + \frac{3 \int e^{-4e-4fx}(c + dx)^3 dx}{8a^3} - \frac{3 \int e^{-2e-2fx}(c + dx)^3 dx}{8a^3} \\
 &= \frac{e^{-6e-6fx}(c + dx)^3}{48a^3f} - \frac{3e^{-4e-4fx}(c + dx)^3}{32a^3f} + \frac{3e^{-2e-2fx}(c + dx)^3}{16a^3f} + \frac{(c + dx)^4}{32a^3d} - \frac{d \int e^{-6e-6fx}(c + dx)^3 dx}{16a^3} \\
 &= \frac{de^{-6e-6fx}(c + dx)^2}{96a^3f^2} - \frac{9de^{-4e-4fx}(c + dx)^2}{128a^3f^2} + \frac{9de^{-2e-2fx}(c + dx)^2}{32a^3f^2} + \frac{e^{-6e-6fx}(c + dx)^3}{48a^3f} - \frac{3e^{-4e-4fx}(c + dx)^3}{16a^3f} \\
 &= \frac{d^2e^{-6e-6fx}(c + dx)}{288a^3f^3} - \frac{9d^2e^{-4e-4fx}(c + dx)}{256a^3f^3} + \frac{9d^2e^{-2e-2fx}(c + dx)}{32a^3f^3} + \frac{de^{-6e-6fx}(c + dx)^2}{96a^3f^2} - \frac{9de^{-4e-4fx}(c + dx)^2}{128a^3f^2} \\
 &= \frac{d^3e^{-6e-6fx}}{1728a^3f^4} - \frac{9d^3e^{-4e-4fx}}{1024a^3f^4} + \frac{9d^3e^{-2e-2fx}}{64a^3f^4} + \frac{d^2e^{-6e-6fx}(c + dx)}{288a^3f^3} - \frac{9d^2e^{-4e-4fx}(c + dx)}{256a^3f^3} + \frac{9d^2e^{-2e-2fx}(c + dx)}{32a^3f^3}
 \end{aligned}$$

Mathematica [A] time = 2.26326, size = 615, normalized size = 1.83

$$\frac{\operatorname{csch}^3(e + fx) \left(81 \left(24c^2d^2f^2(4fx + 3) + 32c^3f^3 + 12cd^2f(8f^2x^2 + 12fx + 7) \right) + d^3 \left(32f^3x^3 + 72f^2x^2 + 84fx + 45 \right) \right) \cos^3(e + fx)}{1728a^3f^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3/(a + a*Coth[e + f*x])^3, x]
```

[Out] $(\text{Csch}[e + f*x]^3*(81*(32*c^3*f^3 + 24*c^2*d*f^2*(3 + 4*f*x) + 12*c*d^2*f*(7 + 12*f*x + 8*f^2*x^2) + d^3*(45 + 84*f*x + 72*f^2*x^2 + 32*f^3*x^3))*\text{Cosh}[e + f*x] + 16*(36*c^3*f^3*(1 + 6*f*x) + 18*c^2*d*f^2*(1 + 6*f*x + 18*f^2*x^2) + 6*c*d^2*f*(1 + 6*f*x + 18*f^2*x^2 + 36*f^3*x^3) + d^3*(1 + 6*f*x + 18*f^2*x^2 + 36*f^3*x^3 + 54*f^4*x^4))*\text{Cosh}[3*(e + f*x)] + 4131*d^3*\text{Sinh}[e + f*x] + 8748*c*d^2*f*\text{Sinh}[e + f*x] + 9720*c^2*d*f^2*\text{Sinh}[e + f*x] + 7776*c^3*f^3*\text{Sinh}[e + f*x] + 8748*d^3*f*x*\text{Sinh}[e + f*x] + 19440*c*d^2*f^2*x*\text{Sinh}[e + f*x] + 23328*c^2*d*f^3*x*\text{Sinh}[e + f*x] + 9720*d^3*f^2*x^2*\text{Sinh}[e + f*x] + 23328*c*d^2*f^3*x^2*\text{Sinh}[e + f*x] + 7776*d^3*f^3*x^3*\text{Sinh}[e + f*x] - 16*d^3*\text{Sinh}[3*(e + f*x)] - 96*c*d^2*f*\text{Sinh}[3*(e + f*x)] - 288*c^2*d*f^2*\text{Sinh}[3*(e + f*x)] - 576*c^3*f^3*\text{Sinh}[3*(e + f*x)] - 96*d^3*f*x*\text{Sinh}[3*(e + f*x)] - 576*c*d^2*f^2*x*\text{Sinh}[3*(e + f*x)] - 1728*c^2*d*f^3*x*\text{Sinh}[3*(e + f*x)] + 3456*c^3*f^4*x*\text{Sinh}[3*(e + f*x)] - 288*d^3*f^2*x^2*\text{Sinh}[3*(e + f*x)] - 1728*c*d^2*f^3*x^2*\text{Sinh}[3*(e + f*x)] + 5184*c^2*d*f^4*x^2*\text{Sinh}[3*(e + f*x)] - 576*d^3*f^3*x^3*\text{Sinh}[3*(e + f*x)] + 3456*c*d^2*f^4*x^3*\text{Sinh}[3*(e + f*x)] + 864*d^3*f^4*x^4*\text{Sinh}[3*(e + f*x)])/(27648*a^3*f^4*(1 + \text{Coth}[e + f*x])^3)$

Maple [A] time = 0.183, size = 368, normalized size = 1.1

$$\frac{d^3x^4}{32a^3} + \frac{cd^2x^3}{8a^3} + \frac{3c^2dx^2}{16a^3} + \frac{c^3x}{8a^3} + \frac{(12d^3x^3f^3 + 36cd^2f^3x^2 + 36c^2df^3x + 18d^3f^2x^2 + 12c^3f^3 + 36cd^2f^2x + 18c^2df^2 + \dots)}{64a^3f^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((d*x+c)^3/(a+a*\text{coth}(f*x+e))^3,x)$

[Out] $1/32/a^3*d^3*x^4+1/8/a^3*c*d^2*x^3+3/16/a^3*c^2*d*x^2+1/8/a^3*c^3*x+3/64*(4*d^3*f^3*x^3+12*c*d^2*f^3*x^2+12*c^2*d*f^3*x+6*d^3*f^2*x^2+4*c^3*f^3+12*c*d^2*f^2*x+6*c^2*d*f^2+6*d^3*f*x+6*c*d^2*f+3*d^3)/a^3/f^4*\text{exp}(-2*f*x-2*e)-3/1024*(32*d^3*f^3*x^3+96*c*d^2*f^3*x^2+96*c^2*d*f^3*x+24*d^3*f^2*x^2+32*c^3*f^3+48*c*d^2*f^2*x+24*c^2*d*f^2+12*d^3*f*x+12*c*d^2*f+3*d^3)/a^3/f^4*\text{exp}(-4*f*x-4*e)+1/1728*(36*d^3*f^3*x^3+108*c*d^2*f^3*x^2+108*c^2*d*f^3*x+18*d^3*f^2*x^2+36*c^3*f^3+36*c*d^2*f^2*x+18*c^2*d*f^2+6*d^3*f*x+6*c*d^2*f+d^3)/a^3/f^4*\text{exp}(-6*f*x-6*e)$

Maxima [A] time = 7.63276, size = 548, normalized size = 1.63

$$\frac{1}{96}c^3\left(\frac{12(fx+e)}{a^3f} + \frac{18e^{(-2fx-2e)} - 9e^{(-4fx-4e)} + 2e^{(-6fx-6e)}}{a^3f}\right) + \frac{(72f^2x^2e^{(6e)} + 108(2fxe^{(4e)} + e^{(4e)})e^{(-2fx)} - 27(4fx^2e^{(4e)} + 3fxe^{(4e)} + e^{(4e)}))e^{(-2fx)}}{384a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3/(a+a*coth(f*x+e))^3,x, algorithm="maxima")

[Out] $\frac{1}{96}c^3\left(\frac{12(fx+e)}{a^3f} + (18e^{-2fx-2e} - 9e^{-4fx-4e}) + 2e^{-6fx-6e}\right)/(a^3f) + \frac{1}{384}(72f^2x^2e^{6e} + 108(2fxe^{4e} + e^{4e}))e^{-2fx} - 27(4fxe^{2e} + e^{2e})e^{-4fx} + 4(6fx+1)e^{-6fx})c^2de^{-6e}/(a^3f^2) + \frac{1}{2304}(288f^3x^3e^{6e} + 648(2f^2x^2e^{4e} + 2fxe^{4e} + e^{4e}))e^{-2fx} - 81(8f^2x^2e^{2e} + 4fxe^{2e} + e^{2e})e^{-4fx} + 8(18f^2x^2 + 6fx+1)e^{-6fx})c^2de^{-6e}/(a^3f^3) + \frac{1}{27648}(864f^4x^4e^{6e} + 1296(4f^3x^3e^{4e} + 6f^2x^2e^{4e} + 6fxe^{4e} + 3e^{4e}))e^{-2fx} - 81(32f^3x^3e^{2e} + 24f^2x^2e^{2e} + 12fxe^{2e} + 3e^{2e})e^{-4fx} + 16(36f^3x^3 + 18f^2x^2 + 6fx+1)e^{-6fx})d^3e^{-6e}/(a^3f^4)$

Fricas [B] time = 2.22475, size = 1877, normalized size = 5.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3/(a+a*coth(f*x+e))^3,x, algorithm="fricas")

[Out] $\frac{1}{27648}(16(54d^3f^4x^4 + 36c^3f^3 + 18c^2d^2f^2 + 6cd^2f + 36(6cd^2f^4 + d^3f^3))x^3 + d^3 + 18(18c^2d^2f^4 + 6cd^2f^3 + d^3f^2)x^2 + 6(36c^3f^4 + 18c^2d^2f^3 + 6cd^2f^2 + d^3f)x)cosh(fx+e)^3 + 48(54d^3f^4x^4 + 36c^3f^3 + 18c^2d^2f^2 + 6cd^2f + 36(6cd^2f^4 + d^3f^3))x^3 + d^3 + 18(18c^2d^2f^4 + 6cd^2f^3 + d^3f^2)x^2 + 6(36c^3f^4 + 18c^2d^2f^3 + 6cd^2f^2 + d^3f)x)cosh(fx+e)sinh(fx+e)^2 + 16(54d^3f^4x^4 - 36c^3f^3 - 18c^2d^2f^2 - 6cd^2f + 36(6cd^2f^4 - d^3f^3))x^3 - d^3 + 18(18c^2d^2f^4 - 6cd^2f^3 - d^3f^2)x^2 + 6(36c^3f^4 - 18c^2d^2f^3 - 6cd^2f^2 - d^3f)x)sinh(fx+e)^3 + 81(32d^3f^3x^3 + 32c^3f^3 + 72c^2d^2f^2 + 84cd^2f + 45d^3 + 24(4cd^2f^3 + 3d^3f^2))x^2 + 12(8c^2d^2f^3 + 12cd^2f^2 + 7d^3f)x)cosh(fx+e) + 3(2592d^3f^3x^3 + 2592c^3f^3 + 3240c^2d^2f^2 + 2916cd^2f + 1377d^3 + 648(12cd^2f^3 + 5d^3f^2))x^2 + 16(54d^3f^4x^4 - 36c^3f^3 - 18c^2d^2f^2 - 6cd^2f + 36(6cd^2f^4 - d^3f^3))x^3 - d^3 + 18(18c^2d^2f^4 - 6cd^2f^3 - d^3f^2)x^2 + 6(36c^3f^4 - 18c^2d^2f^3 - 6cd^2f^2 - d^3f)x)cosh(fx+e)^2 + 324(24c^2d^2f^3 + 20cd^2f^2 + 9d^3f)x)sinh(fx+e))/(a^3f^4cosh(fx+e)^3 + 3a^3f^4cosh(fx+e)^2sinh(fx+e) + 3a^3f^4cosh(fx+e)sinh$

$$(f*x + e)^2 + a^3*f^4*\sinh(f*x + e)^3$$

Sympy [A] time = 13.2819, size = 3925, normalized size = 11.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**3/(a+a*coth(f*x+e))**3,x)

[Out] Piecewise((864*c**3*f**4*x*tanh(e + f*x)**3/(6912*a**3*f**4*tanh(e + f*x)**3 + 20736*a**3*f**4*tanh(e + f*x)**2 + 20736*a**3*f**4*tanh(e + f*x) + 6912*a**3*f**4) + 2592*c**3*f**4*x*tanh(e + f*x)**2/(6912*a**3*f**4*tanh(e + f*x)**3 + 20736*a**3*f**4*tanh(e + f*x)**2 + 20736*a**3*f**4*tanh(e + f*x) + 6912*a**3*f**4) + 2592*c**3*f**4*x*tanh(e + f*x)/(6912*a**3*f**4*tanh(e + f*x)**3 + 20736*a**3*f**4*tanh(e + f*x)**2 + 20736*a**3*f**4*tanh(e + f*x) + 6912*a**3*f**4) + 864*c**3*f**4*x/(6912*a**3*f**4*tanh(e + f*x)**3 + 20736*a**3*f**4*tanh(e + f*x)**2 + 20736*a**3*f**4*tanh(e + f*x) + 6912*a**3*f**4) - 2592*c**3*f**3*tanh(e + f*x)**3/(6912*a**3*f**4*tanh(e + f*x)**3 + 20736*a**3*f**4*tanh(e + f*x)**2 + 20736*a**3*f**4*tanh(e + f*x) + 6912*a**3*f**4) - 1728*c**3*f**3*tanh(e + f*x)**2/(6912*a**3*f**4*tanh(e + f*x)**3 + 20736*a**3*f**4*tanh(e + f*x)**2 + 20736*a**3*f**4*tanh(e + f*x) + 6912*a**3*f**4) + 288*c**3*f**3/(6912*a**3*f**4*tanh(e + f*x)**3 + 20736*a**3*f**4*tanh(e + f*x)**2 + 20736*a**3*f**4*tanh(e + f*x) + 6912*a**3*f**4) + 1296*c**2*d*f**4*x**2*tanh(e + f*x)**3/(6912*a**3*f**4*tanh(e + f*x)**3 + 20736*a**3*f**4*tanh(e + f*x)**2 + 20736*a**3*f**4*tanh(e + f*x) + 6912*a**3*f**4) + 3888*c**2*d*f**4*x**2*tanh(e + f*x)**2/(6912*a**3*f**4*tanh(e + f*x)**3 + 20736*a**3*f**4*tanh(e + f*x)**2 + 20736*a**3*f**4*tanh(e + f*x) + 6912*a**3*f**4) + 3888*c**2*d*f**4*x**2*tanh(e + f*x)/(6912*a**3*f**4*tanh(e + f*x)**3 + 20736*a**3*f**4*tanh(e + f*x)**2 + 20736*a**3*f**4*tanh(e + f*x) + 6912*a**3*f**4) + 1296*c**2*d*f**4*x**2/(6912*a**3*f**4*tanh(e + f*x)**3 + 20736*a**3*f**4*tanh(e + f*x)**2 + 20736*a**3*f**4*tanh(e + f*x) + 6912*a**3*f**4) - 6264*c**2*d*f**3*x*tanh(e + f*x)**3/(6912*a**3*f**4*tanh(e + f*x)**3 + 20736*a**3*f**4*tanh(e + f*x)**2 + 20736*a**3*f**4*tanh(e + f*x) + 6912*a**3*f**4) - 648*c**2*d*f**3*x*tanh(e + f*x)**2/(6912*a**3*f**4*tanh(e + f*x)**3 + 20736*a**3*f**4*tanh(e + f*x)**2 + 20736*a**3*f**4*tanh(e + f*x) + 6912*a**3*f**4) + 4536*c**2*d*f**3*x*tanh(e + f*x)/(6912*a**3*f**4*tanh(e + f*x)**3 + 20736*a**3*f**4*tanh(e + f*x)**2 + 20736*a**3*f**4*tanh(e + f*x) + 6912*a**3*f**4) + 2376*c**2*d*f**3*x/(6912*a**3*f**4*tanh(e + f*x)**3 + 20736*a**3*f**4*tanh(e + f*x)**2 + 20736*a**3*f**4*tanh(e + f*x) + 6912*a**3*f**4) - 3240*c**2*d*f**2*tanh(e + f*x)**3/(6912*a**3*f**4*tanh(e + f*x)**3 + 20736*a**3*f**4*tanh(e + f*x)**2 + 20736*a**3*f**4*tanh(e + f*x) + 69

$$\begin{aligned}
& 12*a^{**3}*f^{**4}) - 3456*c^{**2}*d*f^{**2}*tanh(e + f*x)**2/(6912*a^{**3}*f^{**4}*tanh(e + \\
& f*x)**3 + 20736*a^{**3}*f^{**4}*tanh(e + f*x)**2 + 20736*a^{**3}*f^{**4}*tanh(e + f*x) \\
& + 6912*a^{**3}*f^{**4}) + 792*c^{**2}*d*f^{**2}/(6912*a^{**3}*f^{**4}*tanh(e + f*x)**3 + 2073 \\
& 6*a^{**3}*f^{**4}*tanh(e + f*x)**2 + 20736*a^{**3}*f^{**4}*tanh(e + f*x) + 6912*a^{**3}*f^{** \\
& *4) + 864*c*d^{**2}*f^{**4}*x^{**3}*tanh(e + f*x)**3/(6912*a^{**3}*f^{**4}*tanh(e + f*x)** \\
& 3 + 20736*a^{**3}*f^{**4}*tanh(e + f*x)**2 + 20736*a^{**3}*f^{**4}*tanh(e + f*x) + 6912 \\
& *a^{**3}*f^{**4}) + 2592*c*d^{**2}*f^{**4}*x^{**3}*tanh(e + f*x)**2/(6912*a^{**3}*f^{**4}*tanh(e \\
& + f*x)**3 + 20736*a^{**3}*f^{**4}*tanh(e + f*x)**2 + 20736*a^{**3}*f^{**4}*tanh(e + f* \\
& x) + 6912*a^{**3}*f^{**4}) + 2592*c*d^{**2}*f^{**4}*x^{**3}*tanh(e + f*x)/(6912*a^{**3}*f^{**4}* \\
& tanh(e + f*x)**3 + 20736*a^{**3}*f^{**4}*tanh(e + f*x)**2 + 20736*a^{**3}*f^{**4}*tanh(\\
& e + f*x) + 6912*a^{**3}*f^{**4}) + 864*c*d^{**2}*f^{**4}*x^{**3}/(6912*a^{**3}*f^{**4}*tanh(e + \\
& f*x)**3 + 20736*a^{**3}*f^{**4}*tanh(e + f*x)**2 + 20736*a^{**3}*f^{**4}*tanh(e + f*x) \\
& + 6912*a^{**3}*f^{**4}) - 6264*c*d^{**2}*f^{**3}*x^{**2}*tanh(e + f*x)**3/(6912*a^{**3}*f^{**4}* \\
& tanh(e + f*x)**3 + 20736*a^{**3}*f^{**4}*tanh(e + f*x)**2 + 20736*a^{**3}*f^{**4}*tanh(\\
& e + f*x) + 6912*a^{**3}*f^{**4}) - 648*c*d^{**2}*f^{**3}*x^{**2}*tanh(e + f*x)**2/(6912*a^{** \\
& *3}*f^{**4}*tanh(e + f*x)**3 + 20736*a^{**3}*f^{**4}*tanh(e + f*x)**2 + 20736*a^{**3}*f^{** \\
& *4}*tanh(e + f*x) + 6912*a^{**3}*f^{**4}) + 4536*c*d^{**2}*f^{**3}*x^{**2}*tanh(e + f*x)/(6 \\
& 912*a^{**3}*f^{**4}*tanh(e + f*x)**3 + 20736*a^{**3}*f^{**4}*tanh(e + f*x)**2 + 20736*a^{** \\
& *3}*f^{**4}*tanh(e + f*x) + 6912*a^{**3}*f^{**4}) + 2376*c*d^{**2}*f^{**3}*x^{**2}/(6912*a^{**3} \\
& *f^{**4}*tanh(e + f*x)**3 + 20736*a^{**3}*f^{**4}*tanh(e + f*x)**2 + 20736*a^{**3}*f^{**4} \\
& *tanh(e + f*x) + 6912*a^{**3}*f^{**4}) - 5004*c*d^{**2}*f^{**2}*x*tanh(e + f*x)**3/(691 \\
& 2*a^{**3}*f^{**4}*tanh(e + f*x)**3 + 20736*a^{**3}*f^{**4}*tanh(e + f*x)**2 + 20736*a^{** \\
& 3}*f^{**4}*tanh(e + f*x) + 6912*a^{**3}*f^{**4}) - 2484*c*d^{**2}*f^{**2}*x*tanh(e + f*x)** \\
& 2/(6912*a^{**3}*f^{**4}*tanh(e + f*x)**3 + 20736*a^{**3}*f^{**4}*tanh(e + f*x)**2 + 207 \\
& 36*a^{**3}*f^{**4}*tanh(e + f*x) + 6912*a^{**3}*f^{**4}) + 4428*c*d^{**2}*f^{**2}*x*tanh(e + \\
& f*x)/(6912*a^{**3}*f^{**4}*tanh(e + f*x)**3 + 20736*a^{**3}*f^{**4}*tanh(e + f*x)**2 + \\
& 20736*a^{**3}*f^{**4}*tanh(e + f*x) + 6912*a^{**3}*f^{**4}) + 3060*c*d^{**2}*f^{**2}*x/(6912* \\
& a^{**3}*f^{**4}*tanh(e + f*x)**3 + 20736*a^{**3}*f^{**4}*tanh(e + f*x)**2 + 20736*a^{**3}* \\
& f^{**4}*tanh(e + f*x) + 6912*a^{**3}*f^{**4}) - 2916*c*d^{**2}*f*tanh(e + f*x)**3/(6912 \\
& *a^{**3}*f^{**4}*tanh(e + f*x)**3 + 20736*a^{**3}*f^{**4}*tanh(e + f*x)**2 + 20736*a^{**3} \\
& *f^{**4}*tanh(e + f*x) + 6912*a^{**3}*f^{**4}) - 3744*c*d^{**2}*f*tanh(e + f*x)**2/(691 \\
& 2*a^{**3}*f^{**4}*tanh(e + f*x)**3 + 20736*a^{**3}*f^{**4}*tanh(e + f*x)**2 + 20736*a^{** \\
& 3}*f^{**4}*tanh(e + f*x) + 6912*a^{**3}*f^{**4}) + 1020*c*d^{**2}*f/(6912*a^{**3}*f^{**4}*tanh \\
& (e + f*x)**3 + 20736*a^{**3}*f^{**4}*tanh(e + f*x)**2 + 20736*a^{**3}*f^{**4}*tanh(e + \\
& f*x) + 6912*a^{**3}*f^{**4}) + 216*d^{**3}*f^{**4}*x^{**4}*tanh(e + f*x)**3/(6912*a^{**3}*f^{** \\
& 4}*tanh(e + f*x)**3 + 20736*a^{**3}*f^{**4}*tanh(e + f*x)**2 + 20736*a^{**3}*f^{**4}*tan \\
& h(e + f*x) + 6912*a^{**3}*f^{**4}) + 648*d^{**3}*f^{**4}*x^{**4}*tanh(e + f*x)**2/(6912*a^{** \\
& *3}*f^{**4}*tanh(e + f*x)**3 + 20736*a^{**3}*f^{**4}*tanh(e + f*x)**2 + 20736*a^{**3}*f^{** \\
& *4}*tanh(e + f*x) + 6912*a^{**3}*f^{**4}) + 648*d^{**3}*f^{**4}*x^{**4}*tanh(e + f*x)/(6912 \\
& *a^{**3}*f^{**4}*tanh(e + f*x)**3 + 20736*a^{**3}*f^{**4}*tanh(e + f*x)**2 + 20736*a^{**3} \\
& *f^{**4}*tanh(e + f*x) + 6912*a^{**3}*f^{**4}) + 216*d^{**3}*f^{**4}*x^{**4}/(6912*a^{**3}*f^{**4} \\
& *tanh(e + f*x)**3 + 20736*a^{**3}*f^{**4}*tanh(e + f*x)**2 + 20736*a^{**3}*f^{**4}*tanh(\\
& e + f*x) + 6912*a^{**3}*f^{**4}) - 2088*d^{**3}*f^{**3}*x^{**3}*tanh(e + f*x)**3/(6912*a^{** \\
& 3}*f^{**4}*tanh(e + f*x)**3 + 20736*a^{**3}*f^{**4}*tanh(e + f*x)**2 + 20736*a^{**3}*f^{** \\
& 4}*tanh(e + f*x) + 6912*a^{**3}*f^{**4}) - 216*d^{**3}*f^{**3}*x^{**3}*tanh(e + f*x)**2/(69
\end{aligned}$$

```

12*a**3*f**4*tanh(e + f*x)**3 + 20736*a**3*f**4*tanh(e + f*x)**2 + 20736*a
**3*f**4*tanh(e + f*x) + 6912*a**3*f**4) + 1512*d**3*f**3*x**3*tanh(e + f*x)
/(6912*a**3*f**4*tanh(e + f*x)**3 + 20736*a**3*f**4*tanh(e + f*x)**2 + 2073
6*a**3*f**4*tanh(e + f*x) + 6912*a**3*f**4) + 792*d**3*f**3*x**3/(6912*a**3
*f**4*tanh(e + f*x)**3 + 20736*a**3*f**4*tanh(e + f*x)**2 + 20736*a**3*f**4
*tanh(e + f*x) + 6912*a**3*f**4) - 2502*d**3*f**2*x**2*tanh(e + f*x)**3/(69
12*a**3*f**4*tanh(e + f*x)**3 + 20736*a**3*f**4*tanh(e + f*x)**2 + 20736*a
**3*f**4*tanh(e + f*x) + 6912*a**3*f**4) - 1242*d**3*f**2*x**2*tanh(e + f*x)
**2/(6912*a**3*f**4*tanh(e + f*x)**3 + 20736*a**3*f**4*tanh(e + f*x)**2 + 2
0736*a**3*f**4*tanh(e + f*x) + 6912*a**3*f**4) + 2214*d**3*f**2*x**2*tanh(e
+ f*x)/(6912*a**3*f**4*tanh(e + f*x)**3 + 20736*a**3*f**4*tanh(e + f*x)**2
+ 20736*a**3*f**4*tanh(e + f*x) + 6912*a**3*f**4) + 1530*d**3*f**2*x**2/(6
912*a**3*f**4*tanh(e + f*x)**3 + 20736*a**3*f**4*tanh(e + f*x)**2 + 20736*a
**3*f**4*tanh(e + f*x) + 6912*a**3*f**4) - 2211*d**3*f*x*tanh(e + f*x)**3/(
6912*a**3*f**4*tanh(e + f*x)**3 + 20736*a**3*f**4*tanh(e + f*x)**2 + 20736*
a**3*f**4*tanh(e + f*x) + 6912*a**3*f**4) - 1629*d**3*f*x*tanh(e + f*x)**2/
(6912*a**3*f**4*tanh(e + f*x)**3 + 20736*a**3*f**4*tanh(e + f*x)**2 + 20736
*a**3*f**4*tanh(e + f*x) + 6912*a**3*f**4) + 2115*d**3*f*x*tanh(e + f*x)/(6
912*a**3*f**4*tanh(e + f*x)**3 + 20736*a**3*f**4*tanh(e + f*x)**2 + 20736*a
**3*f**4*tanh(e + f*x) + 6912*a**3*f**4) + 1725*d**3*f*x/(6912*a**3*f**4*ta
nh(e + f*x)**3 + 20736*a**3*f**4*tanh(e + f*x)**2 + 20736*a**3*f**4*tanh(e
+ f*x) + 6912*a**3*f**4) - 1377*d**3*tanh(e + f*x)**3/(6912*a**3*f**4*tanh(
e + f*x)**3 + 20736*a**3*f**4*tanh(e + f*x)**2 + 20736*a**3*f**4*tanh(e + f
*x) + 6912*a**3*f**4) - 1920*d**3*tanh(e + f*x)**2/(6912*a**3*f**4*tanh(e +
f*x)**3 + 20736*a**3*f**4*tanh(e + f*x)**2 + 20736*a**3*f**4*tanh(e + f*x)
+ 6912*a**3*f**4) + 575*d**3/(6912*a**3*f**4*tanh(e + f*x)**3 + 20736*a**3
*f**4*tanh(e + f*x)**2 + 20736*a**3*f**4*tanh(e + f*x) + 6912*a**3*f**4), N
e(f, 0)), ((c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3 + d**3*x**4/4)/(a*coth(e
) + a)**3, True))

```

Giac [A] time = 1.24129, size = 774, normalized size = 2.3

$$(864d^3f^4x^4e^{(6fx+6e)} + 3456cd^2f^4x^3e^{(6fx+6e)} + 5184c^2df^4x^2e^{(6fx+6e)} + 5184d^3f^3x^3e^{(4fx+4e)} - 2592d^3f^3x^3e^{(2fx+2e)} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+a*coth(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/27648*(864*d^3*f^4*x^4*e^(6*f*x + 6*e) + 3456*c*d^2*f^4*x^3*e^(6*f*x + 6*
e) + 5184*c^2*d*f^4*x^2*e^(6*f*x + 6*e) + 5184*d^3*f^3*x^3*e^(4*f*x + 4*e)
- 2592*d^3*f^3*x^3*e^(2*f*x + 2*e) + 576*d^3*f^3*x^3 + 3456*c^3*f^4*x*e^(6*
```

$$\begin{aligned}
& f*x + 6*e) + 15552*c*d^2*f^3*x^2*e^{(4*f*x + 4*e)} - 7776*c*d^2*f^3*x^2*e^{(2* \\
& f*x + 2*e)} + 1728*c*d^2*f^3*x^2 + 15552*c^2*d*f^3*x*e^{(4*f*x + 4*e)} + 7776* \\
& d^3*f^2*x^2*e^{(4*f*x + 4*e)} - 7776*c^2*d*f^3*x*e^{(2*f*x + 2*e)} - 1944*d^3*f \\
& ^2*x^2*e^{(2*f*x + 2*e)} + 1728*c^2*d*f^3*x + 288*d^3*f^2*x^2 + 5184*c^3*f^3* \\
& e^{(4*f*x + 4*e)} + 15552*c*d^2*f^2*x*e^{(4*f*x + 4*e)} - 2592*c^3*f^3*e^{(2*f*x \\
& + 2*e)} - 3888*c*d^2*f^2*x*e^{(2*f*x + 2*e)} + 576*c^3*f^3 + 576*c*d^2*f^2*x \\
& + 7776*c^2*d*f^2*e^{(4*f*x + 4*e)} + 7776*d^3*f*x*e^{(4*f*x + 4*e)} - 1944*c^2* \\
& d*f^2*e^{(2*f*x + 2*e)} - 972*d^3*f*x*e^{(2*f*x + 2*e)} + 288*c^2*d*f^2 + 96*d^ \\
& 3*f*x + 7776*c*d^2*f*e^{(4*f*x + 4*e)} - 972*c*d^2*f*e^{(2*f*x + 2*e)} + 96*c*d \\
& ^2*f + 3888*d^3*e^{(4*f*x + 4*e)} - 243*d^3*e^{(2*f*x + 2*e)} + 16*d^3)*e^{(-6*f \\
& *x - 6*e)}/(a^3*f^4)
\end{aligned}$$

$$3.28 \quad \int \frac{(c+dx)^2}{(a+a \coth(e+fx))^3} dx$$

Optimal. Leaf size=246

$$\frac{d(c+dx)e^{-6e-6fx}}{144a^3f^2} - \frac{3d(c+dx)e^{-4e-4fx}}{64a^3f^2} + \frac{3d(c+dx)e^{-2e-2fx}}{16a^3f^2} + \frac{(c+dx)^2e^{-6e-6fx}}{48a^3f} - \frac{3(c+dx)^2e^{-4e-4fx}}{32a^3f} + \frac{3(c+dx)^2e^{-2e-2fx}}{16a^3f}$$

[Out] (d^2*E^(-6*e - 6*f*x))/(864*a^3*f^3) - (3*d^2*E^(-4*e - 4*f*x))/(256*a^3*f^3) + (3*d^2*E^(-2*e - 2*f*x))/(32*a^3*f^3) + (d*E^(-6*e - 6*f*x)*(c + d*x))/(144*a^3*f^2) - (3*d*E^(-4*e - 4*f*x)*(c + d*x))/(64*a^3*f^2) + (3*d*E^(-2*e - 2*f*x)*(c + d*x))/(16*a^3*f^2) + (E^(-6*e - 6*f*x)*(c + d*x)^2)/(48*a^3*f) - (3*E^(-4*e - 4*f*x)*(c + d*x)^2)/(32*a^3*f) + (3*E^(-2*e - 2*f*x)*(c + d*x)^2)/(16*a^3*f) + (c + d*x)^3/(24*a^3*d)

Rubi [A] time = 0.257589, antiderivative size = 246, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.15$, Rules used = {3729, 2176, 2194}

$$\frac{d(c+dx)e^{-6e-6fx}}{144a^3f^2} - \frac{3d(c+dx)e^{-4e-4fx}}{64a^3f^2} + \frac{3d(c+dx)e^{-2e-2fx}}{16a^3f^2} + \frac{(c+dx)^2e^{-6e-6fx}}{48a^3f} - \frac{3(c+dx)^2e^{-4e-4fx}}{32a^3f} + \frac{3(c+dx)^2e^{-2e-2fx}}{16a^3f}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)^2/(a + a*Coth[e + f*x])^3,x]

[Out] (d^2*E^(-6*e - 6*f*x))/(864*a^3*f^3) - (3*d^2*E^(-4*e - 4*f*x))/(256*a^3*f^3) + (3*d^2*E^(-2*e - 2*f*x))/(32*a^3*f^3) + (d*E^(-6*e - 6*f*x)*(c + d*x))/(144*a^3*f^2) - (3*d*E^(-4*e - 4*f*x)*(c + d*x))/(64*a^3*f^2) + (3*d*E^(-2*e - 2*f*x)*(c + d*x))/(16*a^3*f^2) + (E^(-6*e - 6*f*x)*(c + d*x)^2)/(48*a^3*f) - (3*E^(-4*e - 4*f*x)*(c + d*x)^2)/(32*a^3*f) + (3*E^(-2*e - 2*f*x)*(c + d*x)^2)/(16*a^3*f) + (c + d*x)^3/(24*a^3*d)

Rule 3729

Int[((c_.) + (d_.)*(x_))^(m_)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (1/(2*a) + E^((2*a*(e + f*x))/b)/(2*a))^(n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && ILtQ[n, 0]

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] := Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] && !$UseGamma == True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] := Simp[(F^(c*(a +
b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned} \int \frac{(c + dx)^2}{(a + a \coth(e + fx))^3} dx &= \int \left(\frac{(c + dx)^2}{8a^3} - \frac{e^{-6e-6fx}(c + dx)^2}{8a^3} + \frac{3e^{-4e-4fx}(c + dx)^2}{8a^3} - \frac{3e^{-2e-2fx}(c + dx)^2}{8a^3} \right) dx \\ &= \frac{(c + dx)^3}{24a^3d} - \frac{\int e^{-6e-6fx}(c + dx)^2 dx}{8a^3} + \frac{3 \int e^{-4e-4fx}(c + dx)^2 dx}{8a^3} - \frac{3 \int e^{-2e-2fx}(c + dx)^2 dx}{8a^3} \\ &= \frac{e^{-6e-6fx}(c + dx)^2}{48a^3f} - \frac{3e^{-4e-4fx}(c + dx)^2}{32a^3f} + \frac{3e^{-2e-2fx}(c + dx)^2}{16a^3f} + \frac{(c + dx)^3}{24a^3d} - \frac{d \int e^{-6e-6fx}(c + dx)^2 dx}{24a^3f} \\ &= \frac{de^{-6e-6fx}(c + dx)}{144a^3f^2} - \frac{3de^{-4e-4fx}(c + dx)}{64a^3f^2} + \frac{3de^{-2e-2fx}(c + dx)}{16a^3f^2} + \frac{e^{-6e-6fx}(c + dx)^2}{48a^3f} - \frac{3e^{-4e-4fx}(c + dx)^2}{32a^3f} \\ &= \frac{d^2e^{-6e-6fx}}{864a^3f^3} - \frac{3d^2e^{-4e-4fx}}{256a^3f^3} + \frac{3d^2e^{-2e-2fx}}{32a^3f^3} + \frac{de^{-6e-6fx}(c + dx)}{144a^3f^2} - \frac{3de^{-4e-4fx}(c + dx)}{64a^3f^2} + \frac{3de^{-2e-2fx}(c + dx)}{16a^3f^2} \end{aligned}$$

Mathematica [A] time = 1.37168, size = 371, normalized size = 1.51

$$\frac{\operatorname{csch}^3(e + fx) \left(81 \left(8c^2f^2 + 4cdf(4fx + 3) + d^2(8f^2x^2 + 12fx + 7) \right) \cosh(e + fx) + 8 \left(18c^2f^2(6fx + 1) + 6cdf(18f^2x^2 + 12fx + 7) \right) \operatorname{cosh}^2(e + fx) + 8 \left(18c^2f^2(6fx + 1) + 6cdf(18f^2x^2 + 12fx + 7) \right) \operatorname{cosh}^3(e + fx) \right)}{144a^3f^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2/(a + a*Coth[e + f*x])^3,x]
```

```
[Out] (Csch[e + f*x]^3*(81*(8*c^2*f^2 + 4*c*d*f*(3 + 4*f*x) + d^2*(7 + 12*f*x + 8
*f^2*x^2))*Cosh[e + f*x] + 8*(18*c^2*f^2*(1 + 6*f*x) + 6*c*d*f*(1 + 6*f*x +
18*f^2*x^2) + d^2*(1 + 6*f*x + 18*f^2*x^2 + 36*f^3*x^3))*Cosh[3*(e + f*x)]
+ 729*d^2*Sinh[e + f*x] + 1620*c*d*f*Sinh[e + f*x] + 1944*c^2*f^2*Sinh[e +
f*x] + 1620*d^2*f*x*Sinh[e + f*x] + 3888*c*d*f^2*x*Sinh[e + f*x] + 1944*d^
2*f^2*x^2*Sinh[e + f*x] - 8*d^2*Sinh[3*(e + f*x)] - 48*c*d*f*Sinh[3*(e + f*
x)] - 144*c^2*f^2*Sinh[3*(e + f*x)] - 48*d^2*f*x*Sinh[3*(e + f*x)] - 288*c
```

$$\frac{d^2 f^2 x \operatorname{Sinh}[3(e + f x)] + 864 c^2 f^3 x \operatorname{Sinh}[3(e + f x)] - 144 d^2 f^2 x^2 \operatorname{Sinh}[3(e + f x)] + 864 c d f^3 x^2 \operatorname{Sinh}[3(e + f x)] + 288 d^2 f^3 x^3 \operatorname{Sinh}[3(e + f x)]}{(6912 a^3 f^3 (1 + \operatorname{Coth}[e + f x])^3)}$$

Maple [B] time = 0.125, size = 2098, normalized size = 8.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x+c)^2/(a+a*coth(f*x+e))^3,x)`

[Out] $\frac{1}{a^3 f} \left(-\frac{4}{f^2 d^2} \left(\frac{1}{6} (f x + e)^2 \sinh(f x + e)^3 \cosh(f x + e)^3 - \frac{1}{8} (f x + e)^2 \sinh(f x + e) \cosh(f x + e)^3 + \frac{1}{16} (f x + e)^2 \cosh(f x + e) \sinh(f x + e) + \frac{1}{48} (f x + e)^3 - \frac{1}{18} (f x + e) \sinh(f x + e)^2 \cosh(f x + e)^4 + \frac{13}{144} (f x + e) \sinh(f x + e)^2 \cosh(f x + e)^2 + \frac{1}{36} (f x + e) \cosh(f x + e)^2 + \frac{1}{108} \sinh(f x + e) \cosh(f x + e)^5 - \frac{43}{1728} \cosh(f x + e)^3 \sinh(f x + e) - \frac{7}{1152} \cosh(f x + e) \sinh(f x + e) - \frac{7}{1152} f x - \frac{7}{1152} e \right) + \frac{4}{f^2 d^2} \left(\frac{1}{6} (f x + e)^2 \sinh(f x + e)^2 \cosh(f x + e)^4 - \frac{1}{12} (f x + e)^2 \sinh(f x + e)^2 \cosh(f x + e)^2 - \frac{1}{12} (f x + e)^2 \cosh(f x + e)^2 - \frac{1}{18} (f x + e) \sinh(f x + e) \cosh(f x + e)^5 + \frac{1}{18} (f x + e) \sinh(f x + e) \cosh(f x + e)^3 + \frac{1}{108} \sinh(f x + e)^2 \cosh(f x + e)^4 - \frac{1}{216} \sinh(f x + e)^2 \cosh(f x + e)^2 - \frac{5}{108} \cosh(f x + e)^2 + \frac{1}{12} (f x + e) \cosh(f x + e) \sinh(f x + e) + \frac{1}{24} (f x + e)^2 \right) + \frac{1}{f^2 d^2} \left(\frac{1}{4} (f x + e)^2 \cosh(f x + e) \sinh(f x + e)^3 - \frac{3}{8} (f x + e)^2 \cosh(f x + e) \sinh(f x + e) + \frac{1}{8} (f x + e)^3 - \frac{1}{8} (f x + e) \sinh(f x + e)^2 \cosh(f x + e)^2 + \frac{1}{2} (f x + e) \cosh(f x + e)^2 + \frac{1}{32} \cosh(f x + e)^3 \sinh(f x + e) - \frac{17}{64} \cosh(f x + e) \sinh(f x + e) - \frac{17}{64} f x - \frac{17}{64} e \right) - \frac{3}{f^2 d^2} \left(\frac{1}{4} (f x + e)^2 \sinh(f x + e)^2 \cosh(f x + e)^2 - \frac{1}{4} (f x + e)^2 \cosh(f x + e)^2 - \frac{1}{8} (f x + e) \sinh(f x + e) \cosh(f x + e)^3 + \frac{5}{16} (f x + e) \cosh(f x + e) \sinh(f x + e) + \frac{5}{32} (f x + e)^2 + \frac{1}{32} \sinh(f x + e)^2 \cosh(f x + e)^2 - \frac{1}{8} \cosh(f x + e)^2 \right) + \frac{8}{f^2 d^2} e \left(\frac{1}{6} (f x + e) \sinh(f x + e)^3 \cosh(f x + e)^3 - \frac{1}{8} (f x + e) \sinh(f x + e) \cosh(f x + e)^3 + \frac{1}{16} (f x + e) \cosh(f x + e) \sinh(f x + e) + \frac{1}{32} (f x + e)^2 - \frac{1}{36} \sinh(f x + e)^2 \cosh(f x + e)^4 + \frac{13}{288} \sinh(f x + e)^2 \cosh(f x + e)^2 + \frac{1}{72} \cosh(f x + e)^2 \right) - \frac{8}{f^2 d^2} e \left(\frac{1}{6} (f x + e) \sinh(f x + e)^2 \cosh(f x + e)^4 - \frac{1}{12} (f x + e) \sinh(f x + e)^2 \cosh(f x + e)^2 - \frac{1}{12} (f x + e) \cosh(f x + e)^2 - \frac{1}{36} \sinh(f x + e) \cosh(f x + e)^5 + \frac{1}{36} \cosh(f x + e)^3 \sinh(f x + e) + \frac{1}{24} \cosh(f x + e) \sinh(f x + e) + \frac{1}{24} f x + \frac{1}{24} e \right) - \frac{2}{f^2 d^2} e \left(\frac{1}{4} (f x + e) \cosh(f x + e) \sinh(f x + e)^3 - \frac{3}{8} (f x + e) \cosh(f x + e) \sinh(f x + e) + \frac{3}{16} (f x + e)^2 - \frac{1}{16} \sinh(f x + e)^2 \cosh(f x + e)^2 + \frac{1}{4} \cosh(f x + e)^2 \right) + \frac{6}{f^2 d^2} e \left(\frac{1}{4} (f x + e) \sinh(f x + e)^2 \cosh(f x + e)^2 - \frac{1}{4} (f x + e) \cosh(f x + e)^2 - \frac{1}{16} \cosh(f x + e)^3 \sinh(f x + e) + \frac{5}{32} \cosh(f x + e) \sinh(f x + e) + \frac{5}{32} f x + \frac{5}{32} e \right) - \frac{4}{f^2 d^2} e^2 \left(\frac{1}{6} \sinh(f x + e)^3 \cosh(f x + e)^3 - \frac{1}{8} \cosh(f x + e)^3 \sinh(f x + e) + \frac{1}{16} \cosh(f x + e) \sinh(f x + e) + \frac{1}{16} f x + \frac{1}{16} e \right) + \frac{4}{f^2 d^2} e^2 \left(\frac{1}{6} \sinh(f x + e)^2 \cosh(f x + e)^4 - \frac{1}{12} \sinh(f x + e)^2 \cosh(f x + e)^2 - \frac{1}{12} \cosh(f x + e)^2 \right) + \frac{1}{f^2 d^2} e^2 \left(\left(\frac{1}{4} \sinh(f x + e)^3 - \frac{3}{8} \sinh(f x + e) \right) \cosh \right.$

$(f*x+e)+3/8*f*x+3/8*e)-3/f^2*d^2*e^2*(1/4*\sinh(f*x+e)^2*\cosh(f*x+e)^2-1/4*\cosh(f*x+e)^2)-8*c/f*d*(1/6*(f*x+e)*\sinh(f*x+e)^3*\cosh(f*x+e)^3-1/8*(f*x+e)*\sinh(f*x+e)*\cosh(f*x+e)^3+1/16*(f*x+e)*\cosh(f*x+e)*\sinh(f*x+e)+1/32*(f*x+e)^2-1/36*\sinh(f*x+e)^2*\cosh(f*x+e)^4+13/288*\sinh(f*x+e)^2*\cosh(f*x+e)^2+1/72*\cosh(f*x+e)^2)+8*c/f*d*(1/6*(f*x+e)*\sinh(f*x+e)^2*\cosh(f*x+e)^4-1/12*(f*x+e)*\sinh(f*x+e)^2*\cosh(f*x+e)^2-1/12*(f*x+e)*\cosh(f*x+e)^2-1/36*\sinh(f*x+e)*\cosh(f*x+e)^5+1/36*\cosh(f*x+e)^3*\sinh(f*x+e)+1/24*\cosh(f*x+e)*\sinh(f*x+e)+1/24*f*x+1/24*e)+2*c/f*d*(1/4*(f*x+e)*\cosh(f*x+e)*\sinh(f*x+e)^3-3/8*(f*x+e)*\cosh(f*x+e)*\sinh(f*x+e)+3/16*(f*x+e)^2-1/16*\sinh(f*x+e)^2*\cosh(f*x+e)^2+1/4*\cosh(f*x+e)^2)-6*c/f*d*(1/4*(f*x+e)*\sinh(f*x+e)^2*\cosh(f*x+e)^2-1/4*(f*x+e)*\cosh(f*x+e)^2-1/16*\cosh(f*x+e)^3*\sinh(f*x+e)+5/32*\cosh(f*x+e)*\sinh(f*x+e)+5/32*f*x+5/32*e)+8*c/f*d*e*(1/6*\sinh(f*x+e)^3*\cosh(f*x+e)^3-1/8*\cosh(f*x+e)^3*\sinh(f*x+e)+1/16*\cosh(f*x+e)*\sinh(f*x+e)+1/16*f*x+1/16*e)-8*c/f*d*e*(1/6*\sinh(f*x+e)^2*\cosh(f*x+e)^4-1/12*\sinh(f*x+e)^2*\cosh(f*x+e)^2-1/12*\cosh(f*x+e)^2)-2*c/f*d*e*((1/4*\sinh(f*x+e)^3-3/8*\sinh(f*x+e))*\cosh(f*x+e)+3/8*f*x+3/8*e)+6*c/f*d*e*(1/4*\sinh(f*x+e)^2*\cosh(f*x+e)^2-1/4*\cosh(f*x+e)^2)-4*c^2*(1/6*\sinh(f*x+e)^3*\cosh(f*x+e)^3-1/8*\cosh(f*x+e)^3*\sinh(f*x+e)+1/16*\cosh(f*x+e)*\sinh(f*x+e)+1/16*f*x+1/16*e)+4*c^2*(1/6*\sinh(f*x+e)^2*\cosh(f*x+e)^4-1/12*\sinh(f*x+e)^2*\cosh(f*x+e)^2-1/12*\cosh(f*x+e)^2)+c^2*((1/4*\sinh(f*x+e)^3-3/8*\sinh(f*x+e))*\cosh(f*x+e)+3/8*f*x+3/8*e)-3*c^2*(1/4*\sinh(f*x+e)^2*\cosh(f*x+e)^2-1/4*\cosh(f*x+e)^2))$

Maxima [A] time = 5.60223, size = 343, normalized size = 1.39

$$\frac{1}{96}c^2\left(\frac{12(fx+e)}{a^3f} + \frac{18e^{(-2fx-2e)} - 9e^{(-4fx-4e)} + 2e^{(-6fx-6e)}}{a^3f}\right) + \frac{(72f^2x^2e^{(6e)} + 108(2fxe^{(4e)} + e^{(4e)})e^{(-2fx)} - 27(4fx+1)e^{(-6fx)})*c*d*e^{(-6e)} + 1/6912*(288*f^3*x^3*e^{(6e)} + 648*(2*f^2*x^2*e^{(4e)} + 2*f*x*e^{(4e)} + e^{(4e)})*e^{(-2*f*x)} - 81*(8*f^2*x^2*e^{(2e)} + 4*f*x*e^{(2e)} + e^{(2e)})*e^{(-4*f*x)} + 8*(18*f^2*x^2 + 6*f*x + 1)*e^{(-6*f*x)})*d^2*e^{(-6e)}}{576a^3f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2/(a+a*coth(f*x+e))^3,x, algorithm="maxima")

[Out] 1/96*c^2*(12*(f*x + e)/(a^3*f) + (18*e^(-2*f*x - 2*e) - 9*e^(-4*f*x - 4*e) + 2*e^(-6*f*x - 6*e))/(a^3*f)) + 1/576*(72*f^2*x^2*e^(6*e) + 108*(2*f*x*e^(4*e) + e^(4*e))*e^(-2*f*x) - 27*(4*f*x*e^(2*e) + e^(2*e))*e^(-4*f*x) + 4*(6*f*x + 1)*e^(-6*f*x))*c*d*e^(-6*e)/(a^3*f^2) + 1/6912*(288*f^3*x^3*e^(6*e) + 648*(2*f^2*x^2*e^(4*e) + 2*f*x*e^(4*e) + e^(4*e))*e^(-2*f*x) - 81*(8*f^2*x^2*e^(2*e) + 4*f*x*e^(2*e) + e^(2*e))*e^(-4*f*x) + 8*(18*f^2*x^2 + 6*f*x + 1)*e^(-6*f*x))*d^2*e^(-6*e)/(a^3*f^3)

Fricas [B] time = 2.13223, size = 1215, normalized size = 4.94

$$8(36d^2f^3x^3 + 18c^2f^2 + 6cdf + 18(6cdf^3 + d^2f^2)x^2 + d^2 + 6(18c^2f^3 + 6cdf^2 + d^2f)x) \cosh(fx + e)^3 + 24(36d^2f^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2/(a+a*coth(f*x+e))^3,x, algorithm="fricas")

[Out] 1/6912*(8*(36*d^2*f^3*x^3 + 18*c^2*f^2 + 6*c*d*f + 18*(6*c*d*f^3 + d^2*f^2)*x^2 + d^2 + 6*(18*c^2*f^3 + 6*c*d*f^2 + d^2*f)*x)*cosh(f*x + e)^3 + 24*(36*d^2*f^3*x^3 + 18*c^2*f^2 + 6*c*d*f + 18*(6*c*d*f^3 + d^2*f^2)*x^2 + d^2 + 6*(18*c^2*f^3 + 6*c*d*f^2 + d^2*f)*x)*cosh(f*x + e)*sinh(f*x + e)^2 + 8*(36*d^2*f^3*x^3 - 18*c^2*f^2 - 6*c*d*f + 18*(6*c*d*f^3 - d^2*f^2)*x^2 - d^2 + 6*(18*c^2*f^3 - 6*c*d*f^2 - d^2*f)*x)*sinh(f*x + e)^3 + 81*(8*d^2*f^2*x^2 + 8*c^2*f^2 + 12*c*d*f + 7*d^2 + 4*(4*c*d*f^2 + 3*d^2*f)*x)*cosh(f*x + e) + 3*(648*d^2*f^2*x^2 + 648*c^2*f^2 + 540*c*d*f + 8*(36*d^2*f^3*x^3 - 18*c^2*f^2 - 6*c*d*f + 18*(6*c*d*f^3 - d^2*f^2)*x^2 - d^2 + 6*(18*c^2*f^3 - 6*c*d*f^2 - d^2*f)*x)*cosh(f*x + e)^2 + 243*d^2 + 108*(12*c*d*f^2 + 5*d^2*f)*x)*sinh(f*x + e))/(a^3*f^3*cosh(f*x + e)^3 + 3*a^3*f^3*cosh(f*x + e)^2*sinh(f*x + e) + 3*a^3*f^3*cosh(f*x + e)*sinh(f*x + e)^2 + a^3*f^3*sinh(f*x + e)^3)

Sympy [A] time = 5.25176, size = 2448, normalized size = 9.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**2/(a+a*coth(f*x+e))**3,x)

[Out] Piecewise(((216*c**2*f**3*x*tanh(e + f*x)**3/(1728*a**3*f**3*tanh(e + f*x)**3 + 5184*a**3*f**3*tanh(e + f*x)**2 + 5184*a**3*f**3*tanh(e + f*x) + 1728*a**3*f**3) + 648*c**2*f**3*x*tanh(e + f*x)**2/(1728*a**3*f**3*tanh(e + f*x)**3 + 5184*a**3*f**3*tanh(e + f*x)**2 + 5184*a**3*f**3*tanh(e + f*x) + 1728*a**3*f**3) + 648*c**2*f**3*x*tanh(e + f*x)/(1728*a**3*f**3*tanh(e + f*x)**3 + 5184*a**3*f**3*tanh(e + f*x)**2 + 5184*a**3*f**3*tanh(e + f*x) + 1728*a**3*f**3) + 216*c**2*f**3*x/(1728*a**3*f**3*tanh(e + f*x)**3 + 5184*a**3*f**3*tanh(e + f*x)**2 + 5184*a**3*f**3*tanh(e + f*x) + 1728*a**3*f**3) - 648*c**2*f**2*tanh(e + f*x)**3/(1728*a**3*f**3*tanh(e + f*x)**3 + 5184*a**3*f**3*tanh(e + f*x)**2 + 5184*a**3*f**3*tanh(e + f*x) + 1728*a**3*f**3) - 432*c

$$\begin{aligned}
& *2*f**2*tanh(e + f*x)**2/(1728*a**3*f**3*tanh(e + f*x)**3 + 5184*a**3*f**3* \\
& tanh(e + f*x)**2 + 5184*a**3*f**3*tanh(e + f*x) + 1728*a**3*f**3) + 72*c**2 \\
& *f**2/(1728*a**3*f**3*tanh(e + f*x)**3 + 5184*a**3*f**3*tanh(e + f*x)**2 + \\
& 5184*a**3*f**3*tanh(e + f*x) + 1728*a**3*f**3) + 216*c*d*f**3*x**2*tanh(e + \\
& f*x)**3/(1728*a**3*f**3*tanh(e + f*x)**3 + 5184*a**3*f**3*tanh(e + f*x)**2 \\
& + 5184*a**3*f**3*tanh(e + f*x) + 1728*a**3*f**3) + 648*c*d*f**3*x**2*tanh(\\
& e + f*x)**2/(1728*a**3*f**3*tanh(e + f*x)**3 + 5184*a**3*f**3*tanh(e + f*x) \\
& **2 + 5184*a**3*f**3*tanh(e + f*x) + 1728*a**3*f**3) + 648*c*d*f**3*x**2*ta \\
& nh(e + f*x)/(1728*a**3*f**3*tanh(e + f*x)**3 + 5184*a**3*f**3*tanh(e + f*x) \\
& **2 + 5184*a**3*f**3*tanh(e + f*x) + 1728*a**3*f**3) + 216*c*d*f**3*x**2/(1 \\
& 728*a**3*f**3*tanh(e + f*x)**3 + 5184*a**3*f**3*tanh(e + f*x)**2 + 5184*a** \\
& 3*f**3*tanh(e + f*x) + 1728*a**3*f**3) - 1044*c*d*f**2*x*tanh(e + f*x)**3/(\\
& 1728*a**3*f**3*tanh(e + f*x)**3 + 5184*a**3*f**3*tanh(e + f*x)**2 + 5184*a* \\
& *3*f**3*tanh(e + f*x) + 1728*a**3*f**3) - 108*c*d*f**2*x*tanh(e + f*x)**2/(\\
& 1728*a**3*f**3*tanh(e + f*x)**3 + 5184*a**3*f**3*tanh(e + f*x)**2 + 5184*a* \\
& *3*f**3*tanh(e + f*x) + 1728*a**3*f**3) + 756*c*d*f**2*x*tanh(e + f*x)/(172 \\
& 8*a**3*f**3*tanh(e + f*x)**3 + 5184*a**3*f**3*tanh(e + f*x)**2 + 5184*a**3* \\
& f**3*tanh(e + f*x) + 1728*a**3*f**3) + 396*c*d*f**2*x/(1728*a**3*f**3*tanh(\\
& e + f*x)**3 + 5184*a**3*f**3*tanh(e + f*x)**2 + 5184*a**3*f**3*tanh(e + f*x \\
&) + 1728*a**3*f**3) - 540*c*d*f*tanh(e + f*x)**3/(1728*a**3*f**3*tanh(e + f \\
& x)**3 + 5184*a**3*f**3*tanh(e + f*x)**2 + 5184*a**3*f**3*tanh(e + f*x) + 1 \\
& 728*a**3*f**3) - 576*c*d*f*tanh(e + f*x)**2/(1728*a**3*f**3*tanh(e + f*x)** \\
& 3 + 5184*a**3*f**3*tanh(e + f*x)**2 + 5184*a**3*f**3*tanh(e + f*x) + 1728*a \\
& **3*f**3) + 132*c*d*f/(1728*a**3*f**3*tanh(e + f*x)**3 + 5184*a**3*f**3*tan \\
& h(e + f*x)**2 + 5184*a**3*f**3*tanh(e + f*x) + 1728*a**3*f**3) + 72*d**2*f* \\
& *3*x**3*tanh(e + f*x)**3/(1728*a**3*f**3*tanh(e + f*x)**3 + 5184*a**3*f**3* \\
& tanh(e + f*x)**2 + 5184*a**3*f**3*tanh(e + f*x) + 1728*a**3*f**3) + 216*d** \\
& 2*f**3*x**3*tanh(e + f*x)**2/(1728*a**3*f**3*tanh(e + f*x)**3 + 5184*a**3*f \\
& **3*tanh(e + f*x)**2 + 5184*a**3*f**3*tanh(e + f*x) + 1728*a**3*f**3) + 216 \\
& *d**2*f**3*x**3*tanh(e + f*x)/(1728*a**3*f**3*tanh(e + f*x)**3 + 5184*a**3* \\
& f**3*tanh(e + f*x)**2 + 5184*a**3*f**3*tanh(e + f*x) + 1728*a**3*f**3) + 72 \\
& *d**2*f**3*x**3/(1728*a**3*f**3*tanh(e + f*x)**3 + 5184*a**3*f**3*tanh(e + \\
& f*x)**2 + 5184*a**3*f**3*tanh(e + f*x) + 1728*a**3*f**3) - 522*d**2*f**2*x* \\
& *2*tanh(e + f*x)**3/(1728*a**3*f**3*tanh(e + f*x)**3 + 5184*a**3*f**3*tanh(\\
& e + f*x)**2 + 5184*a**3*f**3*tanh(e + f*x) + 1728*a**3*f**3) - 54*d**2*f**2 \\
& *x**2*tanh(e + f*x)**2/(1728*a**3*f**3*tanh(e + f*x)**3 + 5184*a**3*f**3*ta \\
& nh(e + f*x)**2 + 5184*a**3*f**3*tanh(e + f*x) + 1728*a**3*f**3) + 378*d**2* \\
& f**2*x**2*tanh(e + f*x)/(1728*a**3*f**3*tanh(e + f*x)**3 + 5184*a**3*f**3*ta \\
& nh(e + f*x)**2 + 5184*a**3*f**3*tanh(e + f*x) + 1728*a**3*f**3) + 198*d**2 \\
& *f**2*x**2/(1728*a**3*f**3*tanh(e + f*x)**3 + 5184*a**3*f**3*tanh(e + f*x)* \\
& *2 + 5184*a**3*f**3*tanh(e + f*x) + 1728*a**3*f**3) - 417*d**2*f*x*tanh(e + \\
& f*x)**3/(1728*a**3*f**3*tanh(e + f*x)**3 + 5184*a**3*f**3*tanh(e + f*x)**2 \\
& + 5184*a**3*f**3*tanh(e + f*x) + 1728*a**3*f**3) - 207*d**2*f*x*tanh(e + f \\
& x)**2/(1728*a**3*f**3*tanh(e + f*x)**3 + 5184*a**3*f**3*tanh(e + f*x)**2 + \\
& 5184*a**3*f**3*tanh(e + f*x) + 1728*a**3*f**3) + 369*d**2*f*x*tanh(e + f*x)
\end{aligned}$$

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)/(1728*a**3*f**3*tanh(e + f*x)**3 + 5184*a**3*f**3*tanh(e + f*x)**2 + 5184
*a**3*f**3*tanh(e + f*x) + 1728*a**3*f**3) + 255*d**2*f*x/(1728*a**3*f**3*t
anh(e + f*x)**3 + 5184*a**3*f**3*tanh(e + f*x)**2 + 5184*a**3*f**3*tanh(e +
f*x) + 1728*a**3*f**3) - 243*d**2*tanh(e + f*x)**3/(1728*a**3*f**3*tanh(e
+ f*x)**3 + 5184*a**3*f**3*tanh(e + f*x)**2 + 5184*a**3*f**3*tanh(e + f*x)
+ 1728*a**3*f**3) - 312*d**2*tanh(e + f*x)**2/(1728*a**3*f**3*tanh(e + f*x)
**3 + 5184*a**3*f**3*tanh(e + f*x)**2 + 5184*a**3*f**3*tanh(e + f*x) + 1728
*a**3*f**3) + 85*d**2/(1728*a**3*f**3*tanh(e + f*x)**3 + 5184*a**3*f**3*tan
h(e + f*x)**2 + 5184*a**3*f**3*tanh(e + f*x) + 1728*a**3*f**3), Ne(f, 0)),
((c**2*x + c*d*x**2 + d**2*x**3/3)/(a*coth(e) + a)**3, True))

```

Giac [A] time = 1.25946, size = 447, normalized size = 1.82

$$\frac{(288 d^2 f^3 x^3 e^{(6fx+6e)} + 864 c d f^3 x^2 e^{(6fx+6e)} + 864 c^2 f^3 x e^{(6fx+6e)} + 1296 d^2 f^2 x^2 e^{(4fx+4e)} - 648 d^2 f^2 x^2 e^{(2fx+2e)} + 144 d^2 f^2 x^2 e^{(2fx+2e)} + 144 d^2 f^2 x^2 e^{(2fx+2e)} + 2592 c d f^2 x^2 e^{(4fx+4e)} - 1296 c d f^2 x^2 e^{(2fx+2e)} + 288 c d f^2 x e^{(4fx+4e)} + 1296 c^2 f^2 x e^{(4fx+4e)} + 1296 d^2 f x e^{(4fx+4e)} - 648 c^2 f^2 x e^{(2fx+2e)} - 324 d^2 f x e^{(2fx+2e)} + 144 c^2 f^2 + 48 d^2 f x + 1296 c d f e^{(4fx+4e)} - 324 c d f e^{(2fx+2e)} + 48 c d f + 648 d^2 e^{(4fx+4e)} - 81 d^2 e^{(2fx+2e)} + 8 d^2) e^{(-6fx-6e)}}{(a^3 f^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+a*coth(f*x+e))^3,x, algorithm="giac")
```

```

[Out] 1/6912*(288*d^2*f^3*x^3*e^(6*f*x + 6*e) + 864*c*d*f^3*x^2*e^(6*f*x + 6*e) +
864*c^2*f^3*x*e^(6*f*x + 6*e) + 1296*d^2*f^2*x^2*e^(4*f*x + 4*e) - 648*d^2
*f^2*x^2*e^(2*f*x + 2*e) + 144*d^2*f^2*x^2 + 2592*c*d*f^2*x*e^(4*f*x + 4*e)
- 1296*c*d*f^2*x*e^(2*f*x + 2*e) + 288*c*d*f^2*x + 1296*c^2*f^2*e^(4*f*x +
4*e) + 1296*d^2*f*x*e^(4*f*x + 4*e) - 648*c^2*f^2*e^(2*f*x + 2*e) - 324*d^
2*f*x*e^(2*f*x + 2*e) + 144*c^2*f^2 + 48*d^2*f*x + 1296*c*d*f*e^(4*f*x + 4*
e) - 324*c*d*f*e^(2*f*x + 2*e) + 48*c*d*f + 648*d^2*e^(4*f*x + 4*e) - 81*d^
2*e^(2*f*x + 2*e) + 8*d^2)*e^(-6*f*x - 6*e)/(a^3*f^3)

```

$$3.29 \quad \int \frac{c+dx}{(a+a \coth(e+fx))^3} dx$$

Optimal. Leaf size=183

$$-\frac{c+dx}{8f(a^3 \coth(e+fx)+a^3)} + \frac{x(c+dx)}{8a^3} - \frac{11d}{96f^2(a^3 \coth(e+fx)+a^3)} + \frac{11dx}{96a^3f} - \frac{dx^2}{16a^3} - \frac{c+dx}{8af(a \coth(e+fx)+a)^2}$$

[Out] (11*d*x)/(96*a^3*f) - (d*x^2)/(16*a^3) + (x*(c + d*x))/(8*a^3) - d/(36*f^2*(a + a*Coth[e + f*x])^3) - (c + d*x)/(6*f*(a + a*Coth[e + f*x])^3) - (5*d)/(96*a*f^2*(a + a*Coth[e + f*x])^2) - (c + d*x)/(8*a*f*(a + a*Coth[e + f*x])^2) - (11*d)/(96*f^2*(a^3 + a^3*Coth[e + f*x])) - (c + d*x)/(8*f*(a^3 + a^3*Coth[e + f*x]))

Rubi [A] time = 0.205172, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3479, 8, 3730}

$$-\frac{c+dx}{8f(a^3 \coth(e+fx)+a^3)} + \frac{x(c+dx)}{8a^3} - \frac{11d}{96f^2(a^3 \coth(e+fx)+a^3)} + \frac{11dx}{96a^3f} - \frac{dx^2}{16a^3} - \frac{c+dx}{8af(a \coth(e+fx)+a)^2}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)/(a + a*Coth[e + f*x])^3,x]

[Out] (11*d*x)/(96*a^3*f) - (d*x^2)/(16*a^3) + (x*(c + d*x))/(8*a^3) - d/(36*f^2*(a + a*Coth[e + f*x])^3) - (c + d*x)/(6*f*(a + a*Coth[e + f*x])^3) - (5*d)/(96*a*f^2*(a + a*Coth[e + f*x])^2) - (c + d*x)/(8*a*f*(a + a*Coth[e + f*x])^2) - (11*d)/(96*f^2*(a^3 + a^3*Coth[e + f*x])) - (c + d*x)/(8*f*(a^3 + a^3*Coth[e + f*x]))

Rule 3479

Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(a*(a + b*Tan[c + d*x])^n)/(2*b*d*n), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 3730

Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.),
 x_Symbol] :> With[{u = IntHide[(a + b*Tan[e + f*x])^n, x]}, Dist[(c + d*x)
 ^m, u, x] - Dist[d*m, Int[Dist[(c + d*x)^(m - 1), u, x], x], x]] /; FreeQ[{
 a, b, c, d, e, f}, x] && EqQ[a^2 + b^2, 0] && ILtQ[n, -1] && GtQ[m, 0]

Rubi steps

$$\begin{aligned} \int \frac{c + dx}{(a + a \coth(e + fx))^3} dx &= \frac{x(c + dx)}{8a^3} - \frac{c + dx}{6f(a + a \coth(e + fx))^3} - \frac{c + dx}{8af(a + a \coth(e + fx))^2} - \frac{c + dx}{8f(a^3 + a^3 \coth(e + fx))} \\ &= -\frac{dx^2}{16a^3} + \frac{x(c + dx)}{8a^3} - \frac{c + dx}{6f(a + a \coth(e + fx))^3} - \frac{c + dx}{8af(a + a \coth(e + fx))^2} - \frac{c + dx}{8f(a^3 + a^3 \coth(e + fx))} \\ &= -\frac{dx^2}{16a^3} + \frac{x(c + dx)}{8a^3} - \frac{d}{36f^2(a + a \coth(e + fx))^3} - \frac{c + dx}{6f(a + a \coth(e + fx))^3} - \frac{c + dx}{32af^2(a + a \coth(e + fx))} \\ &= \frac{dx}{16a^3f} - \frac{dx^2}{16a^3} + \frac{x(c + dx)}{8a^3} - \frac{d}{36f^2(a + a \coth(e + fx))^3} - \frac{c + dx}{6f(a + a \coth(e + fx))^3} - \frac{c + dx}{96af^2(a + a \coth(e + fx))} \\ &= \frac{3dx}{32a^3f} - \frac{dx^2}{16a^3} + \frac{x(c + dx)}{8a^3} - \frac{d}{36f^2(a + a \coth(e + fx))^3} - \frac{c + dx}{6f(a + a \coth(e + fx))^3} - \frac{c + dx}{96af^2(a + a \coth(e + fx))} \\ &= \frac{11dx}{96a^3f} - \frac{dx^2}{16a^3} + \frac{x(c + dx)}{8a^3} - \frac{d}{36f^2(a + a \coth(e + fx))^3} - \frac{c + dx}{6f(a + a \coth(e + fx))^3} - \frac{c + dx}{96af^2(a + a \coth(e + fx))} \end{aligned}$$

Mathematica [A] time = 0.701283, size = 185, normalized size = 1.01

$$\frac{\operatorname{csch}^3(e + fx) \left(4(6cf(6fx + 1) + d(18f^2x^2 + 6fx + 1)) \cosh(3(e + fx)) + 27(4cf + d(4fx + 3)) \cosh(e + fx) + 144cf^2 \right)}{1152a^3f^2(1 + \coth(e + fx))}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)/(a + a*Coth[e + f*x])^3,x]

[Out] (Csch[e + f*x]^3*(27*(4*c*f + d*(3 + 4*f*x))*Cosh[e + f*x] + 4*(6*c*f*(1 + 6*f*x) + d*(1 + 6*f*x + 18*f^2*x^2))*Cosh[3*(e + f*x)] + 135*d*Sinh[e + f*x] + 324*c*f*Sinh[e + f*x] + 324*d*f*x*Sinh[e + f*x] - 4*d*Sinh[3*(e + f*x)] - 24*c*f*Sinh[3*(e + f*x)] - 24*d*f*x*Sinh[3*(e + f*x)] + 144*c*f^2*x*Sinh[3*(e + f*x)] + 72*d*f^2*x^2*Sinh[3*(e + f*x)])/(1152*a^3*f^2*(1 + Coth[e + f*x]))

+ f*x])^3)

Maple [B] time = 0.107, size = 791, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)/(a+a*coth(f*x+e))^3,x)

[Out] $\frac{1}{a^3} \frac{1}{f} \left(-\frac{4}{f} d \left(\frac{1}{6} (f*x+e) \sinh(f*x+e)^3 \cosh(f*x+e)^3 - \frac{1}{8} (f*x+e) \sinh(f*x+e) \cosh(f*x+e)^3 + \frac{1}{16} (f*x+e) \cosh(f*x+e) \sinh(f*x+e) + \frac{1}{32} (f*x+e)^2 - \frac{1}{3} 6 \sinh(f*x+e)^2 \cosh(f*x+e)^4 + \frac{13}{288} \sinh(f*x+e)^2 \cosh(f*x+e)^2 + \frac{1}{72} \cosh(f*x+e)^2 \right) + \frac{4}{f} d \left(\frac{1}{6} (f*x+e) \sinh(f*x+e)^2 \cosh(f*x+e)^4 - \frac{1}{12} (f*x+e) \sinh(f*x+e)^2 \cosh(f*x+e)^2 - \frac{1}{12} (f*x+e) \cosh(f*x+e)^2 - \frac{1}{36} \sinh(f*x+e) \cosh(f*x+e)^5 + \frac{1}{36} \cosh(f*x+e)^3 \sinh(f*x+e) + \frac{1}{24} \cosh(f*x+e) \sinh(f*x+e) + \frac{1}{24} f*x + \frac{1}{24} e \right) + \frac{1}{f} d \left(\frac{1}{4} (f*x+e) \cosh(f*x+e) \sinh(f*x+e)^3 - \frac{3}{8} (f*x+e) \cosh(f*x+e) \sinh(f*x+e) + \frac{3}{16} (f*x+e)^2 - \frac{1}{16} \sinh(f*x+e)^2 \cosh(f*x+e)^2 + \frac{1}{4} \cosh(f*x+e)^2 \right) - \frac{3}{f} d \left(\frac{1}{4} (f*x+e) \sinh(f*x+e)^2 \cosh(f*x+e)^2 - \frac{1}{4} (f*x+e) \cosh(f*x+e)^2 - \frac{1}{16} \cosh(f*x+e)^3 \sinh(f*x+e) + \frac{5}{32} \cosh(f*x+e) \sinh(f*x+e) + \frac{5}{32} f*x + \frac{5}{32} e \right) + 4*d*e/f \left(\frac{1}{6} \sinh(f*x+e)^3 \cosh(f*x+e)^3 - \frac{1}{8} \cosh(f*x+e)^3 \sinh(f*x+e) + \frac{1}{16} \cosh(f*x+e) \sinh(f*x+e) + \frac{1}{16} f*x + \frac{1}{16} e \right) - 4*d*e/f \left(\frac{1}{6} \sinh(f*x+e)^2 \cosh(f*x+e)^4 - \frac{1}{12} \sinh(f*x+e)^2 \cosh(f*x+e)^2 - \frac{1}{12} \cosh(f*x+e)^2 \right) - d*e/f \left(\left(\frac{1}{4} \sinh(f*x+e)^3 - \frac{3}{8} \sinh(f*x+e) \right) \cosh(f*x+e) + \frac{3}{8} f*x + \frac{3}{8} e \right) + 3*d*e/f \left(\frac{1}{4} \sinh(f*x+e)^2 \cosh(f*x+e)^2 - \frac{1}{4} \cosh(f*x+e)^2 \right) - 4*c \left(\frac{1}{6} \sinh(f*x+e)^3 \cosh(f*x+e)^3 - \frac{1}{8} \cosh(f*x+e)^3 \sinh(f*x+e) + \frac{1}{16} \cosh(f*x+e) \sinh(f*x+e) + \frac{1}{16} f*x + \frac{1}{16} e \right) + 4*c \left(\frac{1}{6} \sinh(f*x+e)^2 \cosh(f*x+e)^4 - \frac{1}{12} \sinh(f*x+e)^2 \cosh(f*x+e)^2 - \frac{1}{12} \cosh(f*x+e)^2 \right) + c \left(\left(\frac{1}{4} \sinh(f*x+e)^3 - \frac{3}{8} \sinh(f*x+e) \right) \cosh(f*x+e) + \frac{3}{8} f*x + \frac{3}{8} e \right) - 3*c \left(\frac{1}{4} \sinh(f*x+e)^2 \cosh(f*x+e)^2 - \frac{1}{4} \cosh(f*x+e)^2 \right) \right)$

Maxima [A] time = 3.25172, size = 186, normalized size = 1.02

$$\frac{1}{96} c \left(\frac{12(fx+e)}{a^3 f} + \frac{18e^{(-2fx-2e)} - 9e^{(-4fx-4e)} + 2e^{(-6fx-6e)}}{a^3 f} \right) + \frac{(72f^2x^2e^{(6e)} + 108(2fxe^{(4e)} + e^{(4e)})e^{(-2fx)} - 27(4f$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)/(a+a*coth(f*x+e))^3,x, algorithm="maxima")

```
[Out] 1/96*c*(12*(f*x + e)/(a^3*f) + (18*e^(-2*f*x - 2*e) - 9*e^(-4*f*x - 4*e) +
2*e^(-6*f*x - 6*e))/(a^3*f)) + 1/1152*(72*f^2*x^2*e^(6*e) + 108*(2*f*x*e^(4
*e) + e^(4*e))*e^(-2*f*x) - 27*(4*f*x*e^(2*e) + e^(2*e))*e^(-4*f*x) + 4*(6*
f*x + 1)*e^(-6*f*x))*d*e^(-6*e)/(a^3*f^2)
```

Fricas [A] time = 2.09015, size = 709, normalized size = 3.87

$$\frac{4(18df^2x^2 + 6cf + 6(6cf^2 + df)x + d) \cosh(fx + e)^3 + 12(18df^2x^2 + 6cf + 6(6cf^2 + df)x + d) \cosh(fx + e) \sinh(fx + e)^2 + 4(18df^2x^2 - 6cf + 6(6cf^2 - df)x - d) \sinh(fx + e)^3 + 27(4dfx + 4cf + 3d) \cosh(fx + e) + 3(108dfx + 4(18df^2x^2 - 6cf + 6(6cf^2 - df)x - d) \cosh(fx + e)^2 + 108cf + 45d) \sinh(fx + e)}{1152(a^3f^2 \cosh(fx + e)^3 + 3a^3f^2 \cosh(fx + e)^2 \sinh(fx + e) + 3a^3f^2 \cosh(fx + e) \sinh(fx + e)^2 + a^3f^2 \sinh(fx + e)^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+a*coth(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/1152*(4*(18*d*f^2*x^2 + 6*c*f + 6*(6*c*f^2 + d*f)*x + d)*cosh(f*x + e)^3
+ 12*(18*d*f^2*x^2 + 6*c*f + 6*(6*c*f^2 + d*f)*x + d)*cosh(f*x + e)*sinh(f*
x + e)^2 + 4*(18*d*f^2*x^2 - 6*c*f + 6*(6*c*f^2 - d*f)*x - d)*sinh(f*x + e)
^3 + 27*(4*d*f*x + 4*c*f + 3*d)*cosh(f*x + e) + 3*(108*d*f*x + 4*(18*d*f^2*
x^2 - 6*c*f + 6*(6*c*f^2 - d*f)*x - d)*cosh(f*x + e)^2 + 108*c*f + 45*d)*si
nh(f*x + e))/(a^3*f^2*cosh(f*x + e)^3 + 3*a^3*f^2*cosh(f*x + e)^2*sinh(f*x
+ e) + 3*a^3*f^2*cosh(f*x + e)*sinh(f*x + e)^2 + a^3*f^2*sinh(f*x + e)^3)
```

Sympy [A] time = 4.70043, size = 1290, normalized size = 7.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+a*coth(f*x+e))**3,x)
```

```
[Out] Piecewise(((36*c*f**2*x*tanh(e + f*x)**3/(288*a**3*f**2*tanh(e + f*x)**3 + 8
64*a**3*f**2*tanh(e + f*x)**2 + 864*a**3*f**2*tanh(e + f*x) + 288*a**3*f**2
) + 108*c*f**2*x*tanh(e + f*x)**2/(288*a**3*f**2*tanh(e + f*x)**3 + 864*a**
3*f**2*tanh(e + f*x)**2 + 864*a**3*f**2*tanh(e + f*x) + 288*a**3*f**2) + 10
8*c*f**2*x*tanh(e + f*x)/(288*a**3*f**2*tanh(e + f*x)**3 + 864*a**3*f**2*ta
nh(e + f*x)**2 + 864*a**3*f**2*tanh(e + f*x) + 288*a**3*f**2) + 36*c*f**2*x
/(288*a**3*f**2*tanh(e + f*x)**3 + 864*a**3*f**2*tanh(e + f*x)**2 + 864*a**
3*f**2*tanh(e + f*x) + 288*a**3*f**2) - 108*c*f*tanh(e + f*x)**3/(288*a**3*
```



```

f**2*tanh(e + f*x)**3 + 864*a**3*f**2*tanh(e + f*x)**2 + 864*a**3*f**2*tanh
(e + f*x) + 288*a**3*f**2) - 72*c*f*tanh(e + f*x)**2/(288*a**3*f**2*tanh(e
+ f*x)**3 + 864*a**3*f**2*tanh(e + f*x)**2 + 864*a**3*f**2*tanh(e + f*x) +
288*a**3*f**2) + 12*c*f/(288*a**3*f**2*tanh(e + f*x)**3 + 864*a**3*f**2*tan
h(e + f*x)**2 + 864*a**3*f**2*tanh(e + f*x) + 288*a**3*f**2) + 18*d*f**2*x*
**2*tanh(e + f*x)**3/(288*a**3*f**2*tanh(e + f*x)**3 + 864*a**3*f**2*tanh(e
+ f*x)**2 + 864*a**3*f**2*tanh(e + f*x) + 288*a**3*f**2) + 54*d*f**2*x**2*t
anh(e + f*x)**2/(288*a**3*f**2*tanh(e + f*x)**3 + 864*a**3*f**2*tanh(e + f*
x)**2 + 864*a**3*f**2*tanh(e + f*x) + 288*a**3*f**2) + 54*d*f**2*x**2*tanh(
e + f*x)/(288*a**3*f**2*tanh(e + f*x)**3 + 864*a**3*f**2*tanh(e + f*x)**2 +
864*a**3*f**2*tanh(e + f*x) + 288*a**3*f**2) + 18*d*f**2*x**2/(288*a**3*f*
**2*tanh(e + f*x)**3 + 864*a**3*f**2*tanh(e + f*x)**2 + 864*a**3*f**2*tanh(e
+ f*x) + 288*a**3*f**2) - 87*d*f*x*tanh(e + f*x)**3/(288*a**3*f**2*tanh(e
+ f*x)**3 + 864*a**3*f**2*tanh(e + f*x)**2 + 864*a**3*f**2*tanh(e + f*x) +
288*a**3*f**2) - 9*d*f*x*tanh(e + f*x)**2/(288*a**3*f**2*tanh(e + f*x)**3 +
864*a**3*f**2*tanh(e + f*x)**2 + 864*a**3*f**2*tanh(e + f*x) + 288*a**3*f*
**2) + 63*d*f*x*tanh(e + f*x)/(288*a**3*f**2*tanh(e + f*x)**3 + 864*a**3*f**
2*tanh(e + f*x)**2 + 864*a**3*f**2*tanh(e + f*x) + 288*a**3*f**2) + 33*d*f*
x/(288*a**3*f**2*tanh(e + f*x)**3 + 864*a**3*f**2*tanh(e + f*x)**2 + 864*a*
**3*f**2*tanh(e + f*x) + 288*a**3*f**2) - 45*d*tanh(e + f*x)**3/(288*a**3*f*
**2*tanh(e + f*x)**3 + 864*a**3*f**2*tanh(e + f*x)**2 + 864*a**3*f**2*tanh(e
+ f*x) + 288*a**3*f**2) - 48*d*tanh(e + f*x)**2/(288*a**3*f**2*tanh(e + f*
x)**3 + 864*a**3*f**2*tanh(e + f*x)**2 + 864*a**3*f**2*tanh(e + f*x) + 288*
a**3*f**2) + 11*d/(288*a**3*f**2*tanh(e + f*x)**3 + 864*a**3*f**2*tanh(e +
f*x)**2 + 864*a**3*f**2*tanh(e + f*x) + 288*a**3*f**2), Ne(f, 0)), ((c*x +
d*x**2/2)/(a*coth(e) + a)**3, True))

```

Giac [A] time = 1.16859, size = 204, normalized size = 1.11

$$\frac{(72df^2x^2e^{(6fx+6e)} + 144cf^2xe^{(6fx+6e)} + 216dfxe^{(4fx+4e)} - 108dfxe^{(2fx+2e)} + 24dfx + 216cfe^{(4fx+4e)} - 108cfe^{(2fx+2e)})}{1152a^3f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)/(a+a*coth(f*x+e))^3,x, algorithm="giac")

[Out] 1/1152*(72*d*f^2*x^2*e^(6*f*x + 6*e) + 144*c*f^2*x*e^(6*f*x + 6*e) + 216*d*f*x*e^(4*f*x + 4*e) - 108*d*f*x*e^(2*f*x + 2*e) + 24*d*f*x + 216*c*f*e^(4*f*x + 4*e) - 108*c*f*e^(2*f*x + 2*e) + 24*c*f + 108*d*e^(4*f*x + 4*e) - 27*d*e^(2*f*x + 2*e) + 4*d)*e^(-6*f*x - 6*e)/(a^3*f^2)

$$3.30 \quad \int \frac{1}{(c+dx)(a+a \coth(ex+fx))^3} dx$$

Optimal. Leaf size=437

$$\frac{3\text{Chi}\left(2xf + \frac{2cf}{d}\right) \sinh\left(2e - \frac{2cf}{d}\right)}{8a^3d} + \frac{\text{Chi}\left(6xf + \frac{6cf}{d}\right) \sinh\left(6e - \frac{6cf}{d}\right)}{8a^3d} - \frac{3\text{Chi}\left(4xf + \frac{4cf}{d}\right) \sinh\left(4e - \frac{4cf}{d}\right)}{8a^3d} - \frac{3\text{Chi}\left(2xf - \frac{2cf}{d}\right) \sinh\left(2e - \frac{2cf}{d}\right)}{8a^3d}$$

[Out] (-3*Cosh[2*e - (2*c*f)/d]*CoshIntegral[(2*c*f)/d + 2*f*x])/(8*a^3*d) + (3*Cosh[4*e - (4*c*f)/d]*CoshIntegral[(4*c*f)/d + 4*f*x])/(8*a^3*d) - (Cosh[6*e - (6*c*f)/d]*CoshIntegral[(6*c*f)/d + 6*f*x])/(8*a^3*d) + Log[c + d*x]/(8*a^3*d) + (CoshIntegral[(6*c*f)/d + 6*f*x]*Sinh[6*e - (6*c*f)/d])/(8*a^3*d) - (3*CoshIntegral[(4*c*f)/d + 4*f*x]*Sinh[4*e - (4*c*f)/d])/(8*a^3*d) + (3*CoshIntegral[(2*c*f)/d + 2*f*x]*Sinh[2*e - (2*c*f)/d])/(8*a^3*d) + (3*Cosh[2*e - (2*c*f)/d]*SinhIntegral[(2*c*f)/d + 2*f*x])/(8*a^3*d) - (3*Sinh[2*e - (2*c*f)/d]*SinhIntegral[(2*c*f)/d + 2*f*x])/(8*a^3*d) - (3*Cosh[4*e - (4*c*f)/d]*SinhIntegral[(4*c*f)/d + 4*f*x])/(8*a^3*d) + (3*Sinh[4*e - (4*c*f)/d]*SinhIntegral[(4*c*f)/d + 4*f*x])/(8*a^3*d) + (Cosh[6*e - (6*c*f)/d]*SinhIntegral[(6*c*f)/d + 6*f*x])/(8*a^3*d) - (Sinh[6*e - (6*c*f)/d]*SinhIntegral[(6*c*f)/d + 6*f*x])/(8*a^3*d)

Rubi [A] time = 1.76455, antiderivative size = 437, normalized size of antiderivative = 1., number of steps used = 53, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.35$, Rules used = {3728, 3303, 3298, 3301, 3312, 5448, 5470}

$$\frac{3\text{Chi}\left(2xf + \frac{2cf}{d}\right) \sinh\left(2e - \frac{2cf}{d}\right)}{8a^3d} + \frac{\text{Chi}\left(6xf + \frac{6cf}{d}\right) \sinh\left(6e - \frac{6cf}{d}\right)}{8a^3d} - \frac{3\text{Chi}\left(4xf + \frac{4cf}{d}\right) \sinh\left(4e - \frac{4cf}{d}\right)}{8a^3d} - \frac{3\text{Chi}\left(2xf - \frac{2cf}{d}\right) \sinh\left(2e - \frac{2cf}{d}\right)}{8a^3d}$$

Antiderivative was successfully verified.

[In] Int[1/((c + d*x)*(a + a*Coth[e + f*x])^3),x]

[Out] (-3*Cosh[2*e - (2*c*f)/d]*CoshIntegral[(2*c*f)/d + 2*f*x])/(8*a^3*d) + (3*Cosh[4*e - (4*c*f)/d]*CoshIntegral[(4*c*f)/d + 4*f*x])/(8*a^3*d) - (Cosh[6*e - (6*c*f)/d]*CoshIntegral[(6*c*f)/d + 6*f*x])/(8*a^3*d) + Log[c + d*x]/(8*a^3*d) + (CoshIntegral[(6*c*f)/d + 6*f*x]*Sinh[6*e - (6*c*f)/d])/(8*a^3*d) - (3*CoshIntegral[(4*c*f)/d + 4*f*x]*Sinh[4*e - (4*c*f)/d])/(8*a^3*d) + (3*CoshIntegral[(2*c*f)/d + 2*f*x]*Sinh[2*e - (2*c*f)/d])/(8*a^3*d) + (3*Cosh[2*e - (2*c*f)/d]*SinhIntegral[(2*c*f)/d + 2*f*x])/(8*a^3*d) - (3*Sinh[2*e - (2*c*f)/d]*SinhIntegral[(2*c*f)/d + 2*f*x])/(8*a^3*d) - (3*Cosh[4*e - (4*c*f)/d]*SinhIntegral[(4*c*f)/d + 4*f*x])/(8*a^3*d) + (3*Sinh[4*e - (4*c*f)/d]*SinhIntegral[(4*c*f)/d + 4*f*x])/(8*a^3*d) + (Cosh[6*e - (6*c*f)/d]*SinhIntegral[(6*c*f)/d + 6*f*x])/(8*a^3*d) - (Sinh[6*e - (6*c*f)/d]*SinhIntegral[(6*c*f)/d + 6*f*x])/(8*a^3*d)

```
] * SinhIntegral[(4*c*f)/d + 4*f*x]/(8*a^3*d) + (Cosh[6*e - (6*c*f)/d] * SinhIntegral[(6*c*f)/d + 6*f*x]/(8*a^3*d) - (Sinh[6*e - (6*c*f)/d] * SinhIntegral[(6*c*f)/d + 6*f*x]/(8*a^3*d)
```

Rule 3728

```
Int[((c_.) + (d_.)*(x_))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_),
x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, (1/(2*a) + Cos[2*e + 2*f*x]/(2*a) + Sin[2*e + 2*f*x]/(2*b))^(-n), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& EqQ[a^2 + b^2, 0] && ILtQ[m, 0] && ILtQ[n, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a + b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5470

```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]^(p_.)*Sinh[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[(e + f*x)^m, Sinh[a +
b*x]^p*Sinh[c + d*x]^q, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0
] && IGtQ[q, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(c+dx)(a+a \coth(e+fx))^3} dx &= \int \left(\frac{1}{8a^3(c+dx)} - \frac{3 \cosh(2e+2fx)}{8a^3(c+dx)} + \frac{3 \cosh^2(2e+2fx)}{8a^3(c+dx)} - \frac{\cosh^3(2e+2fx)}{8a^3(c+dx)} + \frac{3}{16a^3} \right) dx \\
&= \frac{\log(c+dx)}{8a^3d} - \frac{\int \frac{\cosh^3(2e+2fx)}{c+dx} dx}{8a^3} + \frac{\int \frac{\sinh^3(2e+2fx)}{c+dx} dx}{8a^3} - \frac{3 \int \frac{\sinh(2e+2fx) \sinh(4e+4fx)}{c+dx} dx}{16a^3} \\
&= \frac{\log(c+dx)}{8a^3d} + \frac{i \int \left(\frac{3i \sinh(2e+2fx)}{4(c+dx)} - \frac{i \sinh(6e+6fx)}{4(c+dx)} \right) dx}{8a^3} - \frac{\int \left(\frac{3 \cosh(2e+2fx)}{4(c+dx)} + \frac{\cosh(6e+6fx)}{4(c+dx)} \right) dx}{8a^3} \\
&= -\frac{3 \cosh\left(2e - \frac{2cf}{d}\right) \operatorname{Chi}\left(\frac{2cf}{d} + 2fx\right)}{8a^3d} + \frac{\log(c+dx)}{8a^3d} - \frac{3 \operatorname{Chi}\left(\frac{4cf}{d} + 4fx\right) \sinh\left(4e - \frac{4cf}{d}\right)}{8a^3d} \\
&= -\frac{3 \cosh\left(2e - \frac{2cf}{d}\right) \operatorname{Chi}\left(\frac{2cf}{d} + 2fx\right)}{8a^3d} + \frac{\log(c+dx)}{8a^3d} - \frac{3 \operatorname{Chi}\left(\frac{4cf}{d} + 4fx\right) \sinh\left(4e - \frac{4cf}{d}\right)}{8a^3d} \\
&= -\frac{3 \cosh\left(2e - \frac{2cf}{d}\right) \operatorname{Chi}\left(\frac{2cf}{d} + 2fx\right)}{8a^3d} - \frac{\cosh\left(6e - \frac{6cf}{d}\right) \operatorname{Chi}\left(\frac{6cf}{d} + 6fx\right)}{8a^3d} + \frac{\log(c+dx)}{8a^3d}
\end{aligned}$$

Mathematica [A] time = 0.648873, size = 312, normalized size = 0.71

$$\operatorname{csch}^3(e+fx) (\sinh(fx) + \cosh(fx))^3 \left(\left(\sinh\left(e - \frac{4cf}{d}\right) - \cosh\left(e - \frac{4cf}{d}\right) \right) \left(-\operatorname{Chi}\left(\frac{6f(c+dx)}{d}\right) \sinh\left(2e - \frac{2cf}{d}\right) + \operatorname{Chi}\left(\frac{6f(c+dx)}{d}\right) \right) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + d*x)*(a + a*Coth[e + f*x])^3),x]
```

```
[Out] (Csch[e + f*x]^3*(Cosh[f*x] + Sinh[f*x])^3*(Cosh[3*e]*Log[f*(c + d*x)] + Log[f*(c + d*x)]*Sinh[3*e] + (-Cosh[e - (4*c*f)/d] + Sinh[e - (4*c*f)/d])*(-3*CoshIntegral[(4*f*(c + d*x))/d] + Cosh[2*e - (2*c*f)/d]*CoshIntegral[(6*f*(c + d*x))/d] - CoshIntegral[(6*f*(c + d*x))/d]*Sinh[2*e - (2*c*f)/d] + 3*CoshIntegral[(2*f*(c + d*x))/d]*(Cosh[2*e - (2*c*f)/d] + Sinh[2*e - (2*c*f)/d])
```

d]) - 3*Cosh[2*e - (2*c*f)/d]*SinhIntegral[(2*f*(c + d*x))/d] - 3*Sinh[2*e - (2*c*f)/d]*SinhIntegral[(2*f*(c + d*x))/d] + 3*SinhIntegral[(4*f*(c + d*x))/d] - Cosh[2*e - (2*c*f)/d]*SinhIntegral[(6*f*(c + d*x))/d] + Sinh[2*e - (2*c*f)/d]*SinhIntegral[(6*f*(c + d*x))/d]))/(8*a^3*d*(1 + Coth[e + f*x])^3)

Maple [A] time = 0.477, size = 151, normalized size = 0.4

$$\frac{\ln(dx+c)}{8a^3d} + \frac{1}{8a^3d} e^{6\frac{cf-de}{d}} \text{Ei}\left(1, 6fx + 6e + 6\frac{cf-de}{d}\right) - \frac{3}{8a^3d} e^{4\frac{cf-de}{d}} \text{Ei}\left(1, 4fx + 4e + 4\frac{cf-de}{d}\right) + \frac{3}{8a^3d} e^{2\frac{cf-de}{d}} \text{Ei}\left(1, 2fx + 2e + 2\frac{cf-de}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(d*x+c)/(a+a*coth(f*x+e))^3,x)

[Out] 1/8*ln(d*x+c)/a^3/d+1/8/a^3/d*exp(6*(c*f-d*e)/d)*Ei(1,6*f*x+6*e+6*(c*f-d*e)/d)-3/8/a^3/d*exp(4*(c*f-d*e)/d)*Ei(1,4*f*x+4*e+4*(c*f-d*e)/d)+3/8/a^3/d*exp(2*(c*f-d*e)/d)*Ei(1,2*f*x+2*e+2*(c*f-d*e)/d)

Maxima [A] time = 10.9323, size = 154, normalized size = 0.35

$$\frac{e^{\left(-6e + \frac{6cf}{d}\right)} E_1\left(\frac{6(dx+c)f}{d}\right)}{8a^3d} - \frac{3e^{\left(-4e + \frac{4cf}{d}\right)} E_1\left(\frac{4(dx+c)f}{d}\right)}{8a^3d} + \frac{3e^{\left(-2e + \frac{2cf}{d}\right)} E_1\left(\frac{2(dx+c)f}{d}\right)}{8a^3d} + \frac{\log(dx+c)}{8a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d*x+c)/(a+a*coth(f*x+e))^3,x, algorithm="maxima")

[Out] 1/8*e^(-6*e + 6*c*f/d)*exp_integral_e(1, 6*(d*x + c)*f/d)/(a^3*d) - 3/8*e^(-4*e + 4*c*f/d)*exp_integral_e(1, 4*(d*x + c)*f/d)/(a^3*d) + 3/8*e^(-2*e + 2*c*f/d)*exp_integral_e(1, 2*(d*x + c)*f/d)/(a^3*d) + 1/8*log(d*x + c)/(a^3*d)

Fricas [A] time = 2.22964, size = 439, normalized size = 1.

$$\frac{3 \text{Ei}\left(-\frac{2(dfxc+cf)}{d}\right) \cosh\left(-\frac{2(de-cf)}{d}\right) - 3 \text{Ei}\left(-\frac{4(dfxc+cf)}{d}\right) \cosh\left(-\frac{4(de-cf)}{d}\right) + \text{Ei}\left(-\frac{6(dfxc+cf)}{d}\right) \cosh\left(-\frac{6(de-cf)}{d}\right) + 3 \text{Ei}\left(-\frac{2(dfxc+cf)}{d}\right) \cosh\left(-\frac{2(de-cf)}{d}\right) - 3 \text{Ei}\left(-\frac{4(dfxc+cf)}{d}\right) \cosh\left(-\frac{4(de-cf)}{d}\right) + \text{Ei}\left(-\frac{6(dfxc+cf)}{d}\right) \cosh\left(-\frac{6(de-cf)}{d}\right)}{8a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d*x+c)/(a+a*coth(f*x+e))^3,x, algorithm="fricas")

[Out]
$$-1/8*(3*Ei(-2*(d*f*x + c*f)/d)*\cosh(-2*(d*e - c*f)/d) - 3*Ei(-4*(d*f*x + c*f)/d)*\cosh(-4*(d*e - c*f)/d) + Ei(-6*(d*f*x + c*f)/d)*\cosh(-6*(d*e - c*f)/d) + 3*Ei(-2*(d*f*x + c*f)/d)*\sinh(-2*(d*e - c*f)/d) - 3*Ei(-4*(d*f*x + c*f)/d)*\sinh(-4*(d*e - c*f)/d) + Ei(-6*(d*f*x + c*f)/d)*\sinh(-6*(d*e - c*f)/d) - \log(d*x + c))/(a^3*d)$$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{c \coth^3(e+fx) + 3c \coth^2(e+fx) + 3c \coth(e+fx) + c + dx \coth^3(e+fx) + 3dx \coth^2(e+fx) + 3dx \coth(e+fx) + dx} dx$$

a^3

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d*x+c)/(a+a*coth(f*x+e))^3,x)

[Out] Integral(1/(c*coth(e + f*x)**3 + 3*c*coth(e + f*x)**2 + 3*c*coth(e + f*x) + c + d*x*coth(e + f*x)**3 + 3*d*x*coth(e + f*x)**2 + 3*d*x*coth(e + f*x) + d*x), x)/a**3

Giac [A] time = 1.17015, size = 146, normalized size = 0.33

$$\frac{\left(Ei\left(-\frac{6(dfxcf)}{d}\right) e^{\left(\frac{6cf}{d}\right)} - 3 Ei\left(-\frac{4(dfxcf)}{d}\right) e^{\left(\frac{4cf}{d} + 2e\right)} + 3 Ei\left(-\frac{2(dfxcf)}{d}\right) e^{\left(\frac{2cf}{d} + 4e\right)} - e^{(6e)} \log(dx + c) \right) e^{(-6e)}}{8 a^3 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d*x+c)/(a+a*coth(f*x+e))^3,x, algorithm="giac")

[Out]
$$-1/8*(Ei(-6*(d*f*x + c*f)/d)*e^{(6*c*f/d)} - 3*Ei(-4*(d*f*x + c*f)/d)*e^{(4*c*f/d + 2*e)} + 3*Ei(-2*(d*f*x + c*f)/d)*e^{(2*c*f/d + 4*e)} - e^{(6*e)}*\log(d*x + c))*e^{(-6*e)}/(a^3*d)$$

$$3.31 \quad \int \frac{1}{(c+dx)^2(a+a \coth(e+fx))^3} dx$$

Optimal. Leaf size=692

$$-\frac{3f\text{Chi}\left(6xf + \frac{6cf}{d}\right)\sinh\left(6e - \frac{6cf}{d}\right)}{4a^3d^2} + \frac{3f\text{Chi}\left(4xf + \frac{4cf}{d}\right)\sinh\left(4e - \frac{4cf}{d}\right)}{2a^3d^2} - \frac{3f\text{Chi}\left(2xf + \frac{2cf}{d}\right)\sinh\left(2e - \frac{2cf}{d}\right)}{4a^3d^2} + \dots$$

```
[Out] -1/(8*a^3*d*(c + d*x)) + (9*Cosh[2*e + 2*f*x])/(32*a^3*d*(c + d*x)) - (3*Cosh[2*e + 2*f*x]^2)/(8*a^3*d*(c + d*x)) + Cosh[2*e + 2*f*x]^3/(8*a^3*d*(c + d*x)) + (3*Cosh[6*e + 6*f*x])/(32*a^3*d*(c + d*x)) + (3*f*Cosh[2*e - (2*c*f)/d])*CoshIntegral[(2*c*f)/d + 2*f*x]/(4*a^3*d^2) - (3*f*Cosh[4*e - (4*c*f)/d])*CoshIntegral[(4*c*f)/d + 4*f*x]/(2*a^3*d^2) + (3*f*Cosh[6*e - (6*c*f)/d])*CoshIntegral[(6*c*f)/d + 6*f*x]/(4*a^3*d^2) - (3*f*CoshIntegral[(6*c*f)/d + 6*f*x]*Sinh[6*e - (6*c*f)/d])/(4*a^3*d^2) + (3*f*CoshIntegral[(4*c*f)/d + 4*f*x]*Sinh[4*e - (4*c*f)/d])/(2*a^3*d^2) - (3*f*CoshIntegral[(2*c*f)/d + 2*f*x]*Sinh[2*e - (2*c*f)/d])/(4*a^3*d^2) - (15*Sinh[2*e + 2*f*x])/(32*a^3*d*(c + d*x)) - (3*Sinh[2*e + 2*f*x]^2)/(8*a^3*d*(c + d*x)) - Sinh[2*e + 2*f*x]^3/(8*a^3*d*(c + d*x)) + (3*Sinh[4*e + 4*f*x])/(8*a^3*d*(c + d*x)) - (3*Sinh[6*e + 6*f*x])/(32*a^3*d*(c + d*x)) - (3*f*Cosh[2*e - (2*c*f)/d])*SinhIntegral[(2*c*f)/d + 2*f*x]/(4*a^3*d^2) + (3*f*Sinh[2*e - (2*c*f)/d])*SinhIntegral[(2*c*f)/d + 2*f*x]/(4*a^3*d^2) + (3*f*Cosh[4*e - (4*c*f)/d])*SinhIntegral[(4*c*f)/d + 4*f*x]/(2*a^3*d^2) - (3*f*Sinh[4*e - (4*c*f)/d])*SinhIntegral[(4*c*f)/d + 4*f*x]/(2*a^3*d^2) - (3*f*Cosh[6*e - (6*c*f)/d])*SinhIntegral[(6*c*f)/d + 6*f*x]/(4*a^3*d^2) + (3*f*Sinh[6*e - (6*c*f)/d])*SinhIntegral[(6*c*f)/d + 6*f*x]/(4*a^3*d^2)
```

Rubi [A] time = 1.76195, antiderivative size = 692, normalized size of antiderivative = 1., number of steps used = 60, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.45$, Rules used = {3728, 3297, 3303, 3298, 3301, 3313, 12, 5448, 5470}

$$-\frac{3f\text{Chi}\left(6xf + \frac{6cf}{d}\right)\sinh\left(6e - \frac{6cf}{d}\right)}{4a^3d^2} + \frac{3f\text{Chi}\left(4xf + \frac{4cf}{d}\right)\sinh\left(4e - \frac{4cf}{d}\right)}{2a^3d^2} - \frac{3f\text{Chi}\left(2xf + \frac{2cf}{d}\right)\sinh\left(2e - \frac{2cf}{d}\right)}{4a^3d^2} + \dots$$

Antiderivative was successfully verified.

[In] Int[1/((c + d*x)^2*(a + a*Coth[e + f*x])^3), x]

```
[Out] -1/(8*a^3*d*(c + d*x)) + (9*Cosh[2*e + 2*f*x])/(32*a^3*d*(c + d*x)) - (3*Cosh[2*e + 2*f*x]^2)/(8*a^3*d*(c + d*x)) + Cosh[2*e + 2*f*x]^3/(8*a^3*d*(c + d*x)) + (3*Cosh[6*e + 6*f*x])/(32*a^3*d*(c + d*x)) + (3*f*Cosh[2*e - (2*c*f)
```

```

)/d]*CoshIntegral[(2*c*f)/d + 2*f*x]/(4*a^3*d^2) - (3*f*Cosh[4*e - (4*c*f)
/d]*CoshIntegral[(4*c*f)/d + 4*f*x]/(2*a^3*d^2) + (3*f*Cosh[6*e - (6*c*f)/
d]*CoshIntegral[(6*c*f)/d + 6*f*x]/(4*a^3*d^2) - (3*f*CoshIntegral[(6*c*f)
/d + 6*f*x]*Sinh[6*e - (6*c*f)/d]/(4*a^3*d^2) + (3*f*CoshIntegral[(4*c*f)/d
+ 4*f*x]*Sinh[4*e - (4*c*f)/d]/(2*a^3*d^2) - (3*f*CoshIntegral[(2*c*f)/d
+ 2*f*x]*Sinh[2*e - (2*c*f)/d]/(4*a^3*d^2) - (15*Sinh[2*e + 2*f*x]/(32*a
^3*d*(c + d*x)) - (3*Sinh[2*e + 2*f*x]^2)/(8*a^3*d*(c + d*x)) - Sinh[2*e +
2*f*x]^3/(8*a^3*d*(c + d*x)) + (3*Sinh[4*e + 4*f*x]/(8*a^3*d*(c + d*x)) -
(3*Sinh[6*e + 6*f*x]/(32*a^3*d*(c + d*x)) - (3*f*Cosh[2*e - (2*c*f)/d]*Sin
hIntegral[(2*c*f)/d + 2*f*x]/(4*a^3*d^2) + (3*f*Sinh[2*e - (2*c*f)/d]*Sinh
Integral[(2*c*f)/d + 2*f*x]/(4*a^3*d^2) + (3*f*Cosh[4*e - (4*c*f)/d]*SinhI
ntegral[(4*c*f)/d + 4*f*x]/(2*a^3*d^2) - (3*f*Sinh[4*e - (4*c*f)/d]*SinhIn
tegral[(4*c*f)/d + 4*f*x]/(2*a^3*d^2) - (3*f*Cosh[6*e - (6*c*f)/d]*SinhInt
egral[(6*c*f)/d + 6*f*x]/(4*a^3*d^2) + (3*f*Sinh[6*e - (6*c*f)/d]*SinhInte
gral[(6*c*f)/d + 6*f*x]/(4*a^3*d^2)

```

Rule 3728

```

Int[((c_.) + (d_.)*(x_))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_),
x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, (1/(2*a) + Cos[2*e + 2*f*x]/(
2*a) + Sin[2*e + 2*f*x]/(2*b))^(-n), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& EqQ[a^2 + b^2, 0] && ILtQ[m, 0] && ILtQ[n, 0]

```

Rule 3297

```

Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[((
c + d*x)^(m + 1)*Sin[e + f*x]/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
+ d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]

```

Rule 3303

```

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]

```

Rule 3298

```

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] := Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]

```

Rule 3301


```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:= Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x]
&& EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3313

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol]
:= Simp[((c + d*x)^(m + 1)*Sin[e + f*x]^n)/(d*(m + 1)), x] - Dist[(f*n)/(d*(m + 1)),
Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n - 1), x], x]
]; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] && LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)]
;/; FreeQ[b, x]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol]
:= Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a + b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x]
&& IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5470

```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]^(p_.)*Sinh[(c_.) + (d_.)*(x_)]^(q_.), x_Symbol]
:= Int[ExpandTrigReduce[(e + f*x)^m, Sinh[a + b*x]^p*Sinh[c + d*x]^q, x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& IGtQ[p, 0] && IGtQ[q, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(c+dx)^2(a+a\coth(e+fx))^3} dx &= \int \left(\frac{1}{8a^3(c+dx)^2} - \frac{3\cosh(2e+2fx)}{8a^3(c+dx)^2} + \frac{3\cosh^2(2e+2fx)}{8a^3(c+dx)^2} - \frac{\cosh^3(2e+2fx)}{8a^3(c+dx)^2} + \right. \\
&= -\frac{1}{8a^3d(c+dx)} - \frac{\int \frac{\cosh^3(2e+2fx)}{(c+dx)^2} dx}{8a^3} + \frac{\int \frac{\sinh^3(2e+2fx)}{(c+dx)^2} dx}{8a^3} - \frac{3\int \frac{\sinh(2e+2fx)\sinh(4e+4fx)}{(c+dx)^2} dx}{16a^3} \\
&= -\frac{1}{8a^3d(c+dx)} + \frac{3\cosh(2e+2fx)}{8a^3d(c+dx)} - \frac{3\cosh^2(2e+2fx)}{8a^3d(c+dx)} + \frac{\cosh^3(2e+2fx)}{8a^3d(c+dx)} - \frac{3}{8a^3d(c+dx)} \\
&= -\frac{1}{8a^3d(c+dx)} + \frac{3\cosh(2e+2fx)}{8a^3d(c+dx)} - \frac{3\cosh^2(2e+2fx)}{8a^3d(c+dx)} + \frac{\cosh^3(2e+2fx)}{8a^3d(c+dx)} - \frac{3}{8a^3d(c+dx)} \\
&= -\frac{1}{8a^3d(c+dx)} + \frac{9\cosh(2e+2fx)}{32a^3d(c+dx)} - \frac{3\cosh^2(2e+2fx)}{8a^3d(c+dx)} + \frac{\cosh^3(2e+2fx)}{8a^3d(c+dx)} + \frac{3}{8a^3d(c+dx)} \\
&= -\frac{1}{8a^3d(c+dx)} + \frac{9\cosh(2e+2fx)}{32a^3d(c+dx)} - \frac{3\cosh^2(2e+2fx)}{8a^3d(c+dx)} + \frac{\cosh^3(2e+2fx)}{8a^3d(c+dx)} + \frac{3}{8a^3d(c+dx)} \\
&= -\frac{1}{8a^3d(c+dx)} + \frac{9\cosh(2e+2fx)}{32a^3d(c+dx)} - \frac{3\cosh^2(2e+2fx)}{8a^3d(c+dx)} + \frac{\cosh^3(2e+2fx)}{8a^3d(c+dx)} + \frac{3}{8a^3d(c+dx)}
\end{aligned}$$

Mathematica [A] time = 3.33111, size = 796, normalized size = 1.15

$$\text{csch}^3(e+fx) \left(\cosh\left(\frac{3cf}{d}\right) + \sinh\left(\frac{3cf}{d}\right) \right) \left(3d \cosh\left(e+f\left(x-\frac{3c}{d}\right)\right) - d \cosh\left(3\left(e+f\left(x-\frac{c}{d}\right)\right)\right) + d \cosh\left(3\left(e+f\left(\frac{c}{d}+x\right)\right)\right) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/((c+d*x)^2*(a+a*Coth[e+f*x])^3),x]

[Out] (Csch[e+f*x]^3*(Cosh[(3*c*f)/d]+Sinh[(3*c*f)/d])*(3*d*Cosh[e+f*((-3*c)/d+x)]-d*Cosh[3*(e+f*(-c/d)+x)]+d*Cosh[3*(e+f*(c/d+x))]-3*d*Cosh[e+f*((3*c)/d+x)]+6*c*f*Cosh[3*e-(3*f*(c+d*x))/d]*CoshIntegral[(6*f*(c+d*x))/d]+6*d*f*x*Cosh[3*e-(3*f*(c+d*x))/d]*CoshIntegral[(6*f*(c+d*x))/d]+6*f*(c+d*x)*CoshIntegral[(2*f*(c+d*x))/d]*(Cosh[e-(c*f)/d+3*f*x]+Sinh[e-(c*f)/d+3*f*x])+3*d*Sinh[e+f*((-3*c)/d+x)]-d*Sinh[3*(e+f*(-c/d)+x)]-d*Sinh[3*(e+f*(c/d+x))]+3

```

*d*Sinh[e + f*((3*c)/d + x)] - 6*c*f*CoshIntegral[(6*f*(c + d*x))/d]*Sinh[3
*e - (3*f*(c + d*x))/d] - 6*d*f*x*CoshIntegral[(6*f*(c + d*x))/d]*Sinh[3*e
- (3*f*(c + d*x))/d] + 12*f*(c + d*x)*CoshIntegral[(4*f*(c + d*x))/d]*(-Cos
h[e - (f*(c + 3*d*x))/d] + Sinh[e - (f*(c + 3*d*x))/d]) - 6*c*f*Cosh[e - (c
*f)/d + 3*f*x]*SinhIntegral[(2*f*(c + d*x))/d] - 6*d*f*x*Cosh[e - (c*f)/d +
3*f*x]*SinhIntegral[(2*f*(c + d*x))/d] - 6*c*f*Sinh[e - (c*f)/d + 3*f*x]*S
inhIntegral[(2*f*(c + d*x))/d] - 6*d*f*x*Sinh[e - (c*f)/d + 3*f*x]*SinhInte
gral[(2*f*(c + d*x))/d] + 12*c*f*Cosh[e - (f*(c + 3*d*x))/d]*SinhIntegral[(
4*f*(c + d*x))/d] + 12*d*f*x*Cosh[e - (f*(c + 3*d*x))/d]*SinhIntegral[(4*f*
(c + d*x))/d] - 12*c*f*Sinh[e - (f*(c + 3*d*x))/d]*SinhIntegral[(4*f*(c + d
*x))/d] - 12*d*f*x*Sinh[e - (f*(c + 3*d*x))/d]*SinhIntegral[(4*f*(c + d*x))
/d] - 6*c*f*Cosh[3*e - (3*f*(c + d*x))/d]*SinhIntegral[(6*f*(c + d*x))/d] -
6*d*f*x*Cosh[3*e - (3*f*(c + d*x))/d]*SinhIntegral[(6*f*(c + d*x))/d] + 6*
c*f*Sinh[3*e - (3*f*(c + d*x))/d]*SinhIntegral[(6*f*(c + d*x))/d] + 6*d*f*x
*Sinh[3*e - (3*f*(c + d*x))/d]*SinhIntegral[(6*f*(c + d*x))/d]))/(8*a^3*d^2
*(c + d*x)*(1 + Coth[e + f*x])^3)

```

Maple [A] time = 0.408, size = 239, normalized size = 0.4

$$-\frac{1}{8a^3d(dx+c)} + \frac{fe^{-6fx-6e}}{8a^3d(dfxc+cf)} - \frac{3f}{4a^3d^2}e^{6\frac{cf-de}{d}}\text{Ei}\left(1,6fx+6e+6\frac{cf-de}{d}\right) - \frac{3fe^{-4fx-4e}}{8a^3d(dfxc+cf)} + \frac{3f}{2a^3d^2}e^{4\frac{cf-de}{d}}\text{Ei}\left(1,4fx+4e+4\frac{cf-de}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(d*x+c)^2/(a+a*coth(f*x+e))^3,x)

[Out] $-\frac{1}{8a^3d}/(d*x+c) + \frac{1}{8a^3f}\exp(-6*f*x-6*e)/d/(d*f*x+c*f) - \frac{3}{4a^3f/d^2}\exp(6*(c*f-d*e)/d)*\text{Ei}(1,6*f*x+6*e+6*(c*f-d*e)/d) - \frac{3}{8a^3f}\exp(-4*f*x-4*e)/d/(d*f*x+c*f) + \frac{3}{2a^3f/d^2}\exp(4*(c*f-d*e)/d)*\text{Ei}(1,4*f*x+4*e+4*(c*f-d*e)/d) + \frac{3}{8a^3f}\exp(-2*f*x-2*e)/d/(d*f*x+c*f) - \frac{3}{4a^3f/d^2}\exp(2*(c*f-d*e)/d)*\text{Ei}(1,2*f*x+2*e+2*(c*f-d*e)/d)$

Maxima [A] time = 23.9139, size = 189, normalized size = 0.27

$$-\frac{1}{8(a^3d^2x+a^3cd)} + \frac{e^{(-6e+\frac{6cf}{d})}E_2\left(\frac{6(dx+c)f}{d}\right)}{8(dx+c)a^3d} - \frac{3e^{(-4e+\frac{4cf}{d})}E_2\left(\frac{4(dx+c)f}{d}\right)}{8(dx+c)a^3d} + \frac{3e^{(-2e+\frac{2cf}{d})}E_2\left(\frac{2(dx+c)f}{d}\right)}{8(dx+c)a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+a*coth(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] -1/8/(a^3*d^2*x + a^3*c*d) + 1/8*e^(-6*e + 6*c*f/d)*exp_integral_e(2, 6*(d*
x + c)*f/d)/((d*x + c)*a^3*d) - 3/8*e^(-4*e + 4*c*f/d)*exp_integral_e(2, 4*
(d*x + c)*f/d)/((d*x + c)*a^3*d) + 3/8*e^(-2*e + 2*c*f/d)*exp_integral_e(2,
2*(d*x + c)*f/d)/((d*x + c)*a^3*d)
```

Fricas [A] time = 2.33885, size = 2685, normalized size = 3.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+a*coth(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/4*(3*(d*f*x + c*f)*Ei(-2*(d*f*x + c*f)/d)*cosh(f*x + e)^3*sinh(-2*(d*e -
c*f)/d) - 6*(d*f*x + c*f)*Ei(-4*(d*f*x + c*f)/d)*cosh(f*x + e)^3*sinh(-4*(d
*e - c*f)/d) + 3*(d*f*x + c*f)*Ei(-6*(d*f*x + c*f)/d)*cosh(f*x + e)^3*sinh(
-6*(d*e - c*f)/d) + 3*((d*f*x + c*f)*Ei(-2*(d*f*x + c*f)/d)*cosh(-2*(d*e -
c*f)/d) - 2*(d*f*x + c*f)*Ei(-4*(d*f*x + c*f)/d)*cosh(-4*(d*e - c*f)/d) + (
d*f*x + c*f)*Ei(-6*(d*f*x + c*f)/d)*cosh(-6*(d*e - c*f)/d))*cosh(f*x + e)^3
+ (3*(d*f*x + c*f)*Ei(-2*(d*f*x + c*f)/d)*cosh(-2*(d*e - c*f)/d) - 6*(d*f*
x + c*f)*Ei(-4*(d*f*x + c*f)/d)*cosh(-4*(d*e - c*f)/d) + 3*(d*f*x + c*f)*Ei
(-6*(d*f*x + c*f)/d)*cosh(-6*(d*e - c*f)/d) + 3*(d*f*x + c*f)*Ei(-2*(d*f*x
+ c*f)/d)*sinh(-2*(d*e - c*f)/d) - 6*(d*f*x + c*f)*Ei(-4*(d*f*x + c*f)/d)*s
inh(-4*(d*e - c*f)/d) + 3*(d*f*x + c*f)*Ei(-6*(d*f*x + c*f)/d)*sinh(-6*(d*e
- c*f)/d) - d)*sinh(f*x + e)^3 + 9*((d*f*x + c*f)*Ei(-2*(d*f*x + c*f)/d)*c
osh(f*x + e)*sinh(-2*(d*e - c*f)/d) - 2*(d*f*x + c*f)*Ei(-4*(d*f*x + c*f)/d
)*cosh(f*x + e)*sinh(-4*(d*e - c*f)/d) + (d*f*x + c*f)*Ei(-6*(d*f*x + c*f)/
d)*cosh(f*x + e)*sinh(-6*(d*e - c*f)/d) + ((d*f*x + c*f)*Ei(-2*(d*f*x + c*f
)/d)*cosh(-2*(d*e - c*f)/d) - 2*(d*f*x + c*f)*Ei(-4*(d*f*x + c*f)/d)*cosh(-
4*(d*e - c*f)/d) + (d*f*x + c*f)*Ei(-6*(d*f*x + c*f)/d)*cosh(-6*(d*e - c*f)
/d))*cosh(f*x + e))*sinh(f*x + e)^2 + 3*(3*(d*f*x + c*f)*Ei(-2*(d*f*x + c*f
)/d)*cosh(f*x + e)^2*sinh(-2*(d*e - c*f)/d) - 6*(d*f*x + c*f)*Ei(-4*(d*f*x
+ c*f)/d)*cosh(f*x + e)^2*sinh(-4*(d*e - c*f)/d) + 3*(d*f*x + c*f)*Ei(-6*(d
*f*x + c*f)/d)*cosh(f*x + e)^2*sinh(-6*(d*e - c*f)/d) + (3*(d*f*x + c*f)*Ei
(-2*(d*f*x + c*f)/d)*cosh(-2*(d*e - c*f)/d) - 6*(d*f*x + c*f)*Ei(-4*(d*f*x
+ c*f)/d)*cosh(-4*(d*e - c*f)/d) + 3*(d*f*x + c*f)*Ei(-6*(d*f*x + c*f)/d)*c
osh(-6*(d*e - c*f)/d) - d)*cosh(f*x + e)^2 + d)*sinh(f*x + e))/((a^3*d^3*x
+ a^3*c*d^2)*cosh(f*x + e)^3 + 3*(a^3*d^3*x + a^3*c*d^2)*cosh(f*x + e)^2*si
nh(f*x + e) + 3*(a^3*d^3*x + a^3*c*d^2)*cosh(f*x + e)*sinh(f*x + e)^2 + (a^
```

$$3*d^3*x + a^3*c*d^2)*\sinh(f*x + e)^3)$$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{c^2 \coth^3(e+fx) + 3c^2 \coth^2(e+fx) + 3c^2 \coth(e+fx) + c^2 + 2cdx \coth^3(e+fx) + 6cdx \coth^2(e+fx) + 6cdx \coth(e+fx) + 2cdx + d^2x^2 \coth^3(e+fx) + 3d^2x^2 \coth^2(e+fx) + 3d^2x^2 \coth(e+fx) + d^2x^2} a^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d*x+c)**2/(a+a*coth(f*x+e))**3,x)

[Out] Integral(1/(c**2*coth(e + f*x)**3 + 3*c**2*coth(e + f*x)**2 + 3*c**2*coth(e + f*x) + c**2 + 2*c*d*x*coth(e + f*x)**3 + 6*c*d*x*coth(e + f*x)**2 + 6*c*d*x*coth(e + f*x) + 2*c*d*x + d**2*x**2*coth(e + f*x)**3 + 3*d**2*x**2*coth(e + f*x)**2 + 3*d**2*x**2*coth(e + f*x) + d**2*x**2), x)/a**3

Giac [A] time = 2.38177, size = 358, normalized size = 0.52

$$\frac{6 d f x \operatorname{Ei}\left(-\frac{6(d f x+c f)}{d}\right) e^{\left(\frac{6 c f}{d}-6 e\right)} - 12 d f x \operatorname{Ei}\left(-\frac{4(d f x+c f)}{d}\right) e^{\left(\frac{4 c f}{d}-4 e\right)} + 6 d f x \operatorname{Ei}\left(-\frac{2(d f x+c f)}{d}\right) e^{\left(\frac{2 c f}{d}-2 e\right)} + 6 c f \operatorname{Ei}\left(-\frac{6(d f x+c f)}{d}\right) e^{\left(\frac{6 c f}{d}-6 e\right)}}{8\left(a^3 d^3 x + a^3 c d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d*x+c)^2/(a+a*coth(f*x+e))^3,x, algorithm="giac")

[Out] 1/8*(6*d*f*x*Ei(-6*(d*f*x + c*f)/d)*e^(6*c*f/d - 6*e) - 12*d*f*x*Ei(-4*(d*f*x + c*f)/d)*e^(4*c*f/d - 4*e) + 6*d*f*x*Ei(-2*(d*f*x + c*f)/d)*e^(2*c*f/d - 2*e) + 6*c*f*Ei(-6*(d*f*x + c*f)/d)*e^(6*c*f/d - 6*e) - 12*c*f*Ei(-4*(d*f*x + c*f)/d)*e^(4*c*f/d - 4*e) + 6*c*f*Ei(-2*(d*f*x + c*f)/d)*e^(2*c*f/d - 2*e) + 3*d*e^(-2*f*x - 2*e) - 3*d*e^(-4*f*x - 4*e) + d*e^(-6*f*x - 6*e))/(a^3*d^3*x + a^3*c*d^2) - 1/8/((d*x + c)*a^3*d)

3.32

$$\int (c + dx)^m (a + a \coth(e + fx))^2 dx$$

Optimal. Leaf size=22

$$\text{Unintegrable}((c + dx)^m (a \coth(e + fx) + a)^2, x)$$

[Out] Unintegrable[(c + d*x)^m*(a + a*Coth[e + f*x])^2, x]

Rubi [A] time = 0.0495698, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int (c + dx)^m (a + a \coth(e + fx))^2 dx$$

Verification is Not applicable to the result.

[In] Int[(c + d*x)^m*(a + a*Coth[e + f*x])^2,x]

[Out] Defer[Int][(c + d*x)^m*(a + a*Coth[e + f*x])^2, x]

Rubi steps

$$\int (c + dx)^m (a + a \coth(e + fx))^2 dx = \int (c + dx)^m (a + a \coth(e + fx))^2 dx$$

Mathematica [A] time = 23.6822, size = 0, normalized size = 0.

$$\int (c + dx)^m (a + a \coth(e + fx))^2 dx$$

Verification is Not applicable to the result.

[In] Integrate[(c + d*x)^m*(a + a*Coth[e + f*x])^2,x]

[Out] Integrate[(c + d*x)^m*(a + a*Coth[e + f*x])^2, x]

Maple [A] time = 0.095, size = 0, normalized size = 0.

$$\int (dx + c)^m (a + a \coth(fx + e))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^m*(a+a*coth(f*x+e))^2,x)

[Out] int((d*x+c)^m*(a+a*coth(f*x+e))^2,x)

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^m*(a+a*coth(f*x+e))^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^2 \coth(fx + e)^2 + 2a^2 \coth(fx + e) + a^2\right)(dx + c)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^m*(a+a*coth(f*x+e))^2,x, algorithm="fricas")

[Out] integral((a^2*coth(f*x + e)^2 + 2*a^2*coth(f*x + e) + a^2)*(d*x + c)^m, x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$a^2 \left(\int 2(c + dx)^m \coth(e + fx) dx + \int (c + dx)^m \coth^2(e + fx) dx + \int (c + dx)^m dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*(a+a*coth(f*x+e))**2,x)
```

```
[Out] a**2*(Integral(2*(c + d*x)**m*coth(e + f*x), x) + Integral((c + d*x)**m*cot
h(e + f*x)**2, x) + Integral((c + d*x)**m, x))
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (a \coth(fx + e) + a)^2 (dx + c)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+a*coth(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((a*coth(f*x + e) + a)^2*(d*x + c)^m, x)
```


$$3.33 \quad \int (c + dx)^m (a + a \coth(e + fx)) dx$$

Optimal. Leaf size=20

$$\text{Unintegrable}((c + dx)^m (a \coth(e + fx) + a), x)$$

[Out] Unintegrable[(c + d*x)^m*(a + a*Coth[e + f*x]), x]

Rubi [A] time = 0.0268701, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int (c + dx)^m (a + a \coth(e + fx)) dx$$

Verification is Not applicable to the result.

[In] Int[(c + d*x)^m*(a + a*Coth[e + f*x]), x]

[Out] Defer[Int][(c + d*x)^m*(a + a*Coth[e + f*x]), x]

Rubi steps

$$\int (c + dx)^m (a + a \coth(e + fx)) dx = \int (c + dx)^m (a + a \coth(e + fx)) dx$$

Mathematica [A] time = 13.5735, size = 0, normalized size = 0.

$$\int (c + dx)^m (a + a \coth(e + fx)) dx$$

Verification is Not applicable to the result.

[In] Integrate[(c + d*x)^m*(a + a*Coth[e + f*x]), x]

[Out] Integrate[(c + d*x)^m*(a + a*Coth[e + f*x]), x]

Maple [A] time = 0.085, size = 0, normalized size = 0.

$$\int (dx + c)^m (a + a \coth(fx + e)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^m*(a+a*coth(f*x+e)),x)

[Out] int((d*x+c)^m*(a+a*coth(f*x+e)),x)

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^m*(a+a*coth(f*x+e)),x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}((a \coth(fx + e) + a)(dx + c)^m, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^m*(a+a*coth(f*x+e)),x, algorithm="fricas")

[Out] integral((a*coth(f*x + e) + a)*(d*x + c)^m, x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$a \left(\int (c + dx)^m \coth(e + fx) dx + \int (c + dx)^m dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*(a+a*coth(f*x+e)),x)
```

```
[Out] a*(Integral((c + d*x)**m*coth(e + f*x), x) + Integral((c + d*x)**m, x))
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (a \coth(fx + e) + a)(dx + c)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+a*coth(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((a*coth(f*x + e) + a)*(d*x + c)^m, x)
```

$$3.34 \quad \int \frac{(c+dx)^m}{a+a \coth(e+fx)} dx$$

Optimal. Leaf size=88

$$\frac{2^{-m-2} e^{\frac{2cf}{d}-2e} (c+dx)^m \left(\frac{f(c+dx)}{d}\right)^{-m} \Gamma\left(m+1, \frac{2f(c+dx)}{d}\right)}{af} + \frac{(c+dx)^{m+1}}{2ad(m+1)}$$

[Out] (c + d*x)^(1 + m)/(2*a*d*(1 + m)) + (2^(-2 - m)*E^(-2*e + (2*c*f)/d)*(c + d*x)^m*Gamma[1 + m, (2*f*(c + d*x))/d])/(a*f*((f*(c + d*x))/d)^m)

Rubi [A] time = 0.116941, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$, Rules used = {3727, 2181}

$$\frac{2^{-m-2} e^{\frac{2cf}{d}-2e} (c+dx)^m \left(\frac{f(c+dx)}{d}\right)^{-m} \Gamma\left(m+1, \frac{2f(c+dx)}{d}\right)}{af} + \frac{(c+dx)^{m+1}}{2ad(m+1)}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)^m/(a + a*Coth[e + f*x]),x]

[Out] (c + d*x)^(1 + m)/(2*a*d*(1 + m)) + (2^(-2 - m)*E^(-2*e + (2*c*f)/d)*(c + d*x)^m*Gamma[1 + m, (2*f*(c + d*x))/d])/(a*f*((f*(c + d*x))/d)^m)

Rule 3727

```
Int[((c_.) + (d_.)*(x_))^(m_)/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol]
:> Simp[(c + d*x)^(m + 1)/(2*a*d*(m + 1)), x] + Dist[1/(2*a), Int[(c + d*x)^m*E^((2*a*(e + f*x))/b), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && !IntegerQ[m]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Log[F])/d))*(c + d*x])]/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F])*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !IntegerQ[m]
```

Rubi steps

$$\int \frac{(c+dx)^m}{a+a \coth(e+fx)} dx = \frac{(c+dx)^{1+m}}{2ad(1+m)} + \frac{\int e^{2i\left(i e + \frac{\pi}{2} + i f x\right)} (c+dx)^m dx}{2a}$$

$$= \frac{(c+dx)^{1+m}}{2ad(1+m)} + \frac{2^{-2-m} e^{-2e + \frac{2cf}{d}} (c+dx)^m \left(\frac{f(c+dx)}{d}\right)^{-m} \Gamma\left(1+m, \frac{2f(c+dx)}{d}\right)}{af}$$

Mathematica [B] time = 1.12956, size = 186, normalized size = 2.11

$$\frac{2^{-m-2} (c+dx)^m \operatorname{csch}(e+fx) \left(-\frac{f(c+dx)}{d}\right)^m \left(-\frac{f^2(c+dx)^2}{d^2}\right)^{-m} \left(\sinh\left(f\left(\frac{c}{d}+x\right)\right) + \cosh\left(f\left(\frac{c}{d}+x\right)\right)\right) \left(d(m+1) \left(\cosh\left(e-\frac{cf}{d}\right)\right)\right)}{adf(m+1)(\coth(e+fx))}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)^m/(a + a*Coth[e + f*x]), x]

[Out] (2^(-2 - m)*(c + d*x)^m*(-((f*(c + d*x))/d))^m*Csch[e + f*x]*(d*(1 + m)*Gamma[1 + m, (2*f*(c + d*x))/d]*(Cosh[e - (c*f)/d] - Sinh[e - (c*f)/d]) + 2^(1 + m)*f*(f*(c/d + x))^m*(c + d*x)*(Cosh[e - (c*f)/d] + Sinh[e - (c*f)/d]))*(Cosh[f*(c/d + x)] + Sinh[f*(c/d + x)]))/(a*d*f*(1 + m)*(-(f^2*(c + d*x)^2)/d^2))^m*(1 + Coth[e + f*x]))

Maple [F] time = 0.139, size = 0, normalized size = 0.

$$\int \frac{(dx+c)^m}{a+a \coth(fx+e)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^m/(a+a*coth(f*x+e)), x)

[Out] int((d*x+c)^m/(a+a*coth(f*x+e)), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx + c)^m}{a \coth(fx + e) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^m/(a+a*coth(f*x+e)),x, algorithm="maxima")

[Out] integrate((d*x + c)^m/(a*coth(f*x + e) + a), x)

Fricas [A] time = 2.23782, size = 366, normalized size = 4.16

$$\frac{(dm + d) \cosh\left(\frac{dm \log\left(\frac{2f}{d}\right) + 2de - 2cf}{d}\right) \Gamma\left(m + 1, \frac{2(df x + cf)}{d}\right) - (dm + d) \Gamma\left(m + 1, \frac{2(df x + cf)}{d}\right) \sinh\left(\frac{dm \log\left(\frac{2f}{d}\right) + 2de - 2cf}{d}\right) + 2(df x + cf) \Gamma\left(m + 1, \frac{2(df x + cf)}{d}\right)}{4(adfm + adf)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^m/(a+a*coth(f*x+e)),x, algorithm="fricas")

[Out] 1/4*((d*m + d)*cosh((d*m*log(2*f/d) + 2*d*e - 2*c*f)/d)*gamma(m + 1, 2*(d*f*x + c*f)/d) - (d*m + d)*gamma(m + 1, 2*(d*f*x + c*f)/d)*sinh((d*m*log(2*f/d) + 2*d*e - 2*c*f)/d) + 2*(d*f*x + c*f)*cosh(m*log(d*x + c)) + 2*(d*f*x + c*f)*sinh(m*log(d*x + c)))/(a*d*f*m + a*d*f)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{(c+dx)^m}{\coth(e+fx)+1} dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**m/(a+a*coth(f*x+e)),x)

[Out] Integral((c + d*x)**m/(coth(e + f*x) + 1), x)/a

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx + c)^m}{a \coth(fx + e) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^m/(a+a*coth(f*x+e)),x, algorithm="giac")

[Out] integrate((d*x + c)^m/(a*coth(f*x + e) + a), x)

$$3.35 \quad \int \frac{(c+dx)^m}{(a+a \coth(e+fx))^2} dx$$

Optimal. Leaf size=152

$$\frac{2^{-m-2} e^{\frac{2cf}{d}-2e} (c+dx)^m \left(\frac{f(c+dx)}{d}\right)^{-m} \Gamma\left(m+1, \frac{2f(c+dx)}{d}\right)}{a^2 f} - \frac{4^{-m-2} e^{\frac{4cf}{d}-4e} (c+dx)^m \left(\frac{f(c+dx)}{d}\right)^{-m} \Gamma\left(m+1, \frac{4f(c+dx)}{d}\right)}{a^2 f}$$

[Out] (c + d*x)^(1 + m)/(4*a^2*d*(1 + m)) + (2^(-2 - m)*E^(-2*e + (2*c*f)/d)*(c + d*x)^m*Gamma[1 + m, (2*f*(c + d*x))/d])/(a^2*f*((f*(c + d*x))/d)^m) - (4^(-2 - m)*E^(-4*e + (4*c*f)/d)*(c + d*x)^m*Gamma[1 + m, (4*f*(c + d*x))/d])/(a^2*f*((f*(c + d*x))/d)^m)

Rubi [A] time = 0.172427, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$, Rules used = {3729, 2181}

$$\frac{2^{-m-2} e^{\frac{2cf}{d}-2e} (c+dx)^m \left(\frac{f(c+dx)}{d}\right)^{-m} \Gamma\left(m+1, \frac{2f(c+dx)}{d}\right)}{a^2 f} - \frac{4^{-m-2} e^{\frac{4cf}{d}-4e} (c+dx)^m \left(\frac{f(c+dx)}{d}\right)^{-m} \Gamma\left(m+1, \frac{4f(c+dx)}{d}\right)}{a^2 f}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)^m/(a + a*Coth[e + f*x])^2,x]

[Out] (c + d*x)^(1 + m)/(4*a^2*d*(1 + m)) + (2^(-2 - m)*E^(-2*e + (2*c*f)/d)*(c + d*x)^m*Gamma[1 + m, (2*f*(c + d*x))/d])/(a^2*f*((f*(c + d*x))/d)^m) - (4^(-2 - m)*E^(-4*e + (4*c*f)/d)*(c + d*x)^m*Gamma[1 + m, (4*f*(c + d*x))/d])/(a^2*f*((f*(c + d*x))/d)^m)

Rule 3729

Int[((c_.) + (d_.)*(x_))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, (1/(2*a) + E^((2*a*(e + f*x))/b)/(2*a))^(-n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && ILtQ[n, 0]

Rule 2181

Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol] := -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Log[F]))/d)]*(c + d*x)]/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F]

]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !IntegerQ[m]

Rubi steps

$$\begin{aligned} \int \frac{(c + dx)^m}{(a + a \coth(e + fx))^2} dx &= \int \left(\frac{(c + dx)^m}{4a^2} + \frac{e^{-4e-4fx}(c + dx)^m}{4a^2} - \frac{e^{-2e-2fx}(c + dx)^m}{2a^2} \right) dx \\ &= \frac{(c + dx)^{1+m}}{4a^2 d(1+m)} + \frac{\int e^{-4e-4fx}(c + dx)^m dx}{4a^2} - \frac{\int e^{-2e-2fx}(c + dx)^m dx}{2a^2} \\ &= \frac{(c + dx)^{1+m}}{4a^2 d(1+m)} + \frac{2^{-2-m} e^{-2e + \frac{2cf}{d}} (c + dx)^m \left(\frac{f(c+dx)}{d} \right)^{-m} \Gamma\left(1 + m, \frac{2f(c+dx)}{d}\right)}{a^2 f} - \frac{4^{-2-m} e^{-4e + \frac{4cf}{d}} (c + dx)^m \left(\frac{f(c+dx)}{d} \right)^{-m} \Gamma\left(1 + m, \frac{2f(c+dx)}{d}\right)}{a^2 f} \end{aligned}$$

Mathematica [A] time = 8.10184, size = 194, normalized size = 1.28

$$\frac{4^{-m-2}(c + dx)^m \operatorname{csch}^2(e + fx)(\sinh(2fx) + \cosh(2fx)) \left(-\frac{f(c+dx)}{d} \right)^m \left(-\frac{f^2(c+dx)^2}{d^2} \right)^{-m} \left(-d(m+1)e^{\frac{4cf}{d}} (\cosh(2e) - \sinh(2e)) \right)}{a^2 d f (m+1) (\coth(e + fx))^2}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)^m/(a + a*Coth[e + f*x])^2,x]

[Out] (4^(-2 - m)*(c + d*x)^m*(-((f*(c + d*x))/d))^m*Csch[e + f*x]^2*(2^(2 + m)*d *E^((2*c*f)/d)*(1 + m)*Gamma[1 + m, (2*f*(c + d*x))/d] + 4^(1 + m)*d*((f*(c + d*x))/d)^(1 + m)*(Cosh[e] + Sinh[e])^2 - d*E^((4*c*f)/d)*(1 + m)*Gamma[1 + m, (4*f*(c + d*x))/d]*(Cosh[2*e] - Sinh[2*e]))*(Cosh[2*f*x] + Sinh[2*f*x]))/(a^2*d*f*(1 + m)*(-((f^2*(c + d*x)^2)/d^2))^m*(1 + Coth[e + f*x])^2)

Maple [F] time = 0.128, size = 0, normalized size = 0.

$$\int \frac{(dx + c)^m}{(a + a \operatorname{coth}(fx + e))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^m/(a+a*coth(f*x+e))^2,x)

[Out] $\int (d*x+c)^m/(a+a*\coth(f*x+e))^2, x$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx + c)^m}{(a \coth(fx + e) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)^m/(a+a*coth(f*x+e))^2,x, algorithm="maxima")`

[Out] `integrate((d*x + c)^m/(a*coth(f*x + e) + a)^2, x)`

Fricas [A] time = 2.34152, size = 609, normalized size = 4.01

$$(dm + d) \cosh\left(\frac{dm \log\left(\frac{4f}{d}\right) + 4de - 4cf}{d}\right) \Gamma\left(m + 1, \frac{4(dfx + cf)}{d}\right) - 4(dm + d) \cosh\left(\frac{dm \log\left(\frac{2f}{d}\right) + 2de - 2cf}{d}\right) \Gamma\left(m + 1, \frac{2(dfx + cf)}{d}\right) - (d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)^m/(a+a*coth(f*x+e))^2,x, algorithm="fricas")`

[Out] $-1/16*((d*m + d)*\cosh((d*m*\log(4*f/d) + 4*d*e - 4*c*f)/d)*\gamma(m + 1, 4*(d*f*x + c*f)/d) - 4*(d*m + d)*\cosh((d*m*\log(2*f/d) + 2*d*e - 2*c*f)/d)*\gamma(m + 1, 2*(d*f*x + c*f)/d) - (d*m + d)*\gamma(m + 1, 4*(d*f*x + c*f)/d)*\sinh((d*m*\log(4*f/d) + 4*d*e - 4*c*f)/d) + 4*(d*m + d)*\gamma(m + 1, 2*(d*f*x + c*f)/d)*\sinh((d*m*\log(2*f/d) + 2*d*e - 2*c*f)/d) - 4*(d*f*x + c*f)*\cosh(m*\log(d*x + c)) - 4*(d*f*x + c*f)*\sinh(m*\log(d*x + c)))/(a^2*d*f*m + a^2*d*f)$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(c+dx)^m}{\frac{\coth^2(e+fx)+2\coth(e+fx)+1}{a^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m/(a+a*coth(f*x+e))**2,x)
```

```
[Out] Integral((c + d*x)**m/(coth(e + f*x)**2 + 2*coth(e + f*x) + 1), x)/a**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx + c)^m}{(a \coth(fx + e) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m/(a+a*coth(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m/(a*coth(f*x + e) + a)^2, x)
```

$$3.36 \quad \int \frac{(c+dx)^m}{(a+a \coth(e+fx))^3} dx$$

Optimal. Leaf size=223

$$\frac{3 \cdot 2^{-m-4} e^{\frac{2cf}{d}-2e} (c+dx)^m \left(\frac{f(c+dx)}{d}\right)^{-m} \Gamma\left(m+1, \frac{2f(c+dx)}{d}\right)}{a^3 f} - \frac{3 \cdot 2^{-2m-5} e^{\frac{4cf}{d}-4e} (c+dx)^m \left(\frac{f(c+dx)}{d}\right)^{-m} \Gamma\left(m+1, \frac{4f(c+dx)}{d}\right)}{a^3 f}$$

[Out] (c + d*x)^(1 + m)/(8*a^3*d*(1 + m)) + (3*2^(-4 - m)*E^(-2*e + (2*c*f)/d)*(c + d*x)^m*Gamma[1 + m, (2*f*(c + d*x))/d])/(a^3*f*((f*(c + d*x))/d)^m) - (3*2^(-5 - 2*m)*E^(-4*e + (4*c*f)/d)*(c + d*x)^m*Gamma[1 + m, (4*f*(c + d*x))/d])/(a^3*f*((f*(c + d*x))/d)^m) + (2^(-4 - m)*3^(-1 - m)*E^(-6*e + (6*c*f)/d)*(c + d*x)^m*Gamma[1 + m, (6*f*(c + d*x))/d])/(a^3*f*((f*(c + d*x))/d)^m)

Rubi [A] time = 0.233118, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$, Rules used = {3729, 2181}

$$\frac{3 \cdot 2^{-m-4} e^{\frac{2cf}{d}-2e} (c+dx)^m \left(\frac{f(c+dx)}{d}\right)^{-m} \Gamma\left(m+1, \frac{2f(c+dx)}{d}\right)}{a^3 f} - \frac{3 \cdot 2^{-2m-5} e^{\frac{4cf}{d}-4e} (c+dx)^m \left(\frac{f(c+dx)}{d}\right)^{-m} \Gamma\left(m+1, \frac{4f(c+dx)}{d}\right)}{a^3 f}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)^m/(a + a*Coth[e + f*x])^3,x]

[Out] (c + d*x)^(1 + m)/(8*a^3*d*(1 + m)) + (3*2^(-4 - m)*E^(-2*e + (2*c*f)/d)*(c + d*x)^m*Gamma[1 + m, (2*f*(c + d*x))/d])/(a^3*f*((f*(c + d*x))/d)^m) - (3*2^(-5 - 2*m)*E^(-4*e + (4*c*f)/d)*(c + d*x)^m*Gamma[1 + m, (4*f*(c + d*x))/d])/(a^3*f*((f*(c + d*x))/d)^m) + (2^(-4 - m)*3^(-1 - m)*E^(-6*e + (6*c*f)/d)*(c + d*x)^m*Gamma[1 + m, (6*f*(c + d*x))/d])/(a^3*f*((f*(c + d*x))/d)^m)

Rule 3729

Int[((c_.) + (d_.)*(x_))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, (1/(2*a) + E^((2*a*(e + f*x))/b)/(2*a))^(-n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && ILtQ[n, 0]

Rule 2181

```
Int[(F_)^((g_.)*(e_.) + (f_.)*(x_))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Log[F])/d))*(c + d*x])]/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F])*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !IntegerQ[m]
```

Rubi steps

$$\begin{aligned} \int \frac{(c + dx)^m}{(a + a \coth(e + fx))^3} dx &= \int \left(\frac{(c + dx)^m}{8a^3} - \frac{e^{-6e-6fx}(c + dx)^m}{8a^3} + \frac{3e^{-4e-4fx}(c + dx)^m}{8a^3} - \frac{3e^{-2e-2fx}(c + dx)^m}{8a^3} \right) dx \\ &= \frac{(c + dx)^{1+m}}{8a^3 d(1+m)} - \frac{\int e^{-6e-6fx}(c + dx)^m dx}{8a^3} + \frac{3 \int e^{-4e-4fx}(c + dx)^m dx}{8a^3} - \frac{3 \int e^{-2e-2fx}(c + dx)^m dx}{8a^3} \\ &= \frac{(c + dx)^{1+m}}{8a^3 d(1+m)} + \frac{3 \cdot 2^{-4-m} e^{-2e+\frac{2cf}{d}} (c + dx)^m \left(\frac{f(c+dx)}{d}\right)^{-m} \Gamma\left(1+m, \frac{2f(c+dx)}{d}\right)}{a^3 f} - \frac{3 \cdot 2^{-5-2m} e^{-4e+\frac{2cf}{d}} (c + dx)^m \left(\frac{f(c+dx)}{d}\right)^{-m} \Gamma\left(1+m, \frac{2f(c+dx)}{d}\right)}{a^3 f} \end{aligned}$$

Mathematica [F] time = 180.002, size = 0, normalized size = 0.

\$Aborted

Verification is Not applicable to the result.

[In] Integrate[(c + d*x)^m/(a + a*Coth[e + f*x])^3,x]

[Out] \$Aborted

Maple [F] time = 0.161, size = 0, normalized size = 0.

$$\int \frac{(dx + c)^m}{(a + a \coth(fx + e))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^m/(a+a*coth(f*x+e))^3,x)

[Out] int((d*x+c)^m/(a+a*coth(f*x+e))^3,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx + c)^m}{(a \coth(fx + e) + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^m/(a+a*coth(f*x+e))^3,x, algorithm="maxima")

[Out] integrate((d*x + c)^m/(a*coth(f*x + e) + a)^3, x)

Fricas [A] time = 2.28678, size = 853, normalized size = 3.83

$$2(dm + d) \cosh\left(\frac{dm \log\left(\frac{6f}{d}\right) + 6de - 6cf}{d}\right) \Gamma\left(m + 1, \frac{6(df x + cf)}{d}\right) - 9(dm + d) \cosh\left(\frac{dm \log\left(\frac{4f}{d}\right) + 4de - 4cf}{d}\right) \Gamma\left(m + 1, \frac{4(df x + cf)}{d}\right) + 18$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^m/(a+a*coth(f*x+e))^3,x, algorithm="fricas")

[Out] 1/96*(2*(d*m + d)*cosh((d*m*log(6*f/d) + 6*d*e - 6*c*f)/d)*gamma(m + 1, 6*(d*f*x + c*f)/d) - 9*(d*m + d)*cosh((d*m*log(4*f/d) + 4*d*e - 4*c*f)/d)*gamma(m + 1, 4*(d*f*x + c*f)/d) + 18*(d*m + d)*cosh((d*m*log(2*f/d) + 2*d*e - 2*c*f)/d)*gamma(m + 1, 2*(d*f*x + c*f)/d) - 2*(d*m + d)*gamma(m + 1, 6*(d*f*x + c*f)/d)*sinh((d*m*log(6*f/d) + 6*d*e - 6*c*f)/d) + 9*(d*m + d)*gamma(m + 1, 4*(d*f*x + c*f)/d)*sinh((d*m*log(4*f/d) + 4*d*e - 4*c*f)/d) - 18*(d*m + d)*gamma(m + 1, 2*(d*f*x + c*f)/d)*sinh((d*m*log(2*f/d) + 2*d*e - 2*c*f)/d) + 12*(d*f*x + c*f)*cosh(m*log(d*x + c)) + 12*(d*f*x + c*f)*sinh(m*log(d*x + c)))/(a^3*d*f*m + a^3*d*f)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(c+dx)^m}{\frac{\coth^3(e+fx)+3\coth^2(e+fx)+3\coth(e+fx)+1}{a^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m/(a+a*coth(f*x+e))**3,x)
```

```
[Out] Integral((c + d*x)**m/(coth(e + f*x)**3 + 3*coth(e + f*x)**2 + 3*coth(e + f*x) + 1), x)/a**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx + c)^m}{(a \coth(fx + e) + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m/(a+a*coth(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m/(a*coth(f*x + e) + a)^3, x)
```

3.37 $\int (c + dx)^3 (a + b \coth(e + fx)) dx$

Optimal. Leaf size=133

$$-\frac{3bd^2(c+dx)\text{PolyLog}\left(3, e^{2(e+fx)}\right)}{2f^3} + \frac{3bd(c+dx)^2\text{PolyLog}\left(2, e^{2(e+fx)}\right)}{2f^2} + \frac{3bd^3\text{PolyLog}\left(4, e^{2(e+fx)}\right)}{4f^4} + \frac{a(c+dx)^4}{4d} + \frac{b(c+dx)^4}{4d}$$

[Out] $(a*(c + d*x)^4)/(4*d) - (b*(c + d*x)^4)/(4*d) + (b*(c + d*x)^3*\text{Log}[1 - E^{2*(e + f*x)}])/f + (3*b*d*(c + d*x)^2*\text{PolyLog}[2, E^{2*(e + f*x)}])/(2*f^2) - (3*b*d^2*(c + d*x)*\text{PolyLog}[3, E^{2*(e + f*x)}])/(2*f^3) + (3*b*d^3*\text{PolyLog}[4, E^{2*(e + f*x)}])/(4*f^4)$

Rubi [A] time = 0.259571, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {3722, 3716, 2190, 2531, 6609, 2282, 6589}

$$-\frac{3bd^2(c+dx)\text{PolyLog}\left(3, e^{2(e+fx)}\right)}{2f^3} + \frac{3bd(c+dx)^2\text{PolyLog}\left(2, e^{2(e+fx)}\right)}{2f^2} + \frac{3bd^3\text{PolyLog}\left(4, e^{2(e+fx)}\right)}{4f^4} + \frac{a(c+dx)^4}{4d} + \frac{b(c+dx)^4}{4d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(c + d*x)^3*(a + b*\text{Coth}[e + f*x]), x]$

[Out] $(a*(c + d*x)^4)/(4*d) - (b*(c + d*x)^4)/(4*d) + (b*(c + d*x)^3*\text{Log}[1 - E^{2*(e + f*x)}])/f + (3*b*d*(c + d*x)^2*\text{PolyLog}[2, E^{2*(e + f*x)}])/(2*f^2) - (3*b*d^2*(c + d*x)*\text{PolyLog}[3, E^{2*(e + f*x)}])/(2*f^3) + (3*b*d^3*\text{PolyLog}[4, E^{2*(e + f*x)}])/(4*f^4)$

Rule 3722

$\text{Int}[(c + d*x)^m*(a + b*\text{Tan}[e + f*x])^n, x]$ \rightarrow $\text{Int}[\text{ExpandIntegrand}[(c + d*x)^m, (a + b*\text{Tan}[e + f*x])^n, x], x]$ /; $\text{FreeQ}\{a, b, c, d, e, f, m\}, x$ && $\text{IGtQ}[m, 0]$ && $\text{IGtQ}[n, 0]$

Rule 3716

$\text{Int}[(c + d*x)^m*\text{tan}[e + \text{Pi}*k + (Complex[0, fz])*f*x], x]$ \rightarrow $-\text{Simp}[(I*(c + d*x)^{(m+1)})/(d*(m+1)), x] + \text{Dist}[2*I, \text{Int}[(c + d*x)^m*\text{E}^{2*(-(I*e) + f*fz*x)}]/(\text{E}^{2*I*k*\text{Pi}}*(1 + \text{E}^{2*(-(I*e) + f*fz*x)})/\text{E}^{2*I*k*\text{Pi}})], x, x]$ /; $\text{FreeQ}\{c, d, e, f, fz\}, x$ && $\text{IntegerQ}[4*k]$ && $\text{IGtQ}[m, 0]$

Rule 2190

```
Int[(((F_)^((g_)*(e_) + (f_)*(x_)))^(n_))*((c_) + (d_)*(x_))^(m_)]/
((a_) + (b_)*(F_)^((g_)*(e_) + (f_)*(x_)))^(n_), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_)*(F_)^((c_)*((a_) + (b_)*(x_)))^(n_)]*(f_) + (g_)
*(x_)^(m_), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
))^n])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]), x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_) + (f_)*(x_))^(m_)*PolyLog[n_, (d_)*(F_)^((c_)*((a_) + (b_
)*(x_)))^(p_)], x_Symbol] := Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(
m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_)*((a_) + (b_)*x))*
(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_)*((a_) + (b_)*(x_))^(p_)]/((d_) + (e_)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int (c + dx)^3 (a + b \coth(e + fx)) dx &= \int (a(c + dx)^3 + b(c + dx)^3 \coth(e + fx)) dx \\
&= \frac{a(c + dx)^4}{4d} + b \int (c + dx)^3 \coth(e + fx) dx \\
&= \frac{a(c + dx)^4}{4d} - \frac{b(c + dx)^4}{4d} - (2b) \int \frac{e^{2(e+fx)}(c + dx)^3}{1 - e^{2(e+fx)}} dx \\
&= \frac{a(c + dx)^4}{4d} - \frac{b(c + dx)^4}{4d} + \frac{b(c + dx)^3 \log(1 - e^{2(e+fx)})}{f} - \frac{(3bd) \int (c + dx)^2 \log(1 - e^{2(e+fx)})}{f} \\
&= \frac{a(c + dx)^4}{4d} - \frac{b(c + dx)^4}{4d} + \frac{b(c + dx)^3 \log(1 - e^{2(e+fx)})}{f} + \frac{3bd(c + dx)^2 \text{Li}_2(e^{2(e+fx)})}{2f^2} \\
&= \frac{a(c + dx)^4}{4d} - \frac{b(c + dx)^4}{4d} + \frac{b(c + dx)^3 \log(1 - e^{2(e+fx)})}{f} + \frac{3bd(c + dx)^2 \text{Li}_2(e^{2(e+fx)})}{2f^2} \\
&= \frac{a(c + dx)^4}{4d} - \frac{b(c + dx)^4}{4d} + \frac{b(c + dx)^3 \log(1 - e^{2(e+fx)})}{f} + \frac{3bd(c + dx)^2 \text{Li}_2(e^{2(e+fx)})}{2f^2} \\
&= \frac{a(c + dx)^4}{4d} - \frac{b(c + dx)^4}{4d} + \frac{b(c + dx)^3 \log(1 - e^{2(e+fx)})}{f} + \frac{3bd(c + dx)^2 \text{Li}_2(e^{2(e+fx)})}{2f^2} \\
&= \frac{a(c + dx)^4}{4d} - \frac{b(c + dx)^4}{4d} + \frac{b(c + dx)^3 \log(1 - e^{2(e+fx)})}{f} + \frac{3bd(c + dx)^2 \text{Li}_2(e^{2(e+fx)})}{2f^2}
\end{aligned}$$

Mathematica [A] time = 0.358417, size = 249, normalized size = 1.87

$$\frac{1}{4} \left(-\frac{6bd^2(c + dx)\text{PolyLog}(3, e^{2(e+fx)})}{f^3} + \frac{6bd(c + dx)^2\text{PolyLog}(2, e^{2(e+fx)})}{f^2} + \frac{3bd^3\text{PolyLog}(4, e^{2(e+fx)})}{f^4} + 6ac^2dx^2 + 4a \right)$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)^3*(a + b*Coth[e + f*x]),x]

[Out] (4*a*c^3*x + 6*a*c^2*d*x^2 - 6*b*c^2*d*x^2 + 4*a*c*d^2*x^3 - 4*b*c*d^2*x^3 + a*d^3*x^4 - b*d^3*x^4 + (12*b*c^2*d*x*Log[1 - E^(2*(e + f*x))])/f + (12*b*c*d^2*x^2*Log[1 - E^(2*(e + f*x))])/f + (4*b*d^3*x^3*Log[1 - E^(2*(e + f*x))])/f + (4*b*c^3*Log[Cosh[e + f*x]])/f + (4*b*c^3*Log[Tanh[e + f*x]])/f + (6*b*d*(c + d*x)^2*PolyLog[2, E^(2*(e + f*x))])/f^2 - (6*b*d^2*(c + d*x)*PolyLog[3, E^(2*(e + f*x))])/f^3 + (3*b*d^3*PolyLog[4, E^(2*(e + f*x))])/f^4)/4

Maple [B] time = 0.153, size = 748, normalized size = 5.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x+c)^3*(a+b*coth(f*x+e)),x)`

[Out]
$$\begin{aligned} & -3/2*b/f^4*d^3*e^4+6*b/f^4*d^3*polylog(4,-exp(f*x+e))+6*b/f^4*d^3*polylog(4, \\ & exp(f*x+e))+b/f*c^3*ln(exp(f*x+e)-1)+b/f*c^3*ln(exp(f*x+e)+1)-2*b/f*c^3*ln \\ & (exp(f*x+e))+2*b/f^4*d^3*e^3*ln(exp(f*x+e))+b/f*d^3*ln(1-exp(f*x+e))*x^3+b/ \\ & f^4*d^3*ln(1-exp(f*x+e))*e^3+3*b/f^2*d^3*polylog(2,exp(f*x+e))*x^2+1/4*a*d^ \\ & 3*x^4+a*c*d^2*x^3-b*c*d^2*x^3+3/2*a*c^2*d*x^2-3/2*b*c^2*d*x^2-3*b/f^2*c^2*d \\ & *e*ln(exp(f*x+e)-1)+3*b/f^3*c*d^2*e^2*ln(exp(f*x+e)-1)+3*b/f*c*d^2*ln(1-exp \\ & (f*x+e))*x^2-3*b/f^3*c*d^2*ln(1-exp(f*x+e))*e^2+6*b/f^2*c*d^2*polylog(2,exp \\ & (f*x+e))*x+3*b/f*c*d^2*ln(exp(f*x+e)+1)*x^2+6*b/f^2*c*d^2*polylog(2,-exp(f* \\ & x+e))*x+3*b/f*c^2*d*ln(1-exp(f*x+e))*x+3*b/f^2*c^2*d*ln(1-exp(f*x+e))*e+3*b \\ & /f*c^2*d*ln(exp(f*x+e)+1)*x+6*b/f^2*c^2*d*e*ln(exp(f*x+e))-6*b/f^3*c*d^2*e^ \\ & 2*ln(exp(f*x+e))-1/4*b*d^3*x^4+c^3*a*x+b*c^3*x-3*b/f^2*c^2*d*e^2+4*b/f^3*c* \\ & d^2*e^3-2*b/f^3*d^3*e^3*x-6*b/f^3*c*d^2*polylog(3,exp(f*x+e))-6*b/f^3*c*d^2 \\ & *polylog(3,-exp(f*x+e))-b/f^4*d^3*e^3*ln(exp(f*x+e)-1)+3*b/f^2*c^2*d*polylo \\ & g(2,exp(f*x+e))+3*b/f^2*c^2*d*polylog(2,-exp(f*x+e))+3*b/f^2*d^3*polylog(2, \\ & -exp(f*x+e))*x^2-6*b/f^3*d^3*polylog(3,-exp(f*x+e))*x-6*b/f^3*d^3*polylog(3 \\ & ,exp(f*x+e))*x+b/f*d^3*ln(exp(f*x+e)+1)*x^3-6*b/f*c^2*d*e*x+6*b/f^2*c*d^2*e \\ & ^2*x \end{aligned}$$

Maxima [B] time = 1.41425, size = 566, normalized size = 4.26

$$\frac{1}{4} ad^3x^4 + \frac{1}{4} bd^3x^4 + acd^2x^3 + bcd^2x^3 + \frac{3}{2} ac^2dx^2 + \frac{3}{2} bc^2dx^2 + ac^3x + \frac{bc^3 \log(\sinh(fx + e))}{f} + \frac{3(fx \log(e^{(fx+e)} + 1))}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)^3*(a+b*coth(f*x+e)),x, algorithm="maxima")`

[Out]
$$\begin{aligned} & 1/4*a*d^3*x^4 + 1/4*b*d^3*x^4 + a*c*d^2*x^3 + b*c*d^2*x^3 + 3/2*a*c^2*d*x^2 \\ & + 3/2*b*c^2*d*x^2 + a*c^3*x + b*c^3*log(sinh(f*x + e))/f + 3*(f*x*log(e^(f \\ & *x + e) + 1) + dilog(-e^(f*x + e)))*b*c^2*d/f^2 + 3*(f*x*log(-e^(f*x + e) + \\ & 1) + dilog(e^(f*x + e)))*b*c^2*d/f^2 + 3*(f^2*x^2*log(e^(f*x + e) + 1) + 2 \\ & *f*x*dilog(-e^(f*x + e)) - 2*polylog(3, -e^(f*x + e)))*b*c*d^2/f^3 + 3*(f^2 \\ & *x^2*log(-e^(f*x + e) + 1) + 2*f*x*dilog(e^(f*x + e)) - 2*polylog(3, e^(f*x \\ & + e)))*b*c*d^2/f^3 + (f^3*x^3*log(e^(f*x + e) + 1) + 3*f^2*x^2*dilog(-e^(f \\ & *x + e)) - 6*f*x*polylog(3, -e^(f*x + e)) + 6*polylog(4, -e^(f*x + e)))*b*d \\ & ^3/f^4 + (f^3*x^3*log(-e^(f*x + e) + 1) + 3*f^2*x^2*dilog(e^(f*x + e)) - 6* \\ & f*x*polylog(3, e^(f*x + e)) + 6*polylog(4, e^(f*x + e)))*b*d^3/f^4 - 1/2*(b \end{aligned}$$

$$*d^3*f^4*x^4 + 4*b*c*d^2*f^4*x^3 + 6*b*c^2*d*f^4*x^2)/f^4$$

Fricas [C] time = 2.35873, size = 1207, normalized size = 9.08

$$(a - b)d^3f^4x^4 + 4(a - b)cd^2f^4x^3 + 6(a - b)c^2df^4x^2 + 4(a - b)c^3f^4x + 24bd^3\text{polylog}(4, \cosh(fx + e) + \sinh(fx + e))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*(a+b*coth(f*x+e)),x, algorithm="fricas")

[Out] 1/4*((a - b)*d^3*f^4*x^4 + 4*(a - b)*c*d^2*f^4*x^3 + 6*(a - b)*c^2*d*f^4*x^2 + 4*(a - b)*c^3*f^4*x + 24*b*d^3*polylog(4, cosh(f*x + e) + sinh(f*x + e)) + 24*b*d^3*polylog(4, -cosh(f*x + e) - sinh(f*x + e)) + 12*(b*d^3*f^2*x^2 + 2*b*c*d^2*f^2*x + b*c^2*d*f^2)*dilog(cosh(f*x + e) + sinh(f*x + e)) + 12*(b*d^3*f^2*x^2 + 2*b*c*d^2*f^2*x + b*c^2*d*f^2)*dilog(-cosh(f*x + e) - sinh(f*x + e)) + 4*(b*d^3*f^3*x^3 + 3*b*c*d^2*f^3*x^2 + 3*b*c^2*d*f^3*x + b*c^3*f^3)*log(cosh(f*x + e) + sinh(f*x + e) + 1) - 4*(b*d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2 - b*c^3*f^3)*log(cosh(f*x + e) + sinh(f*x + e) - 1) + 4*(b*d^3*f^3*x^3 + 3*b*c*d^2*f^3*x^2 + 3*b*c^2*d*f^3*x + b*d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2)*log(-cosh(f*x + e) - sinh(f*x + e) + 1) - 24*(b*d^3*f*x + b*c*d^2*f)*polylog(3, cosh(f*x + e) + sinh(f*x + e)) - 24*(b*d^3*f*x + b*c*d^2*f)*polylog(3, -cosh(f*x + e) - sinh(f*x + e)))/f^4

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b \coth(e + fx))(c + dx)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**3*(a+b*coth(f*x+e)),x)

[Out] Integral((a + b*coth(e + f*x))*(c + d*x)**3, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^3 (b \coth(fx + e) + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+b*coth(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*(b*coth(f*x + e) + a), x)
```

3.38 $\int (c + dx)^2 (a + b \coth(e + fx)) dx$

Optimal. Leaf size=101

$$\frac{bd(c + dx)\text{PolyLog}\left(2, e^{2(e+fx)}\right)}{f^2} - \frac{bd^2\text{PolyLog}\left(3, e^{2(e+fx)}\right)}{2f^3} + \frac{a(c + dx)^3}{3d} + \frac{b(c + dx)^2 \log\left(1 - e^{2(e+fx)}\right)}{f} - \frac{b(c + dx)^3}{3d}$$

[Out] (a*(c + d*x)^3)/(3*d) - (b*(c + d*x)^3)/(3*d) + (b*(c + d*x)^2*Log[1 - E^(2*(e + f*x))])/f + (b*d*(c + d*x)*PolyLog[2, E^(2*(e + f*x))])/f^2 - (b*d^2*PolyLog[3, E^(2*(e + f*x))])/(2*f^3)

Rubi [A] time = 0.216669, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3722, 3716, 2190, 2531, 2282, 6589}

$$\frac{bd(c + dx)\text{PolyLog}\left(2, e^{2(e+fx)}\right)}{f^2} - \frac{bd^2\text{PolyLog}\left(3, e^{2(e+fx)}\right)}{2f^3} + \frac{a(c + dx)^3}{3d} + \frac{b(c + dx)^2 \log\left(1 - e^{2(e+fx)}\right)}{f} - \frac{b(c + dx)^3}{3d}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)^2*(a + b*Coth[e + f*x]),x]

[Out] (a*(c + d*x)^3)/(3*d) - (b*(c + d*x)^3)/(3*d) + (b*(c + d*x)^2*Log[1 - E^(2*(e + f*x))])/f + (b*d*(c + d*x)*PolyLog[2, E^(2*(e + f*x))])/f^2 - (b*d^2*PolyLog[3, E^(2*(e + f*x))])/(2*f^3)

Rule 3722

Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, (a + b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]

Rule 3716

Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)], x_Symbol] := -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c + d*x)^m*E^(2*(-I*e) + f*fz*x))/(E^(2*I*k*Pi)*(1 + E^(2*(-I*e) + f*fz*x)))/E^(2*I*k*Pi))], x, x] /; FreeQ[{c, d, e, f, fz}, x] && IntegerQ[4*k] && IGtQ[m, 0]

Rule 2190

```
Int[(((F_)^((g_)*(e_) + (f_)*(x_)))^(n_))*((c_) + (d_)*(x_))^(m_)]/
((a_) + (b_)*(F_)^((g_)*(e_) + (f_)*(x_)))^(n_), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_)*(F_)^((c_)*((a_) + (b_)*(x_)))^(n_)]*((f_) + (g_)
*(x_))^(m_), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
))^n]])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_)*((a_) + (b_)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_)*((a_) + (b_)*(x_))^(p_)]/((d_) + (e_)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int (c + dx)^2 (a + b \coth(e + fx)) dx &= \int (a(c + dx)^2 + b(c + dx)^2 \coth(e + fx)) dx \\
&= \frac{a(c + dx)^3}{3d} + b \int (c + dx)^2 \coth(e + fx) dx \\
&= \frac{a(c + dx)^3}{3d} - \frac{b(c + dx)^3}{3d} - (2b) \int \frac{e^{2(e+fx)}(c + dx)^2}{1 - e^{2(e+fx)}} dx \\
&= \frac{a(c + dx)^3}{3d} - \frac{b(c + dx)^3}{3d} + \frac{b(c + dx)^2 \log(1 - e^{2(e+fx)})}{f} - \frac{(2bd) \int (c + dx) \log(1 - e^{2(e+fx)})}{f} \\
&= \frac{a(c + dx)^3}{3d} - \frac{b(c + dx)^3}{3d} + \frac{b(c + dx)^2 \log(1 - e^{2(e+fx)})}{f} + \frac{bd(c + dx) \text{Li}_2(e^{2(e+fx)})}{f^2} - \frac{bd^2 \text{Li}_2(e^{2(e+fx)})}{f^2} \\
&= \frac{a(c + dx)^3}{3d} - \frac{b(c + dx)^3}{3d} + \frac{b(c + dx)^2 \log(1 - e^{2(e+fx)})}{f} + \frac{bd(c + dx) \text{Li}_2(e^{2(e+fx)})}{f^2} - \frac{bd^2 \text{Li}_2(e^{2(e+fx)})}{f^2} \\
&= \frac{a(c + dx)^3}{3d} - \frac{b(c + dx)^3}{3d} + \frac{b(c + dx)^2 \log(1 - e^{2(e+fx)})}{f} + \frac{bd(c + dx) \text{Li}_2(e^{2(e+fx)})}{f^2} - \frac{bd^2 \text{Li}_2(e^{2(e+fx)})}{f^2}
\end{aligned}$$

Mathematica [A] time = 0.216802, size = 149, normalized size = 1.48

$$\frac{6bdf(c + dx)\text{PolyLog}(2, e^{2(e+fx)}) - 3bd^2\text{PolyLog}(3, e^{2(e+fx)}) + 2f^2(3ac^2fx + 3acdfx^2 + ad^2fx^3 + 3bc^2\log(\tanh(e + fx)))}{6f^3}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)^2*(a + b*Coth[e + f*x]),x]

[Out] (2*f^2*(3*a*c^2*f*x + 3*a*c*d*f*x^2 - 3*b*c*d*f*x^2 + a*d^2*f*x^3 - b*d^2*f*x^3 + 3*b*d*x*(2*c + d*x)*Log[1 - E^(2*(e + f*x))] + 3*b*c^2*Log[Cosh[e + f*x]] + 3*b*c^2*Log[Tanh[e + f*x]]) + 6*b*d*f*(c + d*x)*PolyLog[2, E^(2*(e + f*x))] - 3*b*d^2*PolyLog[3, E^(2*(e + f*x))])/(6*f^3)

Maple [B] time = 0.055, size = 447, normalized size = 4.4

$$\frac{bd^2e^2 \ln(e^{fx+e} - 1)}{f^3} + 2 \frac{bd^2 \text{polylog}(2, e^{fx+e}) x}{f^2} + \frac{bd^2 \ln(e^{fx+e} + 1) x^2}{f} - 2 \frac{bd^2e^2 \ln(e^{fx+e})}{f^3} + 2 \frac{cbd \text{polylog}(2, e^{fx+e})}{f^2} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^2*(a+b*coth(f*x+e)),x)

[Out] $b/f^3*d^2*e^2*\ln(\exp(f*x+e)-1)+2*b/f^2*d^2*\text{polylog}(2,\exp(f*x+e))*x+b/f*d^2*\ln(\exp(f*x+e)+1)*x^2-2*b/f^3*d^2*e^2*\ln(\exp(f*x+e))+2*b/f^2*c*d*\text{polylog}(2,\exp(f*x+e))+2*b/f^2*d^2*\text{polylog}(2,-\exp(f*x+e))*x-b/f^3*d^2*e^2*\ln(1-\exp(f*x+e))+b/f*d^2*\ln(1-\exp(f*x+e))*x^2+2*b/f^2*c*d*\text{polylog}(2,-\exp(f*x+e))-2*b/f^3*d^2*\text{polylog}(3,\exp(f*x+e))-2*b/f^3*d^2*\text{polylog}(3,-\exp(f*x+e))+b/f*c^2*\ln(\exp(f*x+e)+1)-2*b/f*c^2*\ln(\exp(f*x+e))-2*b/f^2*c*d*e^2+2*b/f^2*d^2*e^2*x+b/f*c^2*\ln(\exp(f*x+e)-1)-b*c*d*x^2+a*c*d*x^2+1/3*a*d^2*x^3-1/3*b*d^2*x^3+2*b/f^2*c*d*\ln(1-\exp(f*x+e))*e+4*b/f^2*c*d*e*\ln(\exp(f*x+e))-2*b/f^2*c*d*e*\ln(\exp(f*x+e)-1)+2*b/f*c*d*\ln(1-\exp(f*x+e))*x-4*b/f*c*d*e*x+2*b/f*c*d*\ln(\exp(f*x+e)+1))*x+4/3*b/f^3*d^2*e^3+c^2*a*x+b*c^2*x$

Maxima [B] time = 1.38171, size = 324, normalized size = 3.21

$$\frac{1}{3}ad^2x^3 + \frac{1}{3}bd^2x^3 + acdx^2 + bcdx^2 + ac^2x + \frac{bc^2 \log(\sinh(fx + e))}{f} + \frac{2(fx \log(e^{(fx+e)} + 1) + \text{Li}_2(-e^{(fx+e)}))bcd}{f^2} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2*(a+b*coth(f*x+e)),x, algorithm="maxima")

[Out] $1/3*a*d^2*x^3 + 1/3*b*d^2*x^3 + a*c*d*x^2 + b*c*d*x^2 + a*c^2*x + b*c^2*\log(\sinh(f*x + e))/f + 2*(f*x*\log(e^{(f*x + e)} + 1) + \text{dilog}(-e^{(f*x + e)}))*b*c*d/f^2 + 2*(f*x*\log(-e^{(f*x + e)} + 1) + \text{dilog}(e^{(f*x + e)}))*b*c*d/f^2 + (f^2*x^2*\log(e^{(f*x + e)} + 1) + 2*f*x*\text{dilog}(-e^{(f*x + e)})) - 2*\text{polylog}(3, -e^{(f*x + e)}))*b*d^2/f^3 + (f^2*x^2*\log(-e^{(f*x + e)} + 1) + 2*f*x*\text{dilog}(e^{(f*x + e)})) - 2*\text{polylog}(3, e^{(f*x + e)}))*b*d^2/f^3 - 2/3*(b*d^2*f^3*x^3 + 3*b*c*d*f^3*x^2)/f^3$

Fricas [C] time = 2.24311, size = 782, normalized size = 7.74

$$(a - b)d^2f^3x^3 + 3(a - b)cdf^3x^2 + 3(a - b)c^2f^3x - 6bd^2\text{polylog}(3, \cosh(fx + e) + \sinh(fx + e)) - 6bd^2\text{polylog}(3, \dots)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2*(a+b*coth(f*x+e)),x, algorithm="fricas")

```
[Out] 1/3*((a - b)*d^2*f^3*x^3 + 3*(a - b)*c*d*f^3*x^2 + 3*(a - b)*c^2*f^3*x - 6*
b*d^2*polylog(3, cosh(f*x + e) + sinh(f*x + e)) - 6*b*d^2*polylog(3, -cosh(
f*x + e) - sinh(f*x + e)) + 6*(b*d^2*f*x + b*c*d*f)*dilog(cosh(f*x + e) + s
inh(f*x + e)) + 6*(b*d^2*f*x + b*c*d*f)*dilog(-cosh(f*x + e) - sinh(f*x + e
)) + 3*(b*d^2*f^2*x^2 + 2*b*c*d*f^2*x + b*c^2*f^2)*log(cosh(f*x + e) + sinh
(f*x + e) + 1) + 3*(b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*log(cosh(f*x + e)
+ sinh(f*x + e) - 1) + 3*(b*d^2*f^2*x^2 + 2*b*c*d*f^2*x - b*d^2*e^2 + 2*b*c
*d*e*f)*log(-cosh(f*x + e) - sinh(f*x + e) + 1))/f^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b \coth(e + fx))(c + dx)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*(a+b*coth(f*x+e)),x)
```

```
[Out] Integral((a + b*coth(e + f*x))*(c + d*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^2 (b \coth(fx + e) + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+b*coth(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*(b*coth(f*x + e) + a), x)
```

3.39 $\int (c + dx)(a + b \coth(e + fx)) dx$

Optimal. Leaf size=75

$$\frac{bd \operatorname{PolyLog}\left(2, e^{2(e+fx)}\right)}{2f^2} + \frac{a(c+dx)^2}{2d} + \frac{b(c+dx) \log\left(1 - e^{2(e+fx)}\right)}{f} - \frac{b(c+dx)^2}{2d}$$

[Out] $(a*(c + d*x)^2)/(2*d) - (b*(c + d*x)^2)/(2*d) + (b*(c + d*x)*\operatorname{Log}[1 - E^{2*(e + f*x)}])/f + (b*d*\operatorname{PolyLog}[2, E^{2*(e + f*x)}])/(2*f^2)$

Rubi [A] time = 0.126318, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {3722, 3716, 2190, 2279, 2391}

$$\frac{bd \operatorname{PolyLog}\left(2, e^{2(e+fx)}\right)}{2f^2} + \frac{a(c+dx)^2}{2d} + \frac{b(c+dx) \log\left(1 - e^{2(e+fx)}\right)}{f} - \frac{b(c+dx)^2}{2d}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(c + d*x)*(a + b*\operatorname{Coth}[e + f*x]), x]$

[Out] $(a*(c + d*x)^2)/(2*d) - (b*(c + d*x)^2)/(2*d) + (b*(c + d*x)*\operatorname{Log}[1 - E^{2*(e + f*x)}])/f + (b*d*\operatorname{PolyLog}[2, E^{2*(e + f*x)}])/(2*f^2)$

Rule 3722

$\operatorname{Int}[(c + d*x)^m * \tan(e + f*x)^n, x]$:> $\operatorname{Int}[\operatorname{ExpandIntegrand}[(c + d*x)^m, (a + b*\tan[e + f*x])^n, x], x] /;$ $\operatorname{FreeQ}\{a, b, c, d, e, f, m\}, x$ && $\operatorname{IGtQ}[m, 0]$ && $\operatorname{IGtQ}[n, 0]$

Rule 3716

$\operatorname{Int}[(c + d*x)^m * \tan(e + \pi*k + (Complex[0, fz])*f*x), x]$:> $-\operatorname{Simp}[(I*(c + d*x)^{m+1})/(d*(m+1)), x] + \operatorname{Dist}[2*I, \operatorname{Int}[(c + d*x)^m * E^{2*(-I*e + f*fz*x)}]/(E^{2*I*k*\pi}*(1 + E^{2*(-I*e + f*fz*x)})/E^{2*I*k*\pi}), x, x] /;$ $\operatorname{FreeQ}\{c, d, e, f, fz\}, x$ && $\operatorname{IntegerQ}[4*k]$ && $\operatorname{IGtQ}[m, 0]$

Rule 2190

$\operatorname{Int}[(c + d*x)^m * (F(g + e + f*x))^n, x]$:> $\operatorname{Simp}[(a + b*(F(g + e + f*x))^n), x]$

```

[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a]]/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

```

Rule 2279

```

Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

```

Rule 2391

```

Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]

```

Rubi steps

$$\begin{aligned}
\int (c + dx)(a + b \coth(e + fx)) dx &= \int (a(c + dx) + b(c + dx) \coth(e + fx)) dx \\
&= \frac{a(c + dx)^2}{2d} + b \int (c + dx) \coth(e + fx) dx \\
&= \frac{a(c + dx)^2}{2d} - \frac{b(c + dx)^2}{2d} - (2b) \int \frac{e^{2(e+fx)}(c + dx)}{1 - e^{2(e+fx)}} dx \\
&= \frac{a(c + dx)^2}{2d} - \frac{b(c + dx)^2}{2d} + \frac{b(c + dx) \log(1 - e^{2(e+fx)})}{f} - \frac{(bd) \int \log(1 - e^{2(e+fx)}) dx}{f} \\
&= \frac{a(c + dx)^2}{2d} - \frac{b(c + dx)^2}{2d} + \frac{b(c + dx) \log(1 - e^{2(e+fx)})}{f} - \frac{(bd) \text{Subst}\left(\int \frac{\log(1-x)}{x} dx, x, x\right)}{2f^2} \\
&= \frac{a(c + dx)^2}{2d} - \frac{b(c + dx)^2}{2d} + \frac{b(c + dx) \log(1 - e^{2(e+fx)})}{f} + \frac{bd \text{Li}_2(e^{2(e+fx)})}{2f^2}
\end{aligned}$$

Mathematica [A] time = 0.056351, size = 87, normalized size = 1.16

$$\frac{bd \text{PolyLog}\left(2, e^{2e+2fx}\right)}{2f^2} + acx + \frac{1}{2}adx^2 + \frac{bc(\log(\tanh(e + fx)) + \log(\cosh(e + fx)))}{f} + \frac{bdx \log(1 - e^{2e+2fx})}{f} - \frac{1}{2}bdx^2$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*(a + b*Coth[e + f*x]),x]
```

[Out] $a*c*x + (a*d*x^2)/2 - (b*d*x^2)/2 + (b*d*x*\text{Log}[1 - E^{(2*e + 2*f*x)}])/f + (b*c*(\text{Log}[\text{Cosh}[e + f*x]] + \text{Log}[\text{Tanh}[e + f*x]]))/f + (b*d*\text{PolyLog}[2, E^{(2*e + 2*f*x)}])/(2*f^2)$

Maple [B] time = 0.043, size = 201, normalized size = 2.7

$$\frac{adx^2}{2} - \frac{bdx^2}{2} + acx + bcx - 2 \frac{cb \ln(e^{fx+e})}{f} + \frac{cb \ln(e^{fx+e} + 1)}{f} + \frac{cb \ln(e^{fx+e} - 1)}{f} - 2 \frac{bdex}{f} - \frac{bde^2}{f^2} + \frac{bd \ln(e^{fx+e} + 1)x}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x+c)*(a+b*coth(f*x+e)),x)`

[Out] $1/2*a*d*x^2 - 1/2*b*d*x^2 + a*c*x + b*c*x - 2*b/f*c*\ln(\exp(f*x+e)) + b/f*c*\ln(\exp(f*x+e)+1) + b/f*c*\ln(\exp(f*x+e)-1) - 2*b/f*d*e*x - b/f^2*d*e^2 + b/f*d*\ln(\exp(f*x+e)+1)*x + b/f^2*d*\text{polylog}(2, -\exp(f*x+e)) + b/f*d*\ln(1-\exp(f*x+e))*x + b/f^2*d*\ln(1-\exp(f*x+e))*e + b/f^2*d*\text{polylog}(2, \exp(f*x+e)) + 2*b/f^2*d*e*\ln(\exp(f*x+e)) - b/f^2*d*e*\ln(\exp(f*x+e)-1)$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2} adx^2 + \frac{1}{2} \left(x^2 - 2 \int \frac{x}{e^{(fx+e)} + 1} dx + 2 \int \frac{x}{e^{(fx+e)} - 1} dx \right) bd + acx + \frac{bc \log(\sinh(fx + e))}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)*(a+b*coth(f*x+e)),x, algorithm="maxima")`

[Out] $1/2*a*d*x^2 + 1/2*(x^2 - 2*\text{integrate}(x/(e^{(f*x + e)} + 1), x) + 2*\text{integrate}(x/(e^{(f*x + e)} - 1), x))*b*d + a*c*x + b*c*\log(\sinh(f*x + e))/f$

Fricas [B] time = 2.119, size = 435, normalized size = 5.8

$$(a - b)df^2x^2 + 2(a - b)cf^2x + 2bd\text{Li}_2(\cosh(fx + e) + \sinh(fx + e)) + 2bd\text{Li}_2(-\cosh(fx + e) - \sinh(fx + e)) + 2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+b*coth(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/2*((a - b)*d*f^2*x^2 + 2*(a - b)*c*f^2*x + 2*b*d*dilog(cosh(f*x + e) + sinh(f*x + e)) + 2*b*d*dilog(-cosh(f*x + e) - sinh(f*x + e)) + 2*(b*d*f*x + b*c*f)*log(cosh(f*x + e) + sinh(f*x + e) + 1) - 2*(b*d*e - b*c*f)*log(cosh(f*x + e) + sinh(f*x + e) - 1) + 2*(b*d*f*x + b*d*e)*log(-cosh(f*x + e) - sinh(f*x + e) + 1))/f^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b \coth(e + fx))(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+b*coth(f*x+e)),x)
```

```
[Out] Integral((a + b*coth(e + f*x))*(c + d*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)(b \coth(fx + e) + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+b*coth(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*(b*coth(f*x + e) + a), x)
```

$$3.40 \quad \int \frac{a+b \coth(e+fx)}{c+dx} dx$$

Optimal. Leaf size=20

$$\text{Unintegrable}\left(\frac{a+b \coth(e+fx)}{c+dx}, x\right)$$

[Out] Unintegrable[(a + b*Coth[e + f*x])/(c + d*x), x]

Rubi [A] time = 0.0293813, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{a+b \coth(e+fx)}{c+dx} dx$$

Verification is Not applicable to the result.

[In] Int[(a + b*Coth[e + f*x])/(c + d*x), x]

[Out] Defer[Int] [(a + b*Coth[e + f*x])/(c + d*x), x]

Rubi steps

$$\int \frac{a+b \coth(e+fx)}{c+dx} dx = \int \frac{a+b \coth(e+fx)}{c+dx} dx$$

Mathematica [A] time = 4.99383, size = 0, normalized size = 0.

$$\int \frac{a+b \coth(e+fx)}{c+dx} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b*Coth[e + f*x])/(c + d*x), x]

[Out] Integrate[(a + b*Coth[e + f*x])/(c + d*x), x]

Maple [A] time = 0.14, size = 0, normalized size = 0.

$$\int \frac{a + b \coth(fx + e)}{dx + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*coth(f*x+e))/(d*x+c), x)

[Out] int((a+b*coth(f*x+e))/(d*x+c), x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$b \left(\frac{\log(dx + c)}{d} - \int \frac{1}{dx + (dx e^e + ce^e)e^{(fx)} + c} dx + \int -\frac{1}{dx - (dx e^e + ce^e)e^{(fx)} + c} dx \right) + \frac{a \log(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*coth(f*x+e))/(d*x+c), x, algorithm="maxima")

[Out] b*(log(d*x + c)/d - integrate(1/(d*x + (d*x*e^e + c*e^e)*e^(f*x) + c), x) + integrate(-1/(d*x - (d*x*e^e + c*e^e)*e^(f*x) + c), x)) + a*log(d*x + c)/d

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{b \coth(fx + e) + a}{dx + c}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*coth(f*x+e))/(d*x+c), x, algorithm="fricas")

[Out] integral((b*coth(f*x + e) + a)/(d*x + c), x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b \coth(e + fx)}{c + dx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*coth(f*x+e))/(d*x+c),x)

[Out] Integral((a + b*coth(e + f*x))/(c + d*x), x)

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \coth(fx + e) + a}{dx + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*coth(f*x+e))/(d*x+c),x, algorithm="giac")

[Out] integrate((b*coth(f*x + e) + a)/(d*x + c), x)

$$3.41 \quad \int \frac{a+b \coth(e+fx)}{(c+dx)^2} dx$$

Optimal. Leaf size=20

$$\text{Unintegrable}\left(\frac{a+b \coth(e+fx)}{(c+dx)^2}, x\right)$$

[Out] Unintegrable[(a + b*Coth[e + f*x])/(c + d*x)^2, x]

Rubi [A] time = 0.0288114, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{a+b \coth(e+fx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

[In] Int[(a + b*Coth[e + f*x])/(c + d*x)^2,x]

[Out] Defer[Int] [(a + b*Coth[e + f*x])/(c + d*x)^2, x]

Rubi steps

$$\int \frac{a+b \coth(e+fx)}{(c+dx)^2} dx = \int \frac{a+b \coth(e+fx)}{(c+dx)^2} dx$$

Mathematica [A] time = 20.337, size = 0, normalized size = 0.

$$\int \frac{a+b \coth(e+fx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b*Coth[e + f*x])/(c + d*x)^2,x]

[Out] Integrate[(a + b*Coth[e + f*x])/(c + d*x)^2, x]

Maple [A] time = 0.134, size = 0, normalized size = 0.

$$\int \frac{a + b \coth(fx + e)}{(dx + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*coth(f*x+e))/(d*x+c)^2,x)

[Out] int((a+b*coth(f*x+e))/(d*x+c)^2,x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-b \left(\frac{1}{d^2x + cd} + \int \frac{1}{d^2x^2 + 2cdx + c^2 + (d^2x^2e^e + 2cdxe^e + c^2e^e)e^{(fx)}} dx - \int -\frac{1}{d^2x^2 + 2cdx + c^2 - (d^2x^2e^e + 2cdxe^e + c^2e^e)e^{(fx)}} dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*coth(f*x+e))/(d*x+c)^2,x, algorithm="maxima")

[Out] -b*(1/(d^2*x + c*d) + integrate(1/(d^2*x^2 + 2*c*d*x + c^2 + (d^2*x^2*e^e + 2*c*d*x*e^e + c^2*e^e)*e^(f*x)), x) - integrate(-1/(d^2*x^2 + 2*c*d*x + c^2 - (d^2*x^2*e^e + 2*c*d*x*e^e + c^2*e^e)*e^(f*x)), x)) - a/(d^2*x + c*d)

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{b \coth(fx + e) + a}{d^2x^2 + 2cdx + c^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*coth(f*x+e))/(d*x+c)^2,x, algorithm="fricas")

[Out] integral((b*coth(f*x + e) + a)/(d^2*x^2 + 2*c*d*x + c^2), x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b \coth(e + fx)}{(c + dx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*coth(f*x+e))/(d*x+c)**2,x)

[Out] Integral((a + b*coth(e + f*x))/(c + d*x)**2, x)

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \coth(fx + e) + a}{(dx + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*coth(f*x+e))/(d*x+c)^2,x, algorithm="giac")

[Out] integrate((b*coth(f*x + e) + a)/(d*x + c)^2, x)

3.42 $\int (c + dx)^3 (a + b \coth(e + fx))^2 dx$

Optimal. Leaf size=271

$$-\frac{3abd^2(c+dx)\text{PolyLog}\left(3, e^{2(e+fx)}\right)}{f^3} + \frac{3abd(c+dx)^2\text{PolyLog}\left(2, e^{2(e+fx)}\right)}{f^2} + \frac{3abd^3\text{PolyLog}\left(4, e^{2(e+fx)}\right)}{2f^4} + \frac{3b^2d^2(c+dx)^3}{f^3}$$

```
[Out] -((b^2*(c + d*x)^3)/f) + (a^2*(c + d*x)^4)/(4*d) - (a*b*(c + d*x)^4)/(2*d)
+ (b^2*(c + d*x)^4)/(4*d) - (b^2*(c + d*x)^3*Coth[e + f*x])/f + (3*b^2*d*(c
+ d*x)^2*Log[1 - E^(2*(e + f*x))])/f^2 + (2*a*b*(c + d*x)^3*Log[1 - E^(2*(
e + f*x))])/f + (3*b^2*d^2*(c + d*x)*PolyLog[2, E^(2*(e + f*x))])/f^3 + (3*
a*b*d*(c + d*x)^2*PolyLog[2, E^(2*(e + f*x))])/f^2 - (3*b^2*d^3*PolyLog[3,
E^(2*(e + f*x))])/(2*f^4) - (3*a*b*d^2*(c + d*x)*PolyLog[3, E^(2*(e + f*x))
])/f^3 + (3*a*b*d^3*PolyLog[4, E^(2*(e + f*x))])/(2*f^4)
```

Rubi [A] time = 0.546393, antiderivative size = 271, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.45$, Rules used = {3722, 3716, 2190, 2531, 6609, 2282, 6589, 3720, 32}

$$-\frac{3abd^2(c+dx)\text{PolyLog}\left(3, e^{2(e+fx)}\right)}{f^3} + \frac{3abd(c+dx)^2\text{PolyLog}\left(2, e^{2(e+fx)}\right)}{f^2} + \frac{3abd^3\text{PolyLog}\left(4, e^{2(e+fx)}\right)}{2f^4} + \frac{3b^2d^2(c+dx)^3}{f^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*(a + b*Coth[e + f*x])^2,x]
```

```
[Out] -((b^2*(c + d*x)^3)/f) + (a^2*(c + d*x)^4)/(4*d) - (a*b*(c + d*x)^4)/(2*d)
+ (b^2*(c + d*x)^4)/(4*d) - (b^2*(c + d*x)^3*Coth[e + f*x])/f + (3*b^2*d*(c
+ d*x)^2*Log[1 - E^(2*(e + f*x))])/f^2 + (2*a*b*(c + d*x)^3*Log[1 - E^(2*(
e + f*x))])/f + (3*b^2*d^2*(c + d*x)*PolyLog[2, E^(2*(e + f*x))])/f^3 + (3*
a*b*d*(c + d*x)^2*PolyLog[2, E^(2*(e + f*x))])/f^2 - (3*b^2*d^3*PolyLog[3,
E^(2*(e + f*x))])/(2*f^4) - (3*a*b*d^2*(c + d*x)*PolyLog[3, E^(2*(e + f*x))
])/f^3 + (3*a*b*d^3*PolyLog[4, E^(2*(e + f*x))])/(2*f^4)
```

Rule 3722

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Tan[e + f*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)
*(x_)], x_Symbol] := -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-I*
e) + f*fz*x))/E^(2*I*k*Pi))], x], x] /; FreeQ[{c, d, e, f, fz}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_.) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a]]/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.)
*(x_))))^(p_.)], x_Symbol] := Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]]/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(
m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol]
:> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Dist[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x], x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int (c + dx)^3 (a + b \coth(e + fx))^2 dx &= \int (a^2(c + dx)^3 + 2ab(c + dx)^3 \coth(e + fx) + b^2(c + dx)^3 \coth^2(e + fx)) dx \\
&= \frac{a^2(c + dx)^4}{4d} + (2ab) \int (c + dx)^3 \coth(e + fx) dx + b^2 \int (c + dx)^3 \coth^2(e + fx) dx \\
&= \frac{a^2(c + dx)^4}{4d} - \frac{ab(c + dx)^4}{2d} - \frac{b^2(c + dx)^3 \coth(e + fx)}{f} - (4ab) \int \frac{e^{2(e+fx)}(c + dx)}{1 - e^{2(e+fx)}} dx \\
&= -\frac{b^2(c + dx)^3}{f} + \frac{a^2(c + dx)^4}{4d} - \frac{ab(c + dx)^4}{2d} + \frac{b^2(c + dx)^4}{4d} - \frac{b^2(c + dx)^3 \coth(e + fx)}{f} \\
&= -\frac{b^2(c + dx)^3}{f} + \frac{a^2(c + dx)^4}{4d} - \frac{ab(c + dx)^4}{2d} + \frac{b^2(c + dx)^4}{4d} - \frac{b^2(c + dx)^3 \coth(e + fx)}{f} \\
&= -\frac{b^2(c + dx)^3}{f} + \frac{a^2(c + dx)^4}{4d} - \frac{ab(c + dx)^4}{2d} + \frac{b^2(c + dx)^4}{4d} - \frac{b^2(c + dx)^3 \coth(e + fx)}{f} \\
&= -\frac{b^2(c + dx)^3}{f} + \frac{a^2(c + dx)^4}{4d} - \frac{ab(c + dx)^4}{2d} + \frac{b^2(c + dx)^4}{4d} - \frac{b^2(c + dx)^3 \coth(e + fx)}{f} \\
&= -\frac{b^2(c + dx)^3}{f} + \frac{a^2(c + dx)^4}{4d} - \frac{ab(c + dx)^4}{2d} + \frac{b^2(c + dx)^4}{4d} - \frac{b^2(c + dx)^3 \coth(e + fx)}{f}
\end{aligned}$$

Mathematica [B] time = 9.88911, size = 857, normalized size = 3.16

$$-\frac{ab(\coth(e) - 1)(c + dx)^4}{2d} - \frac{b^2(\coth(e) - 1)(c + dx)^3}{f} - \frac{bc^2(3bd + 2acf)(fx - \log(-\cosh(e + fx) - \sinh(e + fx) + 1))}{f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*(a + b*Coth[e + f*x])^2,x]
```

```
[Out] -((b^2*(c + d*x)^3*(-1 + Coth[e]))/f) - (a*b*(c + d*x)^4*(-1 + Coth[e]))/(2
*d) - (b*c^2*(3*b*d + 2*a*c*f)*(f*x - Log[1 - Cosh[e + f*x] - Sinh[e + f*x]
]))/f^2 + (6*b*c*d*(b*d + a*c*f)*x*Log[1 + Cosh[e + f*x] - Sinh[e + f*x]])/
f^2 + (3*b*d^2*(b*d + 2*a*c*f)*x^2*Log[1 + Cosh[e + f*x] - Sinh[e + f*x]])/
f^2 + (2*a*b*d^3*x^3*Log[1 + Cosh[e + f*x] - Sinh[e + f*x]])/f + (6*b*c*d*(
b*d + a*c*f)*x*Log[1 - Cosh[e + f*x] + Sinh[e + f*x]])/f^2 + (3*b*d^2*(b*d
+ 2*a*c*f)*x^2*Log[1 - Cosh[e + f*x] + Sinh[e + f*x]])/f^2 + (2*a*b*d^3*x^3
*Log[1 - Cosh[e + f*x] + Sinh[e + f*x]])/f - (b*c^2*(3*b*d + 2*a*c*f)*(f*x
- Log[1 + Cosh[e + f*x] + Sinh[e + f*x]]))/f^2 - (6*b*c*d*(b*d + a*c*f)*Pol
yLog[2, Cosh[e + f*x] - Sinh[e + f*x]])/f^3 - (6*b*c*d*(b*d + a*c*f)*PolyLo
g[2, -Cosh[e + f*x] + Sinh[e + f*x]])/f^3 - (6*b*d^2*(b*d + 2*a*c*f)*(f*x*P
olyLog[2, Cosh[e + f*x] - Sinh[e + f*x]] + PolyLog[3, Cosh[e + f*x] - Sinh[
e + f*x]]))/f^4 - (6*b*d^2*(b*d + 2*a*c*f)*(f*x*PolyLog[2, -Cosh[e + f*x] +
Sinh[e + f*x]] + PolyLog[3, -Cosh[e + f*x] + Sinh[e + f*x]]))/f^4 - (6*a*b
*d^3*(f^2*x^2*PolyLog[2, Cosh[e + f*x] - Sinh[e + f*x]] + 2*(f*x*PolyLog[3,
Cosh[e + f*x] - Sinh[e + f*x]] + PolyLog[4, Cosh[e + f*x] - Sinh[e + f*x]
])))/f^4 - (6*a*b*d^3*(f^2*x^2*PolyLog[2, -Cosh[e + f*x] + Sinh[e + f*x]] +
2*(f*x*PolyLog[3, -Cosh[e + f*x] + Sinh[e + f*x]] + PolyLog[4, -Cosh[e + f*
x] + Sinh[e + f*x]])))/f^4 + (Csch[e]*Csch[e + f*x]*(-(a^2 + b^2)*f*x*(4*c
^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3)*Cosh[f*x]) + (a^2 + b^2)*f*x*(4*c
^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3)*Cosh[2*e + f*x] + 2*b*((4*b*(c + d*x)
^3 + a*f*x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3))*Sinh[f*x] + a*f*x*(
4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3)*Sinh[2*e + f*x]))/(8*f)
```

Maple [B] time = 0.168, size = 1393, normalized size = 5.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*(a+b*coth(f*x+e))^2,x)
```

```
[Out] -3*b/f^4*a*d^3*e^4+6*b^2/f^3*d^3*e^2*x-6*b^2/f*c*d^2*x^2-6*b^2/f^3*c*d^2*e^
2-2*a*b*c*d^2*x^3-3*a*b*c^2*d*x^2+4*b^2/f^4*d^3*e^3-2*b^2/f*d^3*x^3-6*b/f^2
*a*c^2*d*e^2+8*b/f^3*a*c*d^2*e^3-4*b/f^3*a*d^3*e^3*x-12*b^2/f^2*c*d^2*e*x+4
*b/f^4*a*d^3*e^3*ln(exp(f*x+e))+12*b^2/f^3*c*d^2*e*ln(exp(f*x+e))-2/f*b^2*(
d^3*x^3+3*c*d^2*x^2+3*c^2*d*x+c^3)/(exp(2*f*x+2*e)-1)+a^2*c*d^2*x^3+b^2*c*d
^2*x^3+6*b^2/f^3*d^3*polylog(2,exp(f*x+e))*x+3*b^2/f^2*d^3*ln(exp(f*x+e)+1)
*x^2+6*b^2/f^3*d^3*polylog(2,-exp(f*x+e))*x+3*b^2/f^4*d^3*e^2*ln(exp(f*x+e)
-1)+12*b/f^4*a*d^3*polylog(4,-exp(f*x+e))+3*b^2/f^2*c^2*d*ln(exp(f*x+e)-1)+
```



```

3*b^2/f^2*c^2*d*ln(exp(f*x+e)+1)+2*b/f*a*c^3*ln(exp(f*x+e)-1)+2*b/f*a*c^3*ln
n(exp(f*x+e)+1)+12*b/f^4*a*d^3*polylog(4,exp(f*x+e))+6*b^2/f^3*c*d^2*polylo
g(2,exp(f*x+e))+6*b^2/f^3*c*d^2*polylog(2,-exp(f*x+e))+3*b^2/f^2*d^3*ln(1-e
xp(f*x+e))*x^2-3*b^2/f^4*d^3*ln(1-exp(f*x+e))*e^2+12*b/f^2*a*c^2*d*e*ln(exp
(f*x+e))-12*b/f^3*a*c*d^2*e^2*ln(exp(f*x+e))+1/4*a^2*d^3*x^4+1/4*b^2*d^3*x^
4+c^3*a^2*x+b^2*c^3*x-12*b/f*a*c^2*d*e*x+12*b/f^2*a*c*d^2*e^2*x-1/2*a*b*d^3
*x^4+3/2*a^2*c^2*d*x^2+3/2*b^2*c^2*d*x^2+2*a*b*c^3*x-6*b^2/f^4*d^3*polylog(
3,exp(f*x+e))-6*b^2/f^4*d^3*polylog(3,-exp(f*x+e))+6*b^2/f^2*c*d^2*ln(exp(f
*x+e)+1)*x-2*b/f^4*a*d^3*e^3*ln(exp(f*x+e)-1)+6*b/f^2*a*c^2*d*polylog(2,-ex
p(f*x+e))+6*b/f^2*a*c^2*d*polylog(2,exp(f*x+e))+2*b/f*a*d^3*ln(1-exp(f*x+e)
)*x^3+2*b/f^4*a*d^3*ln(1-exp(f*x+e))*e^3+6*b/f^2*a*d^3*polylog(2,exp(f*x+e)
)*x^2-12*b/f^3*a*d^3*polylog(3,exp(f*x+e))*x+2*b/f*a*d^3*ln(exp(f*x+e)+1)*x
^3+6*b/f^2*a*d^3*polylog(2,-exp(f*x+e))*x^2-12*b/f^3*a*c*d^2*polylog(3,exp(
f*x+e))-12*b/f^3*a*c*d^2*polylog(3,-exp(f*x+e))-12*b/f^3*a*d^3*polylog(3,-e
xp(f*x+e))*x-6*b^2/f^3*c*d^2*e*ln(exp(f*x+e)-1)+6*b^2/f^2*c*d^2*ln(1-exp(f*
x+e))*x+6*b^2/f^3*c*d^2*ln(1-exp(f*x+e))*e-6*b/f^2*a*c^2*d*e*ln(exp(f*x+e)-
1)+6*b/f^2*ln(1-exp(f*x+e))*a*c^2*d*e+6*b/f^3*a*c*d^2*e^2*ln(exp(f*x+e)-1)+
6*b/f*a*c*d^2*ln(1-exp(f*x+e))*x^2-6*b/f^3*a*c*d^2*ln(1-exp(f*x+e))*e^2+12*
b/f^2*a*c*d^2*polylog(2,exp(f*x+e))*x+6*b/f*a*c*d^2*ln(exp(f*x+e)+1)*x^2+12
*b/f^2*a*c*d^2*polylog(2,-exp(f*x+e))*x+6*b/f*ln(exp(f*x+e)+1)*a*c^2*d*x+6*
b/f*ln(1-exp(f*x+e))*a*c^2*d*x-6*b^2/f^2*c^2*d*ln(exp(f*x+e))-4*b/f*a*c^3*ln
n(exp(f*x+e))-6*b^2/f^4*d^3*e^2*ln(exp(f*x+e))

```

Maxima [B] time = 1.49089, size = 1054, normalized size = 3.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*(a+b*coth(f*x+e))^2,x, algorithm="maxima")

```

[Out] 1/4*a^2*d^3*x^4 + a^2*c*d^2*x^3 + 3/2*a^2*c^2*d*x^2 + a^2*c^3*x - 6*b^2*c^2
*d*x/f + 2*a*b*c^3*log(sinh(f*x + e))/f + 3*b^2*c^2*d*log(e^(f*x + e) + 1)/
f^2 + 3*b^2*c^2*d*log(e^(f*x + e) - 1)/f^2 + 2*(f^3*x^3*log(e^(f*x + e) + 1
) + 3*f^2*x^2*dilog(-e^(f*x + e)) - 6*f*x*polylog(3, -e^(f*x + e)) + 6*poly
log(4, -e^(f*x + e)))*a*b*d^3/f^4 + 2*(f^3*x^3*log(-e^(f*x + e) + 1) + 3*f^
2*x^2*dilog(e^(f*x + e)) - 6*f*x*polylog(3, e^(f*x + e)) + 6*polylog(4, e^(
f*x + e)))*a*b*d^3/f^4 - 1/4*(8*b^2*c^3 + (2*a*b*d^3*f + b^2*d^3*f)*x^4 + 4
*(c^3*f + 6*c^2*d)*b^2*x + 4*(2*a*b*c*d^2*f + (c*d^2*f + 2*d^3)*b^2)*x^3 +
6*(2*a*b*c^2*d*f + (c^2*d*f + 4*c*d^2)*b^2)*x^2 - (4*b^2*c^3*f*x*e^(2*e) +
(2*a*b*d^3*f*e^(2*e) + b^2*d^3*f*e^(2*e))*x^4 + 4*(2*a*b*c*d^2*f*e^(2*e) +
b^2*c*d^2*f*e^(2*e))*x^3 + 6*(2*a*b*c^2*d*f*e^(2*e) + b^2*c^2*d*f*e^(2*e))*

```

$$\begin{aligned}
& x^2) * e^{(2*f*x)}) / (f * e^{(2*f*x + 2*e)} - f) + 6 * (a*b*c^2*d*f + b^2*c*d^2) * (f*x * \\
& \log(e^{(f*x + e)} + 1) + \operatorname{dilog}(e^{(f*x + e)})) / f^3 + 6 * (a*b*c^2*d*f + b^2*c*d^2) * (f*x * \log(-e^{(f*x + e)} + 1) + \\
& \operatorname{dilog}(e^{(f*x + e)})) / f^3 + 3 * (2*a*b*c*d^2*f + b^2*d^3) * (f^2*x^2 * \log(e^{(f*x + e)} + 1) + 2*f*x * \operatorname{dilog}(-e^{(f*x + e)})) - \\
& 2 * \operatorname{polylog}(3, -e^{(f*x + e)}) / f^4 + 3 * (2*a*b*c*d^2*f + b^2*d^3) * (f^2*x^2 * \log(-e^{(f*x + e)} + 1) + 2*f*x * \operatorname{dilog}(e^{(f*x + e)})) - \\
& 2 * \operatorname{polylog}(3, e^{(f*x + e)}) / f^4 - (a*b*d^3*f^4*x^4 + 2 * (2*a*b*c*d^2*f + b^2*d^3) * f^3*x^3 + 6 * (a*b*c^2*d*f^2 + b^2*c*d^2*f) * f^2*x^2) / f^4
\end{aligned}$$

Fricas [C] time = 2.83648, size = 7128, normalized size = 26.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*(a+b*coth(f*x+e))^2,x, algorithm="fricas")

[Out]
$$\begin{aligned}
& -1/4 * ((a^2 - 2*a*b + b^2) * d^3 * f^4 * x^4 + 4 * (a^2 - 2*a*b + b^2) * c * d^2 * f^4 * x^3 \\
& + 6 * (a^2 - 2*a*b + b^2) * c^2 * d * f^4 * x^2 + 4 * a * b * d^3 * e^4 + 4 * (a^2 - 2*a*b + b^2) * c^3 * f^4 * x - 8 * b^2 * d^3 * e^3 - 8 * (2*a*b*c^3 * e - b^2 * c^3) * f^3 + 24 * (a*b*c^2 * d * e^2 - b^2 * c^2 * d * e) * f^2 - ((a^2 - 2*a*b + b^2) * d^3 * f^4 * x^4 + 4 * a * b * d^3 * e^4 - 16 * a * b * c^3 * e * f^3 - 8 * b^2 * d^3 * e^3 - 4 * (2*b^2 * d^3 * f^3 - (a^2 - 2*a*b + b^2) * c * d^2 * f^4) * x^3 + 24 * (a*b*c^2 * d * e^2 - b^2 * c^2 * d * e) * f^2 - 6 * (4*b^2 * c * d^2 * f^3 - (a^2 - 2*a*b + b^2) * c^2 * d * f^4) * x^2 - 8 * (2*a*b*c * d^2 * e^3 - 3*b^2 * c * d^2 * e^2) * f - 4 * (6*b^2 * c^2 * d * f^3 - (a^2 - 2*a*b + b^2) * c^3 * f^4) * x) * \cosh(f*x + e)^2 - 2 * ((a^2 - 2*a*b + b^2) * d^3 * f^4 * x^4 + 4 * a * b * d^3 * e^4 - 16 * a * b * c^3 * e * f^3 - 8 * b^2 * d^3 * e^3 - 4 * (2*b^2 * d^3 * f^3 - (a^2 - 2*a*b + b^2) * c * d^2 * f^4) * x^3 + 24 * (a*b*c^2 * d * e^2 - b^2 * c^2 * d * e) * f^2 - 6 * (4*b^2 * c * d^2 * f^3 - (a^2 - 2*a*b + b^2) * c^2 * d * f^4) * x^2 - 8 * (2*a*b*c * d^2 * e^3 - 3*b^2 * c * d^2 * e^2) * f - 4 * (6*b^2 * c^2 * d * f^3 - (a^2 - 2*a*b + b^2) * c^3 * f^4) * x) * \cosh(f*x + e) * \sinh(f*x + e) - ((a^2 - 2*a*b + b^2) * d^3 * f^4 * x^4 + 4 * a * b * d^3 * e^4 - 16 * a * b * c^3 * e * f^3 - 8 * b^2 * d^3 * e^3 - 4 * (2*b^2 * d^3 * f^3 - (a^2 - 2*a*b + b^2) * c * d^2 * f^4) * x^3 + 24 * (a*b*c^2 * d * e^2 - b^2 * c^2 * d * e) * f^2 - 6 * (4*b^2 * c * d^2 * f^3 - (a^2 - 2*a*b + b^2) * c^2 * d * f^4) * x^2 - 8 * (2*a*b*c * d^2 * e^3 - 3*b^2 * c * d^2 * e^2) * f - 4 * (6*b^2 * c^2 * d * f^3 - (a^2 - 2*a*b + b^2) * c^3 * f^4) * x) * \sinh(f*x + e)^2 - 8 * (2*a*b*c * d^2 * e^3 - 3*b^2 * c * d^2 * e^2) * f + 24 * (a*b*d^3 * f^2 * x^2 + a*b*c^2 * d * f^2 + b^2 * c * d^2 * f - (a*b*d^3 * f^2 * x^2 + a*b*c^2 * d * f^2 + b^2 * c * d^2 * f + (2*a*b*c * d^2 * f^2 + b^2 * d^3 * f) * x) * \cosh(f*x + e)^2 - 2 * (a*b*d^3 * f^2 * x^2 + a*b*c^2 * d * f^2 + b^2 * c * d^2 * f + (2*a*b*c * d^2 * f^2 + b^2 * d^3 * f) * x) * \cosh(f*x + e) * \sinh(f*x + e) - (a*b*d^3 * f^2 * x^2 + a*b*c^2 * d * f^2 + b^2 * c * d^2 * f + (2*a*b*c * d^2 * f^2 + b^2 * d^3 * f) * x) * \sinh(f*x + e))^2 + 24 * (a*b*d^3 * f^2 * x^2 + a*b*c^2 * d * f^2 + b^2 * c * d^2 * f - (a*b*d^3 * f^2 * x^2 + a
\end{aligned}$$

$$\begin{aligned}
& *b^2*c^2*d*f^2 + b^2*c*d^2*f + (2*a*b*c*d^2*f^2 + b^2*d^3*f)*x)*\cosh(f*x + e) \\
& ^2 - 2*(a*b*d^3*f^2*x^2 + a*b*c^2*d*f^2 + b^2*c*d^2*f + (2*a*b*c*d^2*f^2 + \\
& b^2*d^3*f)*x)*\cosh(f*x + e)*\sinh(f*x + e) - (a*b*d^3*f^2*x^2 + a*b*c^2*d*f^2 \\
& ^2 + b^2*c*d^2*f + (2*a*b*c*d^2*f^2 + b^2*d^3*f)*x)*\sinh(f*x + e)^2 + (2*a*b \\
& *c*d^2*f^2 + b^2*d^3*f)*x)*\operatorname{dilog}(-\cosh(f*x + e) - \sinh(f*x + e)) + 4*(2*a*b \\
& *d^3*f^3*x^3 + 2*a*b*c^3*f^3 + 3*b^2*c^2*d*f^2 + 3*(2*a*b*c*d^2*f^3 + b^2*d \\
& ^3*f^2)*x^2 - (2*a*b*d^3*f^3*x^3 + 2*a*b*c^3*f^3 + 3*b^2*c^2*d*f^2 + 3*(2*a \\
& *b*c*d^2*f^3 + b^2*d^3*f^2)*x^2 + 6*(a*b*c^2*d*f^3 + b^2*c*d^2*f^2)*x)*\cosh \\
& (f*x + e)^2 - 2*(2*a*b*d^3*f^3*x^3 + 2*a*b*c^3*f^3 + 3*b^2*c^2*d*f^2 + 3*(2 \\
& *a*b*c*d^2*f^3 + b^2*d^3*f^2)*x^2 + 6*(a*b*c^2*d*f^3 + b^2*c*d^2*f^2)*x)*\co \\
& sh(f*x + e)*\sinh(f*x + e) - (2*a*b*d^3*f^3*x^3 + 2*a*b*c^3*f^3 + 3*b^2*c^2* \\
& d*f^2 + 3*(2*a*b*c*d^2*f^3 + b^2*d^3*f^2)*x^2 + 6*(a*b*c^2*d*f^3 + b^2*c*d^ \\
& ^2*f^2)*x)*\sinh(f*x + e)^2 + 6*(a*b*c^2*d*f^3 + b^2*c*d^2*f^2)*x)*\log(\cosh(f \\
& *x + e) + \sinh(f*x + e) + 1) - 4*(2*a*b*d^3*e^3 - 2*a*b*c^3*f^3 - 3*b^2*d^3 \\
& *e^2 + 3*(2*a*b*c^2*d*e - b^2*c^2*d)*f^2 - (2*a*b*d^3*e^3 - 2*a*b*c^3*f^3 - \\
& 3*b^2*d^3*e^2 + 3*(2*a*b*c^2*d*e - b^2*c^2*d)*f^2 - 6*(a*b*c*d^2*e^2 - b^2 \\
& *c*d^2*e)*f)*\cosh(f*x + e)^2 - 2*(2*a*b*d^3*e^3 - 2*a*b*c^3*f^3 - 3*b^2*d^3 \\
& *e^2 + 3*(2*a*b*c^2*d*e - b^2*c^2*d)*f^2 - 6*(a*b*c*d^2*e^2 - b^2*c*d^2*e)* \\
& f)*\cosh(f*x + e)*\sinh(f*x + e) - (2*a*b*d^3*e^3 - 2*a*b*c^3*f^3 - 3*b^2*d^3 \\
& *e^2 + 3*(2*a*b*c^2*d*e - b^2*c^2*d)*f^2 - 6*(a*b*c*d^2*e^2 - b^2*c*d^2*e)* \\
& f)*\sinh(f*x + e)^2 - 6*(a*b*c*d^2*e^2 - b^2*c*d^2*e)*f)*\log(\cosh(f*x + e) + \\
& \sinh(f*x + e) - 1) + 4*(2*a*b*d^3*f^3*x^3 + 2*a*b*d^3*e^3 + 6*a*b*c^2*d*e* \\
& f^2 - 3*b^2*d^3*e^2 + 3*(2*a*b*c*d^2*f^3 + b^2*d^3*f^2)*x^2 - (2*a*b*d^3*f^ \\
& ^3*x^3 + 2*a*b*d^3*e^3 + 6*a*b*c^2*d*e*f^2 - 3*b^2*d^3*e^2 + 3*(2*a*b*c*d^2* \\
& f^3 + b^2*d^3*f^2)*x^2 - 6*(a*b*c*d^2*e^2 - b^2*c*d^2*e)*f + 6*(a*b*c^2*d*f \\
& ^3 + b^2*c*d^2*f^2)*x)*\cosh(f*x + e)^2 - 2*(2*a*b*d^3*f^3*x^3 + 2*a*b*d^3*e \\
& ^3 + 6*a*b*c^2*d*e*f^2 - 3*b^2*d^3*e^2 + 3*(2*a*b*c*d^2*f^3 + b^2*d^3*f^2)* \\
& x^2 - 6*(a*b*c*d^2*e^2 - b^2*c*d^2*e)*f + 6*(a*b*c^2*d*f^3 + b^2*c*d^2*f^2) \\
& *x)*\cosh(f*x + e)*\sinh(f*x + e) - (2*a*b*d^3*f^3*x^3 + 2*a*b*d^3*e^3 + 6*a* \\
& b*c^2*d*e*f^2 - 3*b^2*d^3*e^2 + 3*(2*a*b*c*d^2*f^3 + b^2*d^3*f^2)*x^2 - 6*(\\
& a*b*c*d^2*e^2 - b^2*c*d^2*e)*f + 6*(a*b*c^2*d*f^3 + b^2*c*d^2*f^2)*x)*\sinh(\\
& f*x + e)^2 - 6*(a*b*c*d^2*e^2 - b^2*c*d^2*e)*f + 6*(a*b*c^2*d*f^3 + b^2*c*d \\
& ^2*f^2)*x)*\log(-\cosh(f*x + e) - \sinh(f*x + e) + 1) - 48*(a*b*d^3*\cosh(f*x + \\
& e)^2 + 2*a*b*d^3*\cosh(f*x + e)*\sinh(f*x + e) + a*b*d^3*\sinh(f*x + e)^2 - a \\
& *b*d^3)*\operatorname{polylog}(4, \cosh(f*x + e) + \sinh(f*x + e)) - 48*(a*b*d^3*\cosh(f*x + \\
& e)^2 + 2*a*b*d^3*\cosh(f*x + e)*\sinh(f*x + e) + a*b*d^3*\sinh(f*x + e)^2 - a \\
& *b*d^3)*\operatorname{polylog}(4, -\cosh(f*x + e) - \sinh(f*x + e)) - 24*(2*a*b*d^3*f*x + 2*a \\
& *b*c*d^2*f + b^2*d^3 - (2*a*b*d^3*f*x + 2*a*b*c*d^2*f + b^2*d^3)*\cosh(f*x + \\
& e)^2 - 2*(2*a*b*d^3*f*x + 2*a*b*c*d^2*f + b^2*d^3)*\cosh(f*x + e)*\sinh(f*x \\
& + e) - (2*a*b*d^3*f*x + 2*a*b*c*d^2*f + b^2*d^3)*\sinh(f*x + e)^2)*\operatorname{polylog}(3 \\
& , \cosh(f*x + e) + \sinh(f*x + e)) - 24*(2*a*b*d^3*f*x + 2*a*b*c*d^2*f + b^2* \\
& d^3 - (2*a*b*d^3*f*x + 2*a*b*c*d^2*f + b^2*d^3)*\cosh(f*x + e)^2 - 2*(2*a*b* \\
& d^3*f*x + 2*a*b*c*d^2*f + b^2*d^3)*\cosh(f*x + e)*\sinh(f*x + e) - (2*a*b*d^3 \\
& *f*x + 2*a*b*c*d^2*f + b^2*d^3)*\sinh(f*x + e)^2)*\operatorname{polylog}(3, -\cosh(f*x + e) \\
& - \sinh(f*x + e)))/(f^4*\cosh(f*x + e)^2 + 2*f^4*\cosh(f*x + e)*\sinh(f*x + e)
\end{aligned}$$

+ f⁴*sinh(f*x + e)² - f⁴)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b \coth(e + fx))^2 (c + dx)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**3*(a+b*coth(f*x+e))**2,x)

[Out] Integral((a + b*coth(e + f*x))**2*(c + d*x)**3, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^3 (b \coth(fx + e) + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*(a+b*coth(f*x+e))^2,x, algorithm="giac")

[Out] integrate((d*x + c)^3*(b*coth(f*x + e) + a)^2, x)

3.43 $\int (c + dx)^2 (a + b \coth(e + fx))^2 dx$

Optimal. Leaf size=209

$$\frac{2abd(c + dx)\text{PolyLog}\left(2, e^{2(e+fx)}\right)}{f^2} - \frac{abd^2\text{PolyLog}\left(3, e^{2(e+fx)}\right)}{f^3} + \frac{b^2d^2\text{PolyLog}\left(2, e^{2(e+fx)}\right)}{f^3} + \frac{a^2(c + dx)^3}{3d} + \frac{2ab(c + d$$

```
[Out] -((b^2*(c + d*x)^2)/f) + (a^2*(c + d*x)^3)/(3*d) - (2*a*b*(c + d*x)^3)/(3*d)
) + (b^2*(c + d*x)^3)/(3*d) - (b^2*(c + d*x)^2*Coth[e + f*x])/f + (2*b^2*d*
(c + d*x)*Log[1 - E^(2*(e + f*x))])/f^2 + (2*a*b*(c + d*x)^2*Log[1 - E^(2*(
e + f*x))])/f + (b^2*d^2*PolyLog[2, E^(2*(e + f*x))])/f^3 + (2*a*b*d*(c + d
*x)*PolyLog[2, E^(2*(e + f*x))])/f^2 - (a*b*d^2*PolyLog[3, E^(2*(e + f*x))
])/f^3
```

Rubi [A] time = 0.404291, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {3722, 3716, 2190, 2531, 2282, 6589, 3720, 2279, 2391, 32}

$$\frac{2abd(c + dx)\text{PolyLog}\left(2, e^{2(e+fx)}\right)}{f^2} - \frac{abd^2\text{PolyLog}\left(3, e^{2(e+fx)}\right)}{f^3} + \frac{b^2d^2\text{PolyLog}\left(2, e^{2(e+fx)}\right)}{f^3} + \frac{a^2(c + dx)^3}{3d} + \frac{2ab(c + d$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*(a + b*Coth[e + f*x])^2, x]
```

```
[Out] -((b^2*(c + d*x)^2)/f) + (a^2*(c + d*x)^3)/(3*d) - (2*a*b*(c + d*x)^3)/(3*d)
) + (b^2*(c + d*x)^3)/(3*d) - (b^2*(c + d*x)^2*Coth[e + f*x])/f + (2*b^2*d*
(c + d*x)*Log[1 - E^(2*(e + f*x))])/f^2 + (2*a*b*(c + d*x)^2*Log[1 - E^(2*(
e + f*x))])/f + (b^2*d^2*PolyLog[2, E^(2*(e + f*x))])/f^3 + (2*a*b*d*(c + d
*x)*PolyLog[2, E^(2*(e + f*x))])/f^2 - (a*b*d^2*PolyLog[3, E^(2*(e + f*x))
])/f^3
```

Rule 3722

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Tan[e + f*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
```

*I, Int[((c + d*x)^m*E^(2*(-I*e) + f*fz*x))/(E^(2*I*k*Pi)*(1 + E^(2*(-I*e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && IntegerQ[4*k] && IGtQ[m, 0]

Rule 2190

Int[(((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)*((c_) + (d_)*(x_))^(m_))/((a_) + (b_)*((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)), x_Symbol] :> Simp[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

Rule 2531

Int[Log[1 + (e_)*((F_)^((c_)*((a_) + (b_)*(x_))))^(n_)]*((f_) + (g_)*(x_))^(m_), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m - 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

Rule 2282

Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_) /; FreeQ[{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_)*((a_) + (b_)*x))* (F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

Rule 6589

Int[PolyLog[n_, (c_)*((a_) + (b_)*(x_))^(p_)]/((d_) + (e_)*(x_)), x_Symbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]

Rule 3720

Int[((c_) + (d_)*(x_))^(m_)*((b_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Dist[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x], x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]

Rule 2279

```
Int[Log[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_))))^(n_)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] :> -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 32

```
Int[((a_) + (b_)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
 \int (c + dx)^2 (a + b \coth(e + fx))^2 dx &= \int (a^2(c + dx)^2 + 2ab(c + dx)^2 \coth(e + fx) + b^2(c + dx)^2 \coth^2(e + fx)) dx \\
 &= \frac{a^2(c + dx)^3}{3d} + (2ab) \int (c + dx)^2 \coth(e + fx) dx + b^2 \int (c + dx)^2 \coth^2(e + fx) dx \\
 &= \frac{a^2(c + dx)^3}{3d} - \frac{2ab(c + dx)^3}{3d} - \frac{b^2(c + dx)^2 \coth(e + fx)}{f} - (4ab) \int \frac{e^{2(e+fx)}(c + dx)}{1 - e^{2(e+fx)}} dx \\
 &= -\frac{b^2(c + dx)^2}{f} + \frac{a^2(c + dx)^3}{3d} - \frac{2ab(c + dx)^3}{3d} + \frac{b^2(c + dx)^3}{3d} - \frac{b^2(c + dx)^2 \coth(e + fx)}{f} \\
 &= -\frac{b^2(c + dx)^2}{f} + \frac{a^2(c + dx)^3}{3d} - \frac{2ab(c + dx)^3}{3d} + \frac{b^2(c + dx)^3}{3d} - \frac{b^2(c + dx)^2 \coth(e + fx)}{f} \\
 &= -\frac{b^2(c + dx)^2}{f} + \frac{a^2(c + dx)^3}{3d} - \frac{2ab(c + dx)^3}{3d} + \frac{b^2(c + dx)^3}{3d} - \frac{b^2(c + dx)^2 \coth(e + fx)}{f} \\
 &= -\frac{b^2(c + dx)^2}{f} + \frac{a^2(c + dx)^3}{3d} - \frac{2ab(c + dx)^3}{3d} + \frac{b^2(c + dx)^3}{3d} - \frac{b^2(c + dx)^2 \coth(e + fx)}{f}
 \end{aligned}$$

Mathematica [B] time = 10.2125, size = 478, normalized size = 2.29

$$\frac{2}{3}b \left(-\frac{3d \operatorname{PolyLog}\left(2, -e^{-e-fx}\right)(2acf + bd)}{f^3} - \frac{3d \operatorname{PolyLog}\left(2, e^{-e-fx}\right)(2acf + bd)}{f^3} - \frac{6ad^2 \left(fx \operatorname{PolyLog}\left(2, -e^{-e-fx}\right) + \operatorname{PolyLog}\left(2, -e^{-e-fx}\right) \right)}{f^3} \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*(a + b*Coth[e + f*x])^2,x]
```

```
[Out] (2*b*((-6*c*(b*d + a*c*f)*x)/((-1 + E^(2*e))*f) - (3*d*(b*d + 2*a*c*f)*x^2)/((-1 + E^(2*e))*f) - (2*a*d^2*x^3)/(-1 + E^(2*e)) + (3*d*(b*d + 2*a*c*f)*x*Log[1 - E^(-e - f*x)]/f^2 + (3*a*d^2*x^2*Log[1 - E^(-e - f*x)]/f + (3*d*(b*d + 2*a*c*f)*x*Log[1 + E^(-e - f*x)]/f^2 + (3*a*d^2*x^2*Log[1 + E^(-e - f*x)]/f - (3*c*(b*d + a*c*f)*(f*x - Log[1 - E^(e + f*x)]))/f^2 - (3*c*(b*d + a*c*f)*(f*x - Log[1 + E^(e + f*x)]))/f^2 - (3*d*(b*d + 2*a*c*f)*PolyLog[2, -E^(-e - f*x)]/f^3 - (3*d*(b*d + 2*a*c*f)*PolyLog[2, E^(-e - f*x)]/f^3 - (6*a*d^2*(f*x*PolyLog[2, -E^(-e - f*x)] + PolyLog[3, -E^(-e - f*x)]))/f^3 - (6*a*d^2*(f*x*PolyLog[2, E^(-e - f*x)] + PolyLog[3, E^(-e - f*x)]))/f^3))/3 + (x*(3*c^2 + 3*c*d*x + d^2*x^2)*Csch[e]*(2*a*b*Cosh[e] + a^2*Sinh[e] + b^2*Sinh[e]))/3 + (Csch[e]*Csch[e + f*x]*(b^2*c^2*Sinh[f*x] + 2*b^2*c*d*x*Sinh[f*x] + b^2*d^2*x^2*Sinh[f*x]))/f
```

Maple [B] time = 0.088, size = 793, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*(a+b*coth(f*x+e))^2,x)
```

```
[Out] 1/3*a^2*d^2*x^3+1/3*b^2*d^2*x^3+c^2*a^2*x+c^2*b^2*x-2*b^2/f*d^2*x^2-2*b^2/f^3*d^2*e^2+2*b^2/f^3*d^2*polylog(2,exp(f*x+e))+2*b^2/f^3*d^2*polylog(2,-exp(f*x+e))+4*b/f^2*ln(1-exp(f*x+e))*a*c*d*e+4*b/f*ln(1-exp(f*x+e))*a*c*d*x+4*b/f*ln(exp(f*x+e)+1)*a*c*d*x-4*b/f^2*a*c*d*e*ln(exp(f*x+e)-1)+8*b/f^2*a*c*d*e*ln(exp(f*x+e))-8*b/f*a*c*d*e*x+8/3*b/f^3*a*d^2*e^3-4*b^2/f^2*d^2*e*x-4*b/f*a*c^2*ln(exp(f*x+e))-4*b^2/f^2*c*d*ln(exp(f*x+e))+4*b^2/f^3*d^2*e*ln(exp(f*x+e))-2/3*a*b*d^2*x^3+2*a*b*c^2*x-2*a*b*c*d*x^2+a^2*c*d*x^2+b^2*c*d*x^2+4*b/f^2*a*c*d*polylog(2,exp(f*x+e))+4*b/f^2*a*c*d*polylog(2,-exp(f*x+e))-2*b/f^3*a*d^2*e^2*ln(1-exp(f*x+e))+2*b/f*a*d^2*ln(1-exp(f*x+e))*x^2+4*b/f^2*a*d^2*polylog(2,exp(f*x+e))*x+2*b/f*a*d^2*ln(exp(f*x+e)+1)*x^2+2*b/f^3*a*d^2*e^2*ln(exp(f*x+e)-1)+4*b/f^2*a*d^2*polylog(2,-exp(f*x+e))*x-4*b/f^2*a*c*d*e^2+4*b/f^2*a*d^2*e^2*x-4*b/f^3*a*d^2*e^2*ln(exp(f*x+e))-2/f*b^2*(d^2*x^2+2*c*d*x+c^2)/(exp(2*f*x+2*e)-1)+2*b^2/f^2*c*d*ln(exp(f*x+e)-1)+2*b^2/f^2*c*d*ln(exp(f*x+e)+1)-4*b/f^3*a*d^2*polylog(3,exp(f*x+e))-4*b/f^3*a*d^2*polylog(3,-exp(f*x+e))+2*b/f*a*c^2*ln(exp(f*x+e)-1)+2*b/f*a*c^2*ln(exp(f*x+e)+1)-2*b^2/f^3*d^2*e*ln(exp(f*x+e)-1)+2*b^2/f^2*d^2*ln(1-exp(f*x+e))*x+2*b^2/f^3*d^2*ln(1-exp(f*x+e))*e+2*b^2/f^2*d^2*ln(exp(f*x+e)+1)*x
```

Maxima [B] time = 1.43785, size = 667, normalized size = 3.19

$$\frac{1}{3} a^2 d^2 x^3 + a^2 c d x^2 + a^2 c^2 x - \frac{4 b^2 c d x}{f} + \frac{2 a b c^2 \log(\sinh(fx + e))}{f} + \frac{2 b^2 c d \log(e^{(fx+e)} + 1)}{f^2} + \frac{2 b^2 c d \log(e^{(fx+e)} - 1)}{f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2*(a+b*coth(f*x+e))^2,x, algorithm="maxima")

[Out] $\frac{1}{3} a^2 d^2 x^3 + a^2 c d x^2 + a^2 c^2 x - \frac{4 b^2 c d x}{f} + \frac{2 a b c^2 \log(\sinh(fx + e))}{f} + \frac{2 b^2 c d \log(e^{(fx+e)} + 1)}{f^2} + \frac{2 b^2 c d \log(e^{(fx+e)} - 1)}{f^2} + 2*(f^2*x^2*\log(e^{(fx+e)} + 1) + 2*f*x*dilog(-e^{(fx+e)} - 1) - 2*polylog(3, -e^{(fx+e)})) * a*b*d^2/f^3 + 2*(f^2*x^2*\log(-e^{(fx+e)} + 1) + 2*f*x*dilog(e^{(fx+e)}) - 2*polylog(3, e^{(fx+e)})) * a*b*d^2/f^3 - 1/3*(6*b^2*c^2 + 3*(c^2*f + 4*c*d)*b^2*x + (2*a*b*d^2*f + b^2*d^2*f)*x^3 + 3*(2*a*b*c*d*f + (c*d*f + 2*d^2)*b^2)*x^2 - (3*b^2*c^2*f*x*e^{(2*e)} + (2*a*b*d^2*f*e^{(2*e)} + b^2*d^2*f*e^{(2*e)})*x^3 + 3*(2*a*b*c*d*f*e^{(2*e)} + b^2*c*d*f*e^{(2*e)})*x^2)*e^{(2*f*x)}/(f*e^{(2*f*x + 2*e)} - f) + 2*(2*a*b*c*d*f + b^2*d^2)*(f*x*\log(e^{(fx+e)} + 1) + dilog(-e^{(fx+e)}))/f^3 + 2*(2*a*b*c*d*f + b^2*d^2)*(f*x*\log(-e^{(fx+e)} + 1) + dilog(e^{(fx+e)}))/f^3 - 2/3*(2*a*b*d^2*f^3*x^3 + 3*(2*a*b*c*d*f + b^2*d^2)*f^2*x^2)/f^3$

Fricas [C] time = 2.66538, size = 4251, normalized size = 20.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2*(a+b*coth(f*x+e))^2,x, algorithm="fricas")

[Out] $-1/3*((a^2 - 2*a*b + b^2)*d^2*f^3*x^3 + 3*(a^2 - 2*a*b + b^2)*c*d*f^3*x^2 - 4*a*b*d^2*e^3 + 3*(a^2 - 2*a*b + b^2)*c^2*f^3*x + 6*b^2*d^2*e^2 - 6*(2*a*b*c^2*e - b^2*c^2)*f^2 - ((a^2 - 2*a*b + b^2)*d^2*f^3*x^3 - 4*a*b*d^2*e^3 - 12*a*b*c^2*e*f^2 + 6*b^2*d^2*e^2 - 3*(2*b^2*d^2*f^2 - (a^2 - 2*a*b + b^2)*c*d*f^3)*x^2 + 12*(a*b*c*d*e^2 - b^2*c*d*e)*f - 3*(4*b^2*c*d*f^2 - (a^2 - 2*a*b + b^2)*c^2*f^3)*x)*cosh(f*x + e)^2 - 2*((a^2 - 2*a*b + b^2)*d^2*f^3*x^3 - 4*a*b*d^2*e^3 - 12*a*b*c^2*e*f^2 + 6*b^2*d^2*e^2 - 3*(2*b^2*d^2*f^2 - (a^2 - 2*a*b + b^2)*c*d*f^3)*x^2 + 12*(a*b*c*d*e^2 - b^2*c*d*e)*f - 3*(4*b^2*c*d*f^2 - (a^2 - 2*a*b + b^2)*c^2*f^3)*x)*cosh(f*x + e)*sinh(f*x + e) - ((a$

```

^2 - 2*a*b + b^2)*d^2*f^3*x^3 - 4*a*b*d^2*e^3 - 12*a*b*c^2*e*f^2 + 6*b^2*d^
2*e^2 - 3*(2*b^2*d^2*f^2 - (a^2 - 2*a*b + b^2)*c*d*f^3)*x^2 + 12*(a*b*c*d*e
^2 - b^2*c*d*e)*f - 3*(4*b^2*c*d*f^2 - (a^2 - 2*a*b + b^2)*c^2*f^3)*x)*sinh
(f*x + e)^2 + 12*(a*b*c*d*e^2 - b^2*c*d*e)*f + 6*(2*a*b*d^2*f*x + 2*a*b*c*d
*f + b^2*d^2 - (2*a*b*d^2*f*x + 2*a*b*c*d*f + b^2*d^2)*cosh(f*x + e)^2 - 2*
(2*a*b*d^2*f*x + 2*a*b*c*d*f + b^2*d^2)*cosh(f*x + e)*sinh(f*x + e) - (2*a*
b*d^2*f*x + 2*a*b*c*d*f + b^2*d^2)*sinh(f*x + e)^2)*dilog(cosh(f*x + e) + s
inh(f*x + e)) + 6*(2*a*b*d^2*f*x + 2*a*b*c*d*f + b^2*d^2 - (2*a*b*d^2*f*x +
2*a*b*c*d*f + b^2*d^2)*cosh(f*x + e)^2 - 2*(2*a*b*d^2*f*x + 2*a*b*c*d*f +
b^2*d^2)*cosh(f*x + e)*sinh(f*x + e) - (2*a*b*d^2*f*x + 2*a*b*c*d*f + b^2*d
^2)*sinh(f*x + e)^2)*dilog(-cosh(f*x + e) - sinh(f*x + e)) + 6*(a*b*d^2*f^2
*x^2 + a*b*c^2*f^2 + b^2*c*d*f - (a*b*d^2*f^2*x^2 + a*b*c^2*f^2 + b^2*c*d*f
+ (2*a*b*c*d*f^2 + b^2*d^2*f)*x)*cosh(f*x + e)^2 - 2*(a*b*d^2*f^2*x^2 + a*
b*c^2*f^2 + b^2*c*d*f + (2*a*b*c*d*f^2 + b^2*d^2*f)*x)*cosh(f*x + e)*sinh(f
*x + e) - (a*b*d^2*f^2*x^2 + a*b*c^2*f^2 + b^2*c*d*f + (2*a*b*c*d*f^2 + b^2
*d^2*f)*x)*sinh(f*x + e)^2 + (2*a*b*c*d*f^2 + b^2*d^2*f)*x*log(cosh(f*x +
e) + sinh(f*x + e) + 1) + 6*(a*b*d^2*e^2 + a*b*c^2*f^2 - b^2*d^2*e - (a*b*d
^2*e^2 + a*b*c^2*f^2 - b^2*d^2*e - (2*a*b*c*d*e - b^2*c*d)*f)*cosh(f*x + e)
^2 - 2*(a*b*d^2*e^2 + a*b*c^2*f^2 - b^2*d^2*e - (2*a*b*c*d*e - b^2*c*d)*f)*
cosh(f*x + e)*sinh(f*x + e) - (a*b*d^2*e^2 + a*b*c^2*f^2 - b^2*d^2*e - (2*a
*b*c*d*e - b^2*c*d)*f)*sinh(f*x + e)^2 - (2*a*b*c*d*e - b^2*c*d)*f*log(cos
h(f*x + e) + sinh(f*x + e) - 1) + 6*(a*b*d^2*f^2*x^2 - a*b*d^2*e^2 + 2*a*b*
c*d*e*f + b^2*d^2*e - (a*b*d^2*f^2*x^2 - a*b*d^2*e^2 + 2*a*b*c*d*e*f + b^2*
d^2*e + (2*a*b*c*d*f^2 + b^2*d^2*f)*x)*cosh(f*x + e)^2 - 2*(a*b*d^2*f^2*x^2
- a*b*d^2*e^2 + 2*a*b*c*d*e*f + b^2*d^2*e + (2*a*b*c*d*f^2 + b^2*d^2*f)*x)
*cosh(f*x + e)*sinh(f*x + e) - (a*b*d^2*f^2*x^2 - a*b*d^2*e^2 + 2*a*b*c*d*
e*f + b^2*d^2*e + (2*a*b*c*d*f^2 + b^2*d^2*f)*x)*sinh(f*x + e)^2 + (2*a*b*c*
d*f^2 + b^2*d^2*f)*x*log(-cosh(f*x + e) - sinh(f*x + e) + 1) + 12*(a*b*d^2
*cosh(f*x + e)^2 + 2*a*b*d^2*cosh(f*x + e)*sinh(f*x + e) + a*b*d^2*sinh(f*x
+ e)^2 - a*b*d^2)*polylog(3, cosh(f*x + e) + sinh(f*x + e)) + 12*(a*b*d^2*
cosh(f*x + e)^2 + 2*a*b*d^2*cosh(f*x + e)*sinh(f*x + e) + a*b*d^2*sinh(f*x
+ e)^2 - a*b*d^2)*polylog(3, -cosh(f*x + e) - sinh(f*x + e)))/(f^3*cosh(f*x
+ e)^2 + 2*f^3*cosh(f*x + e)*sinh(f*x + e) + f^3*sinh(f*x + e)^2 - f^3)

```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b \coth(e + fx))^2 (c + dx)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**2*(a+b*coth(f*x+e))**2,x)

[Out] Integral((a + b*coth(e + f*x))**2*(c + d*x)**2, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^2 (b \coth(fx + e) + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2*(a+b*coth(f*x+e))^2,x, algorithm="giac")

[Out] integrate((d*x + c)^2*(b*coth(f*x + e) + a)^2, x)

3.44 $\int (c + dx)(a + b \coth(e + fx))^2 dx$

Optimal. Leaf size=127

$$\frac{abd \operatorname{PolyLog}\left(2, e^{2(e+fx)}\right)}{f^2} + \frac{a^2(c+dx)^2}{2d} + \frac{2ab(c+dx) \log\left(1 - e^{2(e+fx)}\right)}{f} - \frac{ab(c+dx)^2}{d} - \frac{b^2(c+dx) \coth(e+fx)}{f} + b^2cx$$

[Out] $b^2c*x + (b^2*d*x^2)/2 + (a^2*(c + d*x)^2)/(2*d) - (a*b*(c + d*x)^2)/d - (b^2*(c + d*x)*\operatorname{Coth}[e + f*x])/f + (2*a*b*(c + d*x)*\operatorname{Log}[1 - E^{2*(e + f*x)}])/f + (b^2*d*\operatorname{Log}[\operatorname{Sinh}[e + f*x]])/f^2 + (a*b*d*\operatorname{PolyLog}[2, E^{2*(e + f*x)}])/f^2$

Rubi [A] time = 0.184377, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {3722, 3716, 2190, 2279, 2391, 3720, 3475}

$$\frac{abd \operatorname{PolyLog}\left(2, e^{2(e+fx)}\right)}{f^2} + \frac{a^2(c+dx)^2}{2d} + \frac{2ab(c+dx) \log\left(1 - e^{2(e+fx)}\right)}{f} - \frac{ab(c+dx)^2}{d} - \frac{b^2(c+dx) \coth(e+fx)}{f} + b^2cx$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(c + d*x)*(a + b*\operatorname{Coth}[e + f*x])^2, x]$

[Out] $b^2c*x + (b^2*d*x^2)/2 + (a^2*(c + d*x)^2)/(2*d) - (a*b*(c + d*x)^2)/d - (b^2*(c + d*x)*\operatorname{Coth}[e + f*x])/f + (2*a*b*(c + d*x)*\operatorname{Log}[1 - E^{2*(e + f*x)}])/f + (b^2*d*\operatorname{Log}[\operatorname{Sinh}[e + f*x]])/f^2 + (a*b*d*\operatorname{PolyLog}[2, E^{2*(e + f*x)}])/f^2$

Rule 3722

$\operatorname{Int}[(c + d*x)^m * (a + b*\tan(e + f*x))^n, x]$
 $\operatorname{Int}[\operatorname{ExpandIntegrand}[(c + d*x)^m, (a + b*\tan[e + f*x])^n, x], x] /;$ $\operatorname{FreeQ}\{a, b, c, d, e, f, m, x\}$ && $\operatorname{IGtQ}[m, 0]$ && $\operatorname{IGtQ}[n, 0]$

Rule 3716

$\operatorname{Int}[(c + d*x)^m * \tan(e + \pi*k + (Complex[0, fz])*f*x), x]$
 $\operatorname{Int}[-\operatorname{Simp}[(I*(c + d*x)^{m+1})/(d*(m+1)), x] + \operatorname{Dist}[2*I, \operatorname{Int}[(c + d*x)^m * E^{2*(-I*e + f*fz*x)}]/(E^{2*I*k*\pi}*(1 + E^{2*(-I*e + f*fz*x)})/E^{2*I*k*\pi}), x], x] /;$ $\operatorname{FreeQ}\{c, d, e, f, fz, x\}$ && $\operatorname{IntegerQ}[4*k]$ && $\operatorname{IGtQ}[m, 0]$

Rule 2190

```
Int[(((F_)^((g_)*(e_) + (f_)*(x_)))^(n_))*((c_) + (d_)*(x_))^(m_)]/
((a_) + (b_)*((F_)^((g_)*(e_) + (f_)*(x_)))^(n_)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a]]/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_)))^(n_))], x_Symbol]
:= Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3720

```
Int[((c_) + (d_)*(x_))^(m_)*((b_)*tan[(e_) + (f_)*(x_)])^(n_), x_Symb
ol] := Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_) + (d_)*(x_)], x_Symbol] := -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int (c + dx)(a + b \coth(e + fx))^2 dx &= \int (a^2(c + dx) + 2ab(c + dx) \coth(e + fx) + b^2(c + dx) \coth^2(e + fx)) dx \\
&= \frac{a^2(c + dx)^2}{2d} + (2ab) \int (c + dx) \coth(e + fx) dx + b^2 \int (c + dx) \coth^2(e + fx) dx \\
&= \frac{a^2(c + dx)^2}{2d} - \frac{ab(c + dx)^2}{d} - \frac{b^2(c + dx) \coth(e + fx)}{f} - (4ab) \int \frac{e^{2(e+fx)}(c + dx)}{1 - e^{2(e+fx)}} dx \\
&= b^2cx + \frac{1}{2}b^2dx^2 + \frac{a^2(c + dx)^2}{2d} - \frac{ab(c + dx)^2}{d} - \frac{b^2(c + dx) \coth(e + fx)}{f} + \frac{2ab(c + dx)}{f} \\
&= b^2cx + \frac{1}{2}b^2dx^2 + \frac{a^2(c + dx)^2}{2d} - \frac{ab(c + dx)^2}{d} - \frac{b^2(c + dx) \coth(e + fx)}{f} + \frac{2ab(c + dx)}{f} \\
&= b^2cx + \frac{1}{2}b^2dx^2 + \frac{a^2(c + dx)^2}{2d} - \frac{ab(c + dx)^2}{d} - \frac{b^2(c + dx) \coth(e + fx)}{f} + \frac{2ab(c + dx)}{f}
\end{aligned}$$

Mathematica [A] time = 1.89982, size = 192, normalized size = 1.51

$$\frac{\sinh(e + fx)(a + b \coth(e + fx))^2 (-2abd \sinh(e + fx) \text{PolyLog}(2, e^{-2(e+fx)}) + \sinh(e + fx) (-e + fx) (a^2(-2cf + de - 2f^2)))}{2f^2}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)*(a + b*Coth[e + f*x])^2,x]

[Out] ((a + b*Coth[e + f*x])^2*Sinh[e + f*x]*(-2*b^2*f*(c + d*x)*Cosh[e + f*x] + ((e + f*x)*(-2*a*b*d*(e + f*x) + a^2*(d*e - 2*c*f - d*f*x) + b^2*(d*e - 2*c*f - d*f*x))) + 4*a*b*d*(e + f*x)*Log[1 - E^(-2*(e + f*x))] + 2*b*(b*d - 2*a*d*e + 2*a*c*f)*Log[Sinh[e + f*x]])*Sinh[e + f*x] - 2*a*b*d*PolyLog[2, E^(-2*(e + f*x))*Sinh[e + f*x]])/(2*f^2*(b*Cosh[e + f*x] + a*Sinh[e + f*x])^2)

Maple [B] time = 0.082, size = 318, normalized size = 2.5

$$\frac{a^2dx^2}{2} - abdx^2 + \frac{b^2dx^2}{2} + ca^2x + 2abcx + b^2cx - 2 \frac{b^2(dx + c)}{f(e^{2fx+2e} - 1)} - 2 \frac{b^2d \ln(e^{fx+e})}{f^2} + \frac{b^2d \ln(e^{fx+e} - 1)}{f^2} + \frac{b^2d \ln(e^{fx+e})}{f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x+c)*(a+b*coth(f*x+e))^2,x)`

[Out] $\frac{1}{2}a^2d^2x^2 - a^2bd^2x + \frac{1}{2}b^2d^2x^2 + c^2ax + 2abcx + b^2cx - \frac{2}{f}b^2(d^2x+c) \cdot \frac{1}{\exp(2fx+2e)-1} - \frac{2b^2}{f^2}d^2 \ln(\exp(fx+e)) + \frac{b^2}{f^2}d^2 \ln(\exp(fx+e)-1) + \frac{b^2}{f^2}d^2 \ln(\exp(fx+e)+1) - \frac{4b}{f}a^2c \ln(\exp(fx+e)) + \frac{2b}{f}a^2c \ln(\exp(fx+e)-1) + \frac{2b}{f}a^2c \ln(\exp(fx+e)+1) + \frac{4b}{f^2}d^2ae \ln(\exp(fx+e)) - \frac{2b}{f^2}d^2ae \ln(\exp(fx+e)-1) + \frac{2b}{f}d^2 \ln(\exp(fx+e)+1) * a^2dx + \frac{2b}{f}d^2 \ln(1-\exp(fx+e)) * a^2dx + \frac{2b}{f^2} \ln(1-\exp(fx+e)) * a^2de - \frac{4b}{f}a^2de * x - \frac{2b}{f^2}a^2de^2 + \frac{2b}{f^2}d^2ae \cdot \text{polylog}(2, \exp(fx+e)) + \frac{2b}{f^2}d^2ae \cdot \text{polylog}(2, -\exp(fx+e))$

Maxima [A] time = 1.40955, size = 329, normalized size = 2.59

$$\frac{1}{2} a^2 dx^2 - 2 ab dx^2 + a^2 cx - \frac{2 b^2 dx}{f} + \frac{2 abc \log(\sinh(fx+e))}{f} + \frac{2 \left(fx \log(e^{(fx+e)} + 1) + \text{Li}_2(-e^{(fx+e)}) \right) abd}{f^2} + \frac{2 \left(fx \log(e^{(fx+e)} + 1) + \text{Li}_2(-e^{(fx+e)}) \right) abd}{f^2} + \frac{2 \left(fx \log(e^{(fx+e)} + 1) + \text{Li}_2(-e^{(fx+e)}) \right) abd}{f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)*(a+b*coth(f*x+e))^2,x, algorithm="maxima")`

[Out] $\frac{1}{2}a^2d^2x^2 - 2a^2bd^2x + a^2c^2x - \frac{2b^2d^2x}{f} + \frac{2abc \log(\sinh(fx+e))}{f} + \frac{2 \left(fx \log(e^{(fx+e)} + 1) + \text{dilog}(-e^{(fx+e)}) \right) * a^2bd^2}{f^2} + \frac{2 \left(fx \log(-e^{(fx+e)} + 1) + \text{dilog}(-e^{(fx+e)}) \right) * a^2bd^2}{f^2} + \frac{b^2d^2 \log(e^{(fx+e)} + 1)}{f^2} + \frac{b^2d^2 \log(e^{(fx+e)} - 1)}{f^2} - \frac{1}{2} \left((2cf + 2d)b^2x + 4b^2c + (2abd^2 + b^2d^2) \right) x^2 - \frac{(2b^2c^2fx^2e^{(2e)} + (2abd^2fx^2e^{(2e)} + b^2d^2fx^2e^{(2e)}) * x^2) * e^{(2fx)}}{(f * e^{(2fx+2e)} - f)}$

Fricas [B] time = 2.67433, size = 2142, normalized size = 16.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)*(a+b*coth(f*x+e))^2,x, algorithm="fricas")`

[Out] $-\frac{1}{2} \left((a^2 - 2ab + b^2)d^2fx^2 + 4abd^2e^2 + 2(a^2 - 2ab + b^2)c \right) * f^2x - 4b^2d^2e - \left((a^2 - 2ab + b^2)d^2fx^2 + 4abd^2e^2 - 8abd^2c * ef - 4b^2d^2e - 2(2b^2d^2f - (a^2 - 2ab + b^2)c * f^2) * x \right) * \cosh(fx + e)^2 - 2 \left((a^2 - 2ab + b^2)d^2fx^2 + 4abd^2e^2 - 8abd^2c * ef - 4b^2d^2e - 2(2b^2d^2f - (a^2 - 2ab + b^2)c * f^2) * x \right) * \cosh(fx + e)^2$

```

*d*e - 2*(2*b^2*d*f - (a^2 - 2*a*b + b^2)*c*f^2)*x)*cosh(f*x + e)*sinh(f*x
+ e) - ((a^2 - 2*a*b + b^2)*d*f^2*x^2 + 4*a*b*d*e^2 - 8*a*b*c*e*f - 4*b^2*d
*e - 2*(2*b^2*d*f - (a^2 - 2*a*b + b^2)*c*f^2)*x)*sinh(f*x + e)^2 - 4*(2*a*
b*c*e - b^2*c)*f - 4*(a*b*d*cosh(f*x + e)^2 + 2*a*b*d*cosh(f*x + e)*sinh(f*
x + e) + a*b*d*sinh(f*x + e)^2 - a*b*d)*dilog(cosh(f*x + e) + sinh(f*x + e)
) - 4*(a*b*d*cosh(f*x + e)^2 + 2*a*b*d*cosh(f*x + e)*sinh(f*x + e) + a*b*d*
sinh(f*x + e)^2 - a*b*d)*dilog(-cosh(f*x + e) - sinh(f*x + e)) + 2*(2*a*b*d
*f*x + 2*a*b*c*f + b^2*d - (2*a*b*d*f*x + 2*a*b*c*f + b^2*d)*cosh(f*x + e)^
2 - 2*(2*a*b*d*f*x + 2*a*b*c*f + b^2*d)*cosh(f*x + e)*sinh(f*x + e) - (2*a*
b*d*f*x + 2*a*b*c*f + b^2*d)*sinh(f*x + e)^2)*log(cosh(f*x + e) + sinh(f*x
+ e) + 1) - 2*(2*a*b*d*e - 2*a*b*c*f - b^2*d - (2*a*b*d*e - 2*a*b*c*f - b^2
*d)*cosh(f*x + e)^2 - 2*(2*a*b*d*e - 2*a*b*c*f - b^2*d)*cosh(f*x + e)*sinh(
f*x + e) - (2*a*b*d*e - 2*a*b*c*f - b^2*d)*sinh(f*x + e)^2)*log(cosh(f*x +
e) + sinh(f*x + e) - 1) + 4*(a*b*d*f*x + a*b*d*e - (a*b*d*f*x + a*b*d*e)*co
sh(f*x + e)^2 - 2*(a*b*d*f*x + a*b*d*e)*cosh(f*x + e)*sinh(f*x + e) - (a*b*
d*f*x + a*b*d*e)*sinh(f*x + e)^2)*log(-cosh(f*x + e) - sinh(f*x + e) + 1))/
(f^2*cosh(f*x + e)^2 + 2*f^2*cosh(f*x + e)*sinh(f*x + e) + f^2*sinh(f*x + e
)^2 - f^2)

```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b \coth(e + fx))^2 (c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*(a+b*coth(f*x+e))**2,x)

[Out] Integral((a + b*coth(e + f*x))**2*(c + d*x), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)(b \coth(fx + e) + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*(a+b*coth(f*x+e))^2,x, algorithm="giac")

[Out] integrate((d*x + c)*(b*coth(f*x + e) + a)^2, x)

$$3.45 \quad \int \frac{(a+b \coth(e+fx))^2}{c+dx} dx$$

Optimal. Leaf size=22

$$\text{Unintegrable} \left(\frac{(a + b \coth(e + fx))^2}{c + dx}, x \right)$$

[Out] Unintegrable[(a + b*Coth[e + f*x])^2/(c + d*x), x]

Rubi [A] time = 0.0552581, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{(a + b \coth(e + fx))^2}{c + dx} dx$$

Verification is Not applicable to the result.

[In] Int[(a + b*Coth[e + f*x])^2/(c + d*x), x]

[Out] Defer[Int] [(a + b*Coth[e + f*x])^2/(c + d*x), x]

Rubi steps

$$\int \frac{(a + b \coth(e + fx))^2}{c + dx} dx = \int \frac{(a + b \coth(e + fx))^2}{c + dx} dx$$

Mathematica [A] time = 42.6526, size = 0, normalized size = 0.

$$\int \frac{(a + b \coth(e + fx))^2}{c + dx} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b*Coth[e + f*x])^2/(c + d*x), x]

[Out] Integrate[(a + b*Coth[e + f*x])^2/(c + d*x), x]

Maple [A] time = 0.376, size = 0, normalized size = 0.

$$\int \frac{(a + b \coth(fx + e))^2}{dx + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*coth(f*x+e))^2/(d*x+c),x)

[Out] int((a+b*coth(f*x+e))^2/(d*x+c),x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{a^2 \log(dx + c)}{d} + \frac{2b^2}{dfx + cf - (dfe^{2e} + cfe^{2e})e^{2fx}} + \frac{(2ab + b^2) \log(dx + c)}{d} - \int \frac{2abdfx + 2abc}{d^2fx^2 + 2cdfx + c^2f + (d^2fx^2e^e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*coth(f*x+e))^2/(d*x+c),x, algorithm="maxima")

[Out] a^2*log(d*x + c)/d + 2*b^2/(d*f*x + c*f - (d*f*x*e^(2*e) + c*f*e^(2*e))*e^(2*f*x)) + (2*a*b + b^2)*log(d*x + c)/d - integrate((2*a*b*d*f*x + 2*a*b*c*f - b^2*d)/(d^2*f*x^2 + 2*c*d*f*x + c^2*f + (d^2*f*x^2*e^e + 2*c*d*f*x*e^e + c^2*f*e^e)*e^(f*x)), x) + integrate(-(2*a*b*d*f*x + 2*a*b*c*f - b^2*d)/(d^2*f*x^2 + 2*c*d*f*x + c^2*f - (d^2*f*x^2*e^e + 2*c*d*f*x*e^e + c^2*f*e^e)*e^(f*x)), x)

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{b^2 \coth(fx + e)^2 + 2ab \coth(fx + e) + a^2}{dx + c}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*coth(f*x+e))^2/(d*x+c),x, algorithm="fricas")

[Out] `integral((b^2*coth(f*x + e)^2 + 2*a*b*coth(f*x + e) + a^2)/(d*x + c), x)`

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \coth(e + fx))^2}{c + dx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*coth(f*x+e))**2/(d*x+c),x)`

[Out] `Integral((a + b*coth(e + f*x))**2/(c + d*x), x)`

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \coth(fx + e) + a)^2}{dx + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*coth(f*x+e))^2/(d*x+c),x, algorithm="giac")`

[Out] `integrate((b*coth(f*x + e) + a)^2/(d*x + c), x)`

$$3.46 \quad \int \frac{(a+b \coth(e+fx))^2}{(c+dx)^2} dx$$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{(a+b \coth(e+fx))^2}{(c+dx)^2}, x\right)$$

[Out] Unintegrable[(a + b*Coth[e + f*x])^2/(c + d*x)^2, x]

Rubi [A] time = 0.0525684, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{(a+b \coth(e+fx))^2}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

[In] Int[(a + b*Coth[e + f*x])^2/(c + d*x)^2, x]

[Out] Defer[Int] [(a + b*Coth[e + f*x])^2/(c + d*x)^2, x]

Rubi steps

$$\int \frac{(a+b \coth(e+fx))^2}{(c+dx)^2} dx = \int \frac{(a+b \coth(e+fx))^2}{(c+dx)^2} dx$$

Mathematica [A] time = 30.1259, size = 0, normalized size = 0.

$$\int \frac{(a+b \coth(e+fx))^2}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b*Coth[e + f*x])^2/(c + d*x)^2, x]

[Out] Integrate[(a + b*Coth[e + f*x])^2/(c + d*x)^2, x]

Maple [A] time = 0.589, size = 0, normalized size = 0.

$$\int \frac{(a + b \coth(fx + e))^2}{(dx + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*coth(f*x+e))^2/(d*x+c)^2,x)

[Out] int((a+b*coth(f*x+e))^2/(d*x+c)^2,x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{a^2}{d^2x + cd} - \frac{2abc f + (cf - 2d)b^2 + (2abdf + b^2df)x - (2abcfe^{(2e)} + b^2cfe^{(2e)} + (2abdfe^{(2e)} + b^2dfe^{(2e)})x)e^{(2fx)}}{d^3fx^2 + 2cd^2fx + c^2df - (d^3fx^2e^{(2e)} + 2cd^2fxe^{(2e)} + c^2dfe^{(2e)})e^{(2fx)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*coth(f*x+e))^2/(d*x+c)^2,x, algorithm="maxima")

[Out] $-a^2/(d^2x + cd) - (2abcf + (cf - 2d)b^2 + (2abd f + b^2df)x - (2abcfe^{(2e)} + b^2cfe^{(2e)} + (2abdfe^{(2e)} + b^2dfe^{(2e)})x)e^{(2fx)}) / (d^3fx^2 + 2cd^2fx + c^2df - (d^3fx^2e^{(2e)} + 2cd^2fxe^{(2e)} + c^2dfe^{(2e)})e^{(2fx)}) - \text{integrate}(2(abdfx + abc f - b^2d) / (d^3fx^3 + 3cd^2fx^2 + 3c^2dfx + c^3f + (d^3fx^3e^e + 3cd^2fx^2e^e + 3c^2dfxe^e + c^3fe^e)e^{(fx)}), x) + \text{integrate}(-2(abdfx + abc f - b^2d) / (d^3fx^3 + 3cd^2fx^2 + 3c^2dfx + c^3f - (d^3fx^3e^e + 3cd^2fx^2e^e + 3c^2dfxe^e + c^3fe^e)e^{(fx)}), x)$

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{b^2 \coth(fx + e)^2 + 2ab \coth(fx + e) + a^2}{d^2x^2 + 2cdx + c^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*coth(f*x+e))^2/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*coth(f*x + e)^2 + 2*a*b*coth(f*x + e) + a^2)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \coth(e + fx))^2}{(c + dx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*coth(f*x+e))^2/(d*x+c)^2,x)
```

```
[Out] Integral((a + b*coth(e + f*x))^2/(c + d*x)^2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \coth(fx + e) + a)^2}{(dx + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*coth(f*x+e))^2/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate((b*coth(f*x + e) + a)^2/(d*x + c)^2, x)
```

3.47 $\int (c + dx)^3 (a + b \coth(e + fx))^3 dx$

Optimal. Leaf size=556

$$-\frac{9a^2bd^2(c+dx)\text{PolyLog}\left(3,e^{2(e+fx)}\right)}{2f^3} + \frac{9a^2bd(c+dx)^2\text{PolyLog}\left(2,e^{2(e+fx)}\right)}{2f^2} + \frac{9a^2bd^3\text{PolyLog}\left(4,e^{2(e+fx)}\right)}{4f^4} + \frac{9ab^2d^2(c+dx)^3\text{PolyLog}\left(3,e^{2(e+fx)}\right)}{2f^3}$$

```
[Out] (-3*b^3*d*(c + d*x)^2)/(2*f^2) - (3*a*b^2*(c + d*x)^3)/f + (b^3*(c + d*x)^3)/(2*f) + (a^3*(c + d*x)^4)/(4*d) - (3*a^2*b*(c + d*x)^4)/(4*d) + (3*a*b^2*(c + d*x)^4)/(4*d) - (b^3*(c + d*x)^4)/(4*d) - (3*b^3*d*(c + d*x)^2*Coth[e + f*x])/(2*f^2) - (3*a*b^2*(c + d*x)^3*Coth[e + f*x])/f - (b^3*(c + d*x)^3*Coth[e + f*x]^2)/(2*f) + (3*b^3*d^2*(c + d*x)*Log[1 - E^(2*(e + f*x))])/f^3 + (9*a*b^2*d*(c + d*x)^2*Log[1 - E^(2*(e + f*x))])/f^2 + (3*a^2*b*(c + d*x)^3*Log[1 - E^(2*(e + f*x))])/f + (b^3*(c + d*x)^3*Log[1 - E^(2*(e + f*x))])/f + (3*b^3*d^3*PolyLog[2, E^(2*(e + f*x))])/(2*f^4) + (9*a*b^2*d^2*(c + d*x)*PolyLog[2, E^(2*(e + f*x))])/f^3 + (9*a^2*b*d*(c + d*x)^2*PolyLog[2, E^(2*(e + f*x))])/(2*f^2) + (3*b^3*d*(c + d*x)^2*PolyLog[2, E^(2*(e + f*x))])/(2*f^2) - (9*a*b^2*d^3*PolyLog[3, E^(2*(e + f*x))])/(2*f^4) - (9*a^2*b*d^2*(c + d*x)*PolyLog[3, E^(2*(e + f*x))])/(2*f^3) - (3*b^3*d^2*(c + d*x)*PolyLog[3, E^(2*(e + f*x))])/(2*f^3) + (9*a^2*b*d^3*PolyLog[4, E^(2*(e + f*x))])/(4*f^4) + (3*b^3*d^3*PolyLog[4, E^(2*(e + f*x))])/(4*f^4)
```

Rubi [A] time = 1.04909, antiderivative size = 556, normalized size of antiderivative = 1., number of steps used = 28, number of rules used = 11, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.55$, Rules used = {3722, 3716, 2190, 2531, 6609, 2282, 6589, 3720, 32, 2279, 2391}

$$-\frac{9a^2bd^2(c+dx)\text{PolyLog}\left(3,e^{2(e+fx)}\right)}{2f^3} + \frac{9a^2bd(c+dx)^2\text{PolyLog}\left(2,e^{2(e+fx)}\right)}{2f^2} + \frac{9a^2bd^3\text{PolyLog}\left(4,e^{2(e+fx)}\right)}{4f^4} + \frac{9ab^2d^2(c+dx)^3\text{PolyLog}\left(3,e^{2(e+fx)}\right)}{2f^3}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)^3*(a + b*Coth[e + f*x])^3,x]

```
[Out] (-3*b^3*d*(c + d*x)^2)/(2*f^2) - (3*a*b^2*(c + d*x)^3)/f + (b^3*(c + d*x)^3)/(2*f) + (a^3*(c + d*x)^4)/(4*d) - (3*a^2*b*(c + d*x)^4)/(4*d) + (3*a*b^2*(c + d*x)^4)/(4*d) - (b^3*(c + d*x)^4)/(4*d) - (3*b^3*d*(c + d*x)^2*Coth[e + f*x])/(2*f^2) - (3*a*b^2*(c + d*x)^3*Coth[e + f*x])/f - (b^3*(c + d*x)^3*Coth[e + f*x]^2)/(2*f) + (3*b^3*d^2*(c + d*x)*Log[1 - E^(2*(e + f*x))])/f^3 + (9*a*b^2*d*(c + d*x)^2*Log[1 - E^(2*(e + f*x))])/f^2 + (3*a^2*b*(c + d*x)^3*Log[1 - E^(2*(e + f*x))])/f + (b^3*(c + d*x)^3*Log[1 - E^(2*(e + f*x))])/f + (3*b^3*d^3*PolyLog[2, E^(2*(e + f*x))])/(2*f^4) + (9*a*b^2*d^2*(c + d*x)*PolyLog[2, E^(2*(e + f*x))])/f^3 + (9*a^2*b*d*(c + d*x)^2*PolyLog[2, E^(2*(e + f*x))])/(2*f^2) + (3*b^3*d*(c + d*x)^2*PolyLog[2, E^(2*(e + f*x))])/(2*f^2) - (9*a*b^2*d^3*PolyLog[3, E^(2*(e + f*x))])/(2*f^4) - (9*a^2*b*d^2*(c + d*x)*PolyLog[3, E^(2*(e + f*x))])/(2*f^3) - (3*b^3*d^2*(c + d*x)*PolyLog[3, E^(2*(e + f*x))])/(2*f^3) + (9*a^2*b*d^3*PolyLog[4, E^(2*(e + f*x))])/(4*f^4) + (3*b^3*d^3*PolyLog[4, E^(2*(e + f*x))])/(4*f^4)
```

```
*x)*PolyLog[2, E^(2*(e + f*x))]/f^3 + (9*a^2*b*d*(c + d*x)^2*PolyLog[2, E^(2*(e + f*x))]/(2*f^2) + (3*b^3*d*(c + d*x)^2*PolyLog[2, E^(2*(e + f*x))]/(2*f^2) - (9*a*b^2*d^3*PolyLog[3, E^(2*(e + f*x))]/(2*f^4) - (9*a^2*b*d^2*(c + d*x)*PolyLog[3, E^(2*(e + f*x))]/(2*f^3) - (3*b^3*d^2*(c + d*x)*PolyLog[3, E^(2*(e + f*x))]/(2*f^3) + (9*a^2*b*d^3*PolyLog[4, E^(2*(e + f*x))]/(4*f^4) + (3*b^3*d^3*PolyLog[4, E^(2*(e + f*x))]/(4*f^4)
```

Rule 3722

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, (a + b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)], x_Symbol] := -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c + d*x)^m*E^(2*(-I*e) + f*fz*x))]/(E^(2*I*k*Pi)*(1 + E^(2*(-I*e) + f*fz*x))/E^(2*I*k*Pi)), x], x] /; FreeQ[{c, d, e, f, fz}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] := Simp[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a]]/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x))))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m - 1)*PolyLog[2, -(e*(F^(c*(a + b*x))))^n], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(p_.)], x_Symbol] := Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p]]/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c, d, e, f, n, p}, x] && GtQ[m, 0]
```


Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x],
Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Dist[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x], x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] := Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol] := Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int (c + dx)^3 (a + b \coth(e + fx))^3 dx &= \int \left(a^3 (c + dx)^3 + 3a^2 b (c + dx)^3 \coth(e + fx) + 3ab^2 (c + dx)^3 \coth^2(e + fx) + b^3 (c + dx)^3 \coth^3(e + fx) \right) dx \\
&= \frac{a^3 (c + dx)^4}{4d} + (3a^2 b) \int (c + dx)^3 \coth(e + fx) dx + (3ab^2) \int (c + dx)^3 \coth^2(e + fx) dx + b^3 \int (c + dx)^3 \coth^3(e + fx) dx \\
&= \frac{a^3 (c + dx)^4}{4d} - \frac{3a^2 b (c + dx)^4}{4d} - \frac{3ab^2 (c + dx)^3 \coth(e + fx)}{f} - \frac{b^3 (c + dx)^3 \coth^2(e + fx)}{2f} \\
&= -\frac{3ab^2 (c + dx)^3}{f} + \frac{a^3 (c + dx)^4}{4d} - \frac{3a^2 b (c + dx)^4}{4d} + \frac{3ab^2 (c + dx)^4}{4d} - \frac{b^3 (c + dx)^4}{4d} - \frac{3b^3 d (c + dx)^2}{2f^2} \\
&= -\frac{3b^3 d (c + dx)^2}{2f^2} - \frac{3ab^2 (c + dx)^3}{f} + \frac{b^3 (c + dx)^3}{2f} + \frac{a^3 (c + dx)^4}{4d} - \frac{3a^2 b (c + dx)^4}{4d} + \frac{3ab^2 (c + dx)^4}{4d} \\
&= -\frac{3b^3 d (c + dx)^2}{2f^2} - \frac{3ab^2 (c + dx)^3}{f} + \frac{b^3 (c + dx)^3}{2f} + \frac{a^3 (c + dx)^4}{4d} - \frac{3a^2 b (c + dx)^4}{4d} + \frac{3ab^2 (c + dx)^4}{4d} \\
&= -\frac{3b^3 d (c + dx)^2}{2f^2} - \frac{3ab^2 (c + dx)^3}{f} + \frac{b^3 (c + dx)^3}{2f} + \frac{a^3 (c + dx)^4}{4d} - \frac{3a^2 b (c + dx)^4}{4d} + \frac{3ab^2 (c + dx)^4}{4d} \\
&= -\frac{3b^3 d (c + dx)^2}{2f^2} - \frac{3ab^2 (c + dx)^3}{f} + \frac{b^3 (c + dx)^3}{2f} + \frac{a^3 (c + dx)^4}{4d} - \frac{3a^2 b (c + dx)^4}{4d} + \frac{3ab^2 (c + dx)^4}{4d} \\
&= -\frac{3b^3 d (c + dx)^2}{2f^2} - \frac{3ab^2 (c + dx)^3}{f} + \frac{b^3 (c + dx)^3}{2f} + \frac{a^3 (c + dx)^4}{4d} - \frac{3a^2 b (c + dx)^4}{4d} + \frac{3ab^2 (c + dx)^4}{4d}
\end{aligned}$$

Mathematica [B] time = 14.0006, size = 2043, normalized size = 3.67

Result too large to show

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)^3*(a + b*Coth[e + f*x])^3,x]

[Out]
$$\begin{aligned}
& \left(-(b^3 c^3) - 3b^3 c^2 d x - 3b^3 c d^2 x^2 - b^3 d^3 x^3 \right) \operatorname{Csch}[e + f x] \\
& - \frac{(b E^{2e}) (24 b^2 c d^2 x + 72 a b c^2 d f x + 24 a^2 c^3 f^2 x + 8 b^2 c^3 f^2 x + 12 b^2 d^3 x^2 + 72 a b c d^2 f x^2 + 36 a^2 c^2 d f^2 x^2 + 12 b^2 c^2 d f^2 x^2 + 24 a b d^3 f x^3 + 24 a^2 c d^2 f^2 x^3 + 8 b^2 c d^2 f^2 x^3 + 6 a^2 d^3 f^2 x^4 + 2 b^2 d^3 f^2 x^4 - 36 a b c^2 d \operatorname{Log}[1 - E^{2(e + f x)}]) + (36 a b c^2 d \operatorname{Log}[1 - E^{2(e + f x)}])}{E^{2e}} \\
& - \frac{(12 b^2 c d^2 \operatorname{Log}[1 - E^{2(e + f x)}])}{f} + \frac{(12 b^2 c d^2 \operatorname{Log}[1 - E^{2(e + f x)}])}{(E^{2e} f) - 12 a^2 c^3 f \operatorname{Log}[1 - E^{2(e + f x)}] - 4 b^2 c^3 f \operatorname{Log}[1 - E^{2(e + f x)}] + (12 a^2 c^3 f \operatorname{Log}[1 - E^{2(e + f x)}])}{E^{2e}} \\
& + \frac{(4 b^2 c^3 f \operatorname{Log}[1 - E^{2(e + f x)}])}{E^{2e}} - 72 a b c d^2 x \operatorname{Log}[1 - E^{2(e + f x)}]
\end{aligned}$$

$$\begin{aligned}
& \left((2*(e + f*x)) \right) + (72*a*b*c*d^2*x*\text{Log}[1 - E^{(2*(e + f*x))}])/E^{(2*e)} - (12*b \\
& ^2*d^3*x*\text{Log}[1 - E^{(2*(e + f*x))}])/f + (12*b^2*d^3*x*\text{Log}[1 - E^{(2*(e + f*x))}]) \\
&)/(E^{(2*e)}*f) - 36*a^2*c^2*d*f*x*\text{Log}[1 - E^{(2*(e + f*x))}] - 12*b^2*c^2*d* \\
& f*x*\text{Log}[1 - E^{(2*(e + f*x))}] + (36*a^2*c^2*d*f*x*\text{Log}[1 - E^{(2*(e + f*x))}])/ \\
& E^{(2*e)} + (12*b^2*c^2*d*f*x*\text{Log}[1 - E^{(2*(e + f*x))}])/E^{(2*e)} - 36*a*b*d^3* \\
& x^2*\text{Log}[1 - E^{(2*(e + f*x))}] + (36*a*b*d^3*x^2*\text{Log}[1 - E^{(2*(e + f*x))}])/E^{(2*e)} \\
& - 36*a^2*c*d^2*f*x^2*\text{Log}[1 - E^{(2*(e + f*x))}] - 12*b^2*c*d^2*f*x^2*\text{Lo} \\
& g[1 - E^{(2*(e + f*x))}] + (36*a^2*c*d^2*f*x^2*\text{Log}[1 - E^{(2*(e + f*x))}])/E^{(2 \\
& *e)} + (12*b^2*c*d^2*f*x^2*\text{Log}[1 - E^{(2*(e + f*x))}])/E^{(2*e)} - 12*a^2*d^3*f* \\
& x^3*\text{Log}[1 - E^{(2*(e + f*x))}] - 4*b^2*d^3*f*x^3*\text{Log}[1 - E^{(2*(e + f*x))}] + (\\
& 12*a^2*d^3*f*x^3*\text{Log}[1 - E^{(2*(e + f*x))}])/E^{(2*e)} + (4*b^2*d^3*f*x^3*\text{Log}[1 \\
& - E^{(2*(e + f*x))}])/E^{(2*e)} - (6*d*(-1 + E^{(2*e)})*(6*a*b*d*f*(c + d*x) + 3 \\
& *a^2*f^2*(c + d*x)^2 + b^2*(d^2 + c^2*f^2 + 2*c*d*f^2*x + d^2*f^2*x^2))*\text{Pol} \\
& y\text{Log}[2, E^{(2*(e + f*x))}]/(E^{(2*e)}*f^2) + (6*d^2*(-1 + E^{(2*e)})*(3*a*b*d + \\
& 3*a^2*f*(c + d*x) + b^2*f*(c + d*x))*\text{PolyLog}[3, E^{(2*(e + f*x))}]/(E^{(2*e)}* \\
& f^2) - (9*a^2*d^3*\text{PolyLog}[4, E^{(2*(e + f*x))}])/f^2 - (3*b^2*d^3*\text{PolyLog}[4, \\
& E^{(2*(e + f*x))}])/f^2 + (9*a^2*d^3*\text{PolyLog}[4, E^{(2*(e + f*x))}]/(E^{(2*e)}*f^ \\
& 2) + (3*b^2*d^3*\text{PolyLog}[4, E^{(2*(e + f*x))}]/(E^{(2*e)}*f^2)))/(4*(-1 + E^{(2* \\
& e)})*f^2) + (3*x^2*(-(a^3*c^2*d) + 3*a^2*b*c^2*d - 3*a*b^2*c^2*d + b^3*c^2*d \\
& + a^3*c^2*d*\text{Cosh}[2*e] + 3*a^2*b*c^2*d*\text{Cosh}[2*e] + 3*a*b^2*c^2*d*\text{Cosh}[2*e] \\
& + b^3*c^2*d*\text{Cosh}[2*e] + a^3*c^2*d*\text{Sinh}[2*e] + 3*a^2*b*c^2*d*\text{Sinh}[2*e] + 3*a \\
& *b^2*c^2*d*\text{Sinh}[2*e] + b^3*c^2*d*\text{Sinh}[2*e]))/(2*(-1 + \text{Cosh}[2*e] + \text{Sinh}[2*e] \\
&)) + (x^3*(-(a^3*c*d^2) + 3*a^2*b*c*d^2 - 3*a*b^2*c*d^2 + b^3*c*d^2 + a^3*c \\
& *d^2*\text{Cosh}[2*e] + 3*a^2*b*c*d^2*\text{Cosh}[2*e] + 3*a*b^2*c*d^2*\text{Cosh}[2*e] + b^3*c \\
& *d^2*\text{Cosh}[2*e] + a^3*c*d^2*\text{Sinh}[2*e] + 3*a^2*b*c*d^2*\text{Sinh}[2*e] + 3*a*b^2*c*d \\
& ^2*\text{Sinh}[2*e] + b^3*c*d^2*\text{Sinh}[2*e]))/(-1 + \text{Cosh}[2*e] + \text{Sinh}[2*e]) + (x^4*(- \\
& (a^3*d^3) + 3*a^2*b*d^3 - 3*a*b^2*d^3 + b^3*d^3 + a^3*d^3*\text{Cosh}[2*e] + 3*a^2 \\
& *b*d^3*\text{Cosh}[2*e] + 3*a*b^2*d^3*\text{Cosh}[2*e] + b^3*d^3*\text{Cosh}[2*e] + a^3*d^3*\text{Sinh} \\
& [2*e] + 3*a^2*b*d^3*\text{Sinh}[2*e] + 3*a*b^2*d^3*\text{Sinh}[2*e] + b^3*d^3*\text{Sinh}[2*e])) \\
& /((4*(-1 + \text{Cosh}[2*e] + \text{Sinh}[2*e])) + x*(a^3*c^3 + 3*a*b^2*c^3 + (3*a^2*b*c^3 \\
&)/(-1 + \text{Cosh}[2*e] + \text{Sinh}[2*e]) + (3*a^2*b*c^3*\text{Cosh}[2*e] + 3*a^2*b*c^3*\text{Sinh}[\\
& 2*e])/(-1 + \text{Cosh}[2*e] + \text{Sinh}[2*e]) + (2*b^3*c^3*\text{Cosh}[2*e] + 2*b^3*c^3*\text{Sinh}[\\
& 2*e])/((-1 + \text{Cosh}[2*e] + \text{Sinh}[2*e]))*(1 + \text{Cosh}[2*e] + \text{Cosh}[4*e] + \text{Sinh}[2*e] \\
& + \text{Sinh}[4*e])) + (2*b^3*c^3*\text{Cosh}[4*e] + 2*b^3*c^3*\text{Sinh}[4*e])/((-1 + \text{Cosh}[2*e] \\
& + \text{Sinh}[2*e]))*(1 + \text{Cosh}[2*e] + \text{Cosh}[4*e] + \text{Sinh}[2*e] + \text{Sinh}[4*e])) + (b^3* \\
& c^3)/(-1 + \text{Cosh}[6*e] + \text{Sinh}[6*e]) + (b^3*c^3*\text{Cosh}[6*e] + b^3*c^3*\text{Sinh}[6*e]) \\
& /(-1 + \text{Cosh}[6*e] + \text{Sinh}[6*e])) + (3*\text{Csch}[e]*\text{Csch}[e + f*x]*(b^3*c^2*d*\text{Sinh}[f \\
& *x] + 2*a*b^2*c^3*f*\text{Sinh}[f*x] + 2*b^3*c*d^2*x*\text{Sinh}[f*x] + 6*a*b^2*c^2*d*f*x \\
& *\text{Sinh}[f*x] + b^3*d^3*x^2*\text{Sinh}[f*x] + 6*a*b^2*c*d^2*f*x^2*\text{Sinh}[f*x] + 2*a*b^ \\
& 2*d^3*f*x^3*\text{Sinh}[f*x]))/(2*f^2)
\end{aligned}$$

Maple [B] time = 0.239, size = 2777, normalized size = 5.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((d*x+c)^3*(a+b*\text{coth}(f*x+e))^3,x)$

[Out]
$$\begin{aligned} & \frac{3}{2}a^3c^2d^2x^2 - \frac{3}{2}b^3c^2d^2x^2 + 3a^2b^3c^3x + 3ab^2c^3x - \frac{1}{4}b^3d^3 \\ & x^4 + c^3a^3x + c^3b^3x + \frac{1}{4}a^3d^3x^4 - 36b^2/f^2ac^2d^2ex + 18b/f^2a^2 \\ & c^2d^2e^2x - 18b/f^2a^2c^2d^2ex - 18b/f^3a^2c^2d^2e^2\ln(\exp(f*x+e)) + 36 \\ & b^2/f^3ac^2d^2e^2\ln(\exp(f*x+e)) + 18b/f^2a^2c^2d^2e^2\ln(\exp(f*x+e)) - 3a^2 \\ & b^3c^2d^2x^3 + 3ab^2c^2d^2x^3 - 9/2a^2b^3c^2d^2x^2 + 9/2ab^2c^2d^2x^2 + 6b^3 \\ & /f^2\text{polylog}(2, -\exp(f*x+e)) * c^2d^2x + 3b^3/f\ln(1-\exp(f*x+e)) * c^2d^2x^2 + 3b \\ & /f^2a^2d^3\ln(1-\exp(f*x+e)) * x^3 + 3b/f^4a^2d^3\ln(1-\exp(f*x+e)) * e^3 + 9b/f^2 \\ & a^2d^3\text{polylog}(2, \exp(f*x+e)) * x^2 - 18b/f^3a^2d^3\text{polylog}(3, \exp(f*x+e)) * \\ & x + 3b/f^2a^2d^3\ln(\exp(f*x+e)+1) * x^3 + 9b/f^2a^2d^3\text{polylog}(2, -\exp(f*x+e)) \\ & * x^2 - 18b/f^3a^2d^3\text{polylog}(3, -\exp(f*x+e)) * x - 18b/f^3a^2c^2d^2\text{polylog}(3 \\ & , \exp(f*x+e)) - 18b/f^3a^2c^2d^2\text{polylog}(3, -\exp(f*x+e)) + 9b/f^2a^2c^2d^2\text{poly} \\ & \text{log}(2, \exp(f*x+e)) + 9b/f^2a^2c^2d^2\text{polylog}(2, -\exp(f*x+e)) + 6b^3/f^2\text{poly} \\ & \text{log}(2, \exp(f*x+e)) * c^2d^2x + 3b^3/f\ln(1-\exp(f*x+e)) * c^2d^2x + 3b^3/f^2\ln(1-\exp \\ & (f*x+e)) * c^2d^2e + 3b^3/f\ln(\exp(f*x+e)+1) * c^2d^2x - 3b^3/f^4a^2d^3e^3\ln(\exp \\ & (f*x+e)-1) + 3b^3/f^3c^2d^2e^2\ln(\exp(f*x+e)-1) - b^3c^2d^2x^3 + a^3c^2d^2x^3 \\ & - 3/2b^3/f^4d^3e^4 - 3b^3/f^2d^3x^2 - 3b^3/f^4d^3e^2 - 2b^3/f^3c^3\ln(\exp \\ & (f*x+e)) + 18b^2/f^2\ln(1-\exp(f*x+e)) * ac^2d^2x + 18b^2/f^2\ln(\exp(f*x+e)+ \\ & 1) * ac^2d^2x + 9b/f\ln(1-\exp(f*x+e)) * a^2c^2d^2x^2 + 18b/f^2\text{polylog}(2, \exp(f \\ & x+e)) * a^2c^2d^2x + 9b/f\ln(\exp(f*x+e)+1) * a^2c^2d^2x^2 + 18b/f^2\text{polylog}(2, - \\ & \exp(f*x+e)) * a^2c^2d^2x + 9b/f^3a^2c^2d^2e^2\ln(\exp(f*x+e)-1) + 18b^2/f^3\ln \\ & (1-\exp(f*x+e)) * ac^2d^2e - 3/4a^2b^3d^3x^4 + 3/4ab^2d^3x^4 - b^2(6ad^3f \\ & x^3\exp(2fx+2e) + 2bd^3fx^3\exp(2fx+2e) + 18ad^2fx^2\exp(2fx+2e) + 6b^2c^2d^2fx^2 \\ & \exp(2fx+2e) + 18a^2c^2d^2fx\exp(2fx+2e) - 6ad^3fx^3 + 6b^2c^2d^2fx\exp(2fx+2e) \\ & + 3bd^3x^2\exp(2fx+2e) + 6a^2c^3fx\exp(2fx+2e) - 18ad^2fx^2 + 2b^2c^3fx\exp(2fx+2e) \\ & + 6b^2c^2d^2x\exp(2fx+2e) - 18a^2c^2d^2fx + 3b^2c^2d^2\exp(2fx+2e) - 3bd^3x^2 - 6a^2c^3f \\ & - 6b^2c^2d^2x - 3b^2c^2d) / f^2 / (\exp(2fx+2e)-1)^2 + 3b^3/f^3c^2d^2\ln(\exp(f*x+e)+1) \\ & + b^3/f^3c^3\ln(\exp(f*x+e)-1) + b^3/f^3c^3\ln(\exp(f*x+e)+1) + 6b^3/f^4d^3\text{polylog} \\ & (4, \exp(f*x+e)) + 6b^3/f^4d^3\text{polylog}(4, -\exp(f*x+e)) + 3b^3/f^4d^3\text{polylog}(2 \\ & , \exp(f*x+e)) + 3b^3/f^4d^3\text{polylog}(2, -\exp(f*x+e)) + 18b^2/f^3ac^2d^2\text{polylo} \\ & \text{g}(2, \exp(f*x+e)) + 18b^2/f^3ac^2d^2\text{polylog}(2, -\exp(f*x+e)) - 3b^3/f^3\ln(1-\exp \\ & (f*x+e)) * c^2d^2e^2 - 3b^3/f^2c^2d^2e^2\ln(\exp(f*x+e)-1) + 9b^2/f^2ac^2d^2\ln \\ & (\exp(f*x+e)-1) + 9b^2/f^2ac^2d^2\ln(\exp(f*x+e)+1) + 9b^2/f^4ad^3e^2\ln(\exp \\ & (f*x+e)-1) - 9b^2/f^4ad^3e^2\ln(1-\exp(f*x+e)) + 9b^2/f^2ad^3\ln(1-\exp(f \\ & x+e)) * x^2 + 18b^2/f^3ad^3\text{polylog}(2, \exp(f*x+e)) * x + 9b^2/f^2ad^3\ln(\exp \\ & (f*x+e)+1) * x^2 + 18b^2/f^3ad^3\text{polylog}(2, -\exp(f*x+e)) * x + 3b^3/f\ln(\exp(f*x+ \end{aligned}$$

$$\begin{aligned}
& e)+1)*c*d^2*x^2+9*b/f*\ln(\exp(f*x+e)+1)*a^2*c^2*d*x+9*b/f*\ln(1-\exp(f*x+e))*a \\
& ^2*c^2*d*x+9*b/f^2*\ln(1-\exp(f*x+e))*a^2*c^2*d*e-18*b^2/f^3*a*c*d^2*e*\ln(\exp \\
& (f*x+e)-1)-9*b/f^3*\ln(1-\exp(f*x+e))*a^2*c*d^2*e^2-9*b/f^2*a^2*c^2*d*e*\ln(\exp \\
& (f*x+e)-1)+3*b^3/f^2*c^2*d*polylog(2,\exp(f*x+e))+3*b^3/f^2*c^2*d*polylog(2 \\
& ,-\exp(f*x+e))-18*b^2/f^4*a*d^3*polylog(3,\exp(f*x+e))-18*b^2/f^4*a*d^3*polylog \\
& (3,-\exp(f*x+e))-3*b^3/f^4*d^3*e*\ln(\exp(f*x+e)-1)-b^3/f^4*d^3*e^3*\ln(\exp(f \\
& *x+e)-1)+3*b^3/f^3*d^3*\ln(1-\exp(f*x+e))*x+3*b^3/f^4*d^3*\ln(1-\exp(f*x+e))*e+ \\
& 3*b^3/f^3*d^3*\ln(\exp(f*x+e)+1)*x+b^3/f*d^3*\ln(1-\exp(f*x+e))*x^3+b^3/f^4*d^3 \\
& *\ln(1-\exp(f*x+e))*e^3+3*b^3/f^2*d^3*polylog(2,\exp(f*x+e))*x^2-6*b^3/f^3*d^3 \\
& *polylog(3,\exp(f*x+e))*x+b^3/f*d^3*\ln(\exp(f*x+e)+1)*x^3+3*b^3/f^2*d^3*polylog \\
& (2,-\exp(f*x+e))*x^2-6*b^3/f^3*d^3*polylog(3,-\exp(f*x+e))*x+18*b/f^4*a^2*d \\
& ^3*polylog(4,\exp(f*x+e))+18*b/f^4*a^2*d^3*polylog(4,-\exp(f*x+e))-6*b^3/f^3*c \\
& *d^2*polylog(3,\exp(f*x+e))-6*b^3/f^3*c*d^2*polylog(3,-\exp(f*x+e))+3*b/f*a^ \\
& 2*c^3*\ln(\exp(f*x+e)-1)+3*b/f*a^2*c^3*\ln(\exp(f*x+e)+1)+3*b^3/f^3*c*d^2*\ln(\exp \\
& (f*x+e)-1)+4*b^3/f^3*c*d^2*e^3+12*b^2/f^4*a*d^3*e^3-2*b^3/f^3*d^3*e^3*x-6*b \\
& ^2/f*a*d^3*x^3-6*b^3/f^3*d^3*e*x-9/2*b/f^4*a^2*d^3*e^4-3*b^3/f^2*c^2*d*e^2 \\
& +2*b^3/f^4*d^3*e^3*\ln(\exp(f*x+e))-6*b/f*a^2*c^3*\ln(\exp(f*x+e))-6*b^3/f^3*c \\
& d^2*\ln(\exp(f*x+e))+6*b^3/f^4*d^3*e*\ln(\exp(f*x+e))-18*b^2/f^3*a*c*d^2*e^2+12 \\
& *b/f^3*a^2*c*d^2*e^3-6*b^3/f*c^2*d*e*x-9*b/f^2*a^2*c^2*d*e^2+6*b^3/f^2*c*d^ \\
& 2*e^2*x-18*b^2/f*a*c*d^2*x^2-6*b/f^3*a^2*d^3*e^3*x+18*b^2/f^3*a*d^3*e^2*x+6 \\
& *b/f^4*a^2*d^3*e^3*\ln(\exp(f*x+e))-6*b^3/f^3*c*d^2*e^2*\ln(\exp(f*x+e))-18*b^2 \\
& /f^4*a*d^3*e^2*\ln(\exp(f*x+e))+6*b^3/f^2*c^2*d*e*\ln(\exp(f*x+e))-18*b^2/f^2*a \\
& *c^2*d*\ln(\exp(f*x+e))
\end{aligned}$$

Maxima [B] time = 1.66127, size = 2067, normalized size = 3.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*(a+b*coth(f*x+e))^3,x, algorithm="maxima")

[Out]
$$\begin{aligned}
& 1/4*a^3*d^3*x^4 + a^3*c*d^2*x^3 + 3/2*a^3*c^2*d*x^2 + a^3*c^3*x + 3*a^2*b*c \\
& ^3*\log(\sinh(f*x + e))/f + 1/4*(24*a*b^2*c^3*f + 12*b^3*c^2*d + (3*a^2*b*d^3 \\
& *f^2 + 3*a*b^2*d^3*f^2 + b^3*d^3*f^2)*x^4 + 4*(3*a^2*b*c*d^2*f^2 + b^3*c*d^ \\
& 2*f^2 + 3*(c*d^2*f^2 + 2*d^3*f)*a*b^2)*x^3 + 6*(3*a^2*b*c^2*d*f^2 + 3*(c^2* \\
& d*f^2 + 4*c*d^2*f)*a*b^2 + (c^2*d*f^2 + 2*d^3)*b^3)*x^2 + 4*(3*(c^3*f^2 + 6 \\
& *c^2*d*f)*a*b^2 + (c^3*f^2 + 6*c*d^2)*b^3)*x + ((3*a^2*b*d^3*f^2*e^(4*e) + \\
& 3*a*b^2*d^3*f^2*e^(4*e) + b^3*d^3*f^2*e^(4*e))*x^4 + 4*(3*a^2*b*c*d^2*f^2*e \\
& ^4 + 3*a*b^2*c*d^2*f^2*e^(4*e) + b^3*c*d^2*f^2*e^(4*e))*x^3 + 6*(3*a^2* \\
& b*c^2*d*f^2*e^(4*e) + 3*a*b^2*c^2*d*f^2*e^(4*e) + b^3*c^2*d*f^2*e^(4*e))*x^ \\
& 2 + 4*(3*a*b^2*c^3*f^2*e^(4*e) + b^3*c^3*f^2*e^(4*e))*x)*e^(4*f*x) - 2*(12*
\end{aligned}$$

$$\begin{aligned}
& a^2 b^2 c^3 f e^{(2e)} + (3 a^2 b d^3 f^2 e^{(2e)} + 3 a b^2 d^3 f^2 e^{(2e)} + \\
& b^3 d^3 f^2 e^{(2e)}) x^4 + 2 (2 c^3 f e^{(2e)} + 3 c^2 d e^{(2e)}) b^3 + 4 (3 \\
& a^2 b c d^2 f^2 e^{(2e)} + 3 (c d^2 f^2 e^{(2e)} + d^3 f e^{(2e)}) a b^2 + (c \\
& d^2 f^2 e^{(2e)} + d^3 f e^{(2e)}) b^3) x^3 + 6 (3 a^2 b c^2 d f^2 e^{(2e)} + \\
& 3 (c^2 d f^2 e^{(2e)} + 2 c d^2 f e^{(2e)}) a b^2 + (c^2 d f^2 e^{(2e)} + 2 c \\
& d^2 f e^{(2e)} + d^3 e^{(2e)}) b^3) x^2 + 4 (3 (c^3 f^2 e^{(2e)} + 3 c^2 d f e^{(2e)} \\
& e^{(2e)}) a b^2 + (c^3 f^2 e^{(2e)} + 3 c^2 d f e^{(2e)} + 3 c d^2 e^{(2e)}) b^3 \\
&) x) e^{(2 f x)} / (f^2 e^{(4 f x + 4 e)} - 2 f^2 e^{(2 f x + 2 e)} + f^2) - 2 (9 \\
& a b^2 c^2 d f + (c^3 f^2 + 3 c d^2) b^3) x / f^2 + (9 a b^2 c^2 d f + (c^3 f^2 + \\
& 3 c d^2) b^3) \log(e^{(f x + e)} + 1) / f^3 + (9 a b^2 c^2 d f + (c^3 f^2 + \\
& 3 c d^2) b^3) \log(e^{(f x + e)} - 1) / f^3 + (f^3 x^3 \log(e^{(f x + e)} + 1) + 3 \\
& f^2 x^2 \operatorname{dilog}(-e^{(f x + e)}) - 6 f x \operatorname{polylog}(3, -e^{(f x + e)}) + 6 \operatorname{polylog}(4 \\
& , -e^{(f x + e)})) (3 a^2 b d^3 + b^3 d^3) / f^4 + (f^3 x^3 \log(-e^{(f x + e)} + \\
& 1) + 3 f^2 x^2 \operatorname{dilog}(e^{(f x + e)}) - 6 f x \operatorname{polylog}(3, e^{(f x + e)}) + 6 \operatorname{polylog}(4 \\
& , e^{(f x + e)})) (3 a^2 b d^3 + b^3 d^3) / f^4 + 3 (3 a^2 b c d^2 f + b^3 c \\
& d^2 f + 3 a b^2 d^3) (f^2 x^2 \log(e^{(f x + e)} + 1) + 2 f x \operatorname{dilog}(-e^{(f x \\
& + e)}) - 2 \operatorname{polylog}(3, -e^{(f x + e)})) / f^4 + 3 (3 a^2 b c d^2 f + b^3 c d^2 f \\
& + 3 a b^2 d^3) (f^2 x^2 \log(-e^{(f x + e)} + 1) + 2 f x \operatorname{dilog}(e^{(f x + e)}) - \\
& 2 \operatorname{polylog}(3, e^{(f x + e)})) / f^4 + 3 (3 a^2 b c^2 d f^2 + 6 a b^2 c d^2 f + (c^2 d f^2 \\
& + d^3) b^3) (f x \log(e^{(f x + e)} + 1) + \operatorname{dilog}(-e^{(f x + e)})) / f^4 \\
& + 3 (3 a^2 b c^2 d f^2 + 6 a b^2 c d^2 f + (c^2 d f^2 + d^3) b^3) (f x \log(- \\
& e^{(f x + e)} + 1) + \operatorname{dilog}(e^{(f x + e)})) / f^4 - 1/2 ((3 a^2 b d^3 + b^3 d^3) \\
& f^4 x^4 + 4 (3 a^2 b c d^2 f + b^3 c d^2 f + 3 a b^2 d^3) f^3 x^3 + 6 (3 a^2 \\
& 2 b c^2 d f^2 + 6 a b^2 c d^2 f + (c^2 d f^2 + d^3) b^3) f^2 x^2) / f^4
\end{aligned}$$

Fricas [C] time = 4.17985, size = 23616, normalized size = 42.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*(a+b*coth(f*x+e))^3,x, algorithm="fricas")

[Out] $1/4 * ((a^3 - 3 a^2 b + 3 a b^2 - b^3) d^3 f^4 x^4 + 4 (a^3 - 3 a^2 b + 3 a b^2 - b^3) c d^2 f^4 x^3 + 6 (a^3 - 3 a^2 b + 3 a b^2 - b^3) c^2 d f^4 x^2 - 24 a b^2 d^3 e^3 + 4 (a^3 - 3 a^2 b + 3 a b^2 - b^3) c^3 f^4 x + 12 b^3 d^3 e^2 + 2 (3 a^2 b + b^3) d^3 e^4 + ((a^3 - 3 a^2 b + 3 a b^2 - b^3) d^3 f^4 x^4 - 24 a b^2 d^3 e^3 + 12 b^3 d^3 e^2 + 2 (3 a^2 b + b^3) d^3 e^4 - 8 (3 a^2 b + b^3) c^3 e f^3 - 4 (6 a b^2 d^3 f^3 - (a^3 - 3 a^2 b + 3 a b^2 - b^3) c d^2 f^4) x^3 - 12 (6 a b^2 c^2 d e - (3 a^2 b + b^3) c^2 d e^2) f^2 - 6 (12 a b^2 c d^2 f^3 + 2 b^3 d^3 f^2 - (a^3 - 3 a^2 b + 3 a b^2 - b^3) c^2 d f^4) x^2 + 8 (9 a b^2 c d^2 e^2 - 3 b^3 c d^2 e - (3 a^2 b + b^3) c d^2$

$$\begin{aligned}
& 2e^3 * f - 4 * (18 * a * b^2 * c^2 * d * f^3 + 6 * b^3 * c * d^2 * f^2 - (a^3 - 3 * a^2 * b + 3 * a * b^2 - b^3) * c^3 * f^4) * x) * \cosh(f * x + e)^4 + 4 * ((a^3 - 3 * a^2 * b + 3 * a * b^2 - b^3) * d^3 * f^4 * x^4 - 24 * a * b^2 * d^3 * e^3 + 12 * b^3 * d^3 * e^2 + 2 * (3 * a^2 * b + b^3) * d^3 * e^4 - 8 * (3 * a^2 * b + b^3) * c^3 * e * f^3 - 4 * (6 * a * b^2 * d^3 * f^3 - (a^3 - 3 * a^2 * b + 3 * a * b^2 - b^3) * c * d^2 * f^4) * x^3 - 12 * (6 * a * b^2 * c^2 * d * e - (3 * a^2 * b + b^3) * c^2 * d * e^2) * f^2 - 6 * (12 * a * b^2 * c * d^2 * f^3 + 2 * b^3 * d^3 * f^2 - (a^3 - 3 * a^2 * b + 3 * a * b^2 - b^3) * c^2 * d * f^4) * x^2 + 8 * (9 * a * b^2 * c * d^2 * e^2 - 3 * b^3 * c * d^2 * e - (3 * a^2 * b + b^3) * c * d^2 * e^3) * f - 4 * (18 * a * b^2 * c^2 * d * f^3 + 6 * b^3 * c * d^2 * f^2 - (a^3 - 3 * a^2 * b + 3 * a * b^2 - b^3) * c^3 * f^4) * x) * \cosh(f * x + e) * \sinh(f * x + e)^3 + ((a^3 - 3 * a^2 * b + 3 * a * b^2 - b^3) * d^3 * f^4 * x^4 - 24 * a * b^2 * d^3 * e^3 + 12 * b^3 * d^3 * e^2 + 2 * (3 * a^2 * b + b^3) * d^3 * e^4 - 8 * (3 * a^2 * b + b^3) * c^3 * e * f^3 - 4 * (6 * a * b^2 * d^3 * f^3 - (a^3 - 3 * a^2 * b + 3 * a * b^2 - b^3) * c * d^2 * f^4) * x^3 - 12 * (6 * a * b^2 * c^2 * d * e - (3 * a^2 * b + b^3) * c^2 * d * e^2) * f^2 - 6 * (12 * a * b^2 * c * d^2 * f^3 + 2 * b^3 * d^3 * f^2 - (a^3 - 3 * a^2 * b + 3 * a * b^2 - b^3) * c^2 * d * f^4) * x^2 + 8 * (9 * a * b^2 * c * d^2 * e^2 - 3 * b^3 * c * d^2 * e - (3 * a^2 * b + b^3) * c * d^2 * e^3) * f - 4 * (18 * a * b^2 * c^2 * d * f^3 + 6 * b^3 * c * d^2 * f^2 - (a^3 - 3 * a^2 * b + 3 * a * b^2 - b^3) * c^3 * f^4) * x) * \sinh(f * x + e)^4 + 8 * (3 * a * b^2 * c^3 - (3 * a^2 * b + b^3) * c^3 * e) * f^3 - 12 * (6 * a * b^2 * c^2 * d * e - b^3 * c^2 * d - (3 * a^2 * b + b^3) * c^2 * d * e^2) * f^2 - 2 * ((a^3 - 3 * a^2 * b + 3 * a * b^2 - b^3) * d^3 * f^4 * x^4 - 24 * a * b^2 * d^3 * e^3 + 12 * b^3 * d^3 * e^2 + 2 * (3 * a^2 * b + b^3) * d^3 * e^4 - 4 * (2 * (3 * a^2 * b + b^3) * c^3 * e - (3 * a * b^2 + b^3) * c^3) * f^3 + 4 * ((a^3 - 3 * a^2 * b + 3 * a * b^2 - b^3) * c * d^2 * f^4 - (3 * a * b^2 - b^3) * d^3 * f^3) * x^3 - 6 * (12 * a * b^2 * c^2 * d * e - b^3 * c^2 * d - 2 * (3 * a^2 * b + b^3) * c^2 * d * e^2) * f^2 - 6 * (b^3 * d^3 * f^2 - (a^3 - 3 * a^2 * b + 3 * a * b^2 - b^3) * c^2 * d * f^4 + 2 * (3 * a * b^2 - b^3) * c * d^2 * f^3) * x^2 + 8 * (9 * a * b^2 * c * d^2 * e^2 - 3 * b^3 * c * d^2 * e - (3 * a^2 * b + b^3) * c * d^2 * e^3) * f - 4 * (3 * b^3 * c * d^2 * f^2 - (a^3 - 3 * a^2 * b + 3 * a * b^2 - b^3) * c^3 * f^4 + 3 * (3 * a * b^2 - b^3) * c^2 * d * f^3) * x) * \cosh(f * x + e)^2 - 2 * ((a^3 - 3 * a^2 * b + 3 * a * b^2 - b^3) * d^3 * f^4 * x^4 - 24 * a * b^2 * d^3 * e^3 + 12 * b^3 * d^3 * e^2 + 2 * (3 * a^2 * b + b^3) * d^3 * e^4 - 4 * (2 * (3 * a^2 * b + b^3) * c^3 * e - (3 * a * b^2 + b^3) * c^3) * f^3 + 4 * ((a^3 - 3 * a^2 * b + 3 * a * b^2 - b^3) * c * d^2 * f^4 - (3 * a * b^2 - b^3) * d^3 * f^3) * x^3 - 6 * (12 * a * b^2 * c^2 * d * e - b^3 * c^2 * d - 2 * (3 * a^2 * b + b^3) * c^2 * d * e^2) * f^2 - 6 * (b^3 * d^3 * f^2 - (a^3 - 3 * a^2 * b + 3 * a * b^2 - b^3) * c^2 * d * f^4 + 2 * (3 * a * b^2 - b^3) * c * d^2 * f^3) * x^2 - 3 * ((a^3 - 3 * a^2 * b + 3 * a * b^2 - b^3) * d^3 * f^4 * x^4 - 24 * a * b^2 * d^3 * e^3 + 12 * b^3 * d^3 * e^2 + 2 * (3 * a^2 * b + b^3) * d^3 * e^4 - 8 * (3 * a^2 * b + b^3) * c^3 * e * f^3 - 4 * (6 * a * b^2 * d^3 * f^3 - (a^3 - 3 * a^2 * b + 3 * a * b^2 - b^3) * c * d^2 * f^4) * x^3 - 12 * (6 * a * b^2 * c^2 * d * e - (3 * a^2 * b + b^3) * c^2 * d * e^2) * f^2 - 6 * (12 * a * b^2 * c * d^2 * f^3 + 2 * b^3 * d^3 * f^2 - (a^3 - 3 * a^2 * b + 3 * a * b^2 - b^3) * c^2 * d * f^4) * x^2 + 8 * (9 * a * b^2 * c * d^2 * e^2 - 3 * b^3 * c * d^2 * e - (3 * a^2 * b + b^3) * c * d^2 * e^3) * f - 4 * (18 * a * b^2 * c^2 * d * f^3 + 6 * b^3 * c * d^2 * f^2 - (a^3 - 3 * a^2 * b + 3 * a * b^2 - b^3) * c^3 * f^4) * x) * \cosh(f * x + e)^2 + 8 * (9 * a * b^2 * c * d^2 * e^2 - 3 * b^3 * c * d^2 * e - (3 * a^2 * b + b^3) * c * d^2 * e^3) * f - 4 * (3 * b^3 * c * d^2 * f^2 - (a^3 - 3 * a^2 * b + 3 * a * b^2 - b^3) * c^3 * f^4 + 3 * (3 * a * b^2 - b^3) * c^2 * d * f^3) * x) * \sinh(f * x + e)^2 + 8 * (9 * a * b^2 * c * d^2 * e^2 - 3 * b^3 * c * d^2 * e - (3 * a^2 * b + b^3) * c * d^2 * e^3) * f + 12 * ((3 * a^2 * b + b^3) * d^3 * f^2 * x^2 + 6 * a * b^2 * c * d^2 * f + b^3 * d^3 + (3 * a^2 * b + b^3) * c^2 * d * f^2 + ((3 * a^2 * b + b^3) * d^3 * f^2 * x^2 + 6 * a * b^2 * c * d^2 * f + b^3 * d^3 + (3 * a^2 * b + b^3) * c^2 * d * f^2 + 2 * (3 * a * b^2 * d^3 * f + (3 * a^2 * b + b^3) * c * d^2 * f^2) * x) * \cosh(f * x + e)^4 + 4 * ((3 * a^2 * b + b^3) * d^3 * f^2 * x^2 + 6 * a * b
\end{aligned}$$

$$\begin{aligned}
&^2*c*d^2*f + b^3*d^3 + (3*a^2*b + b^3)*c^2*d*f^2 + 2*(3*a*b^2*d^3*f + (3*a^2*b + b^3)*c*d^2*f^2)*x)*\cosh(f*x + e)*\sinh(f*x + e)^3 + ((3*a^2*b + b^3)*d^3*f^2*x^2 + 6*a*b^2*c*d^2*f + b^3*d^3 + (3*a^2*b + b^3)*c^2*d*f^2 + 2*(3*a*b^2*d^3*f + (3*a^2*b + b^3)*c*d^2*f^2)*x)*\sinh(f*x + e)^4 - 2*((3*a^2*b + b^3)*d^3*f^2*x^2 + 6*a*b^2*c*d^2*f + b^3*d^3 + (3*a^2*b + b^3)*c^2*d*f^2 + 2*(3*a*b^2*d^3*f + (3*a^2*b + b^3)*c*d^2*f^2)*x)*\cosh(f*x + e)^2 - 2*((3*a^2*b + b^3)*d^3*f^2*x^2 + 6*a*b^2*c*d^2*f + b^3*d^3 + (3*a^2*b + b^3)*c^2*d*f^2 - 3*((3*a^2*b + b^3)*d^3*f^2*x^2 + 6*a*b^2*c*d^2*f + b^3*d^3 + (3*a^2*b + b^3)*c^2*d*f^2 + 2*(3*a*b^2*d^3*f + (3*a^2*b + b^3)*c*d^2*f^2)*x)*\cosh(f*x + e)^2 + 2*(3*a*b^2*d^3*f + (3*a^2*b + b^3)*c*d^2*f^2)*x)*\sinh(f*x + e)^2 + 2*(3*a*b^2*d^3*f + (3*a^2*b + b^3)*c*d^2*f^2)*x + 4*((3*a^2*b + b^3)*d^3*f^2*x^2 + 6*a*b^2*c*d^2*f + b^3*d^3 + (3*a^2*b + b^3)*c^2*d*f^2 + 2*(3*a*b^2*d^3*f + (3*a^2*b + b^3)*c*d^2*f^2)*x)*\cosh(f*x + e)^3 - ((3*a^2*b + b^3)*d^3*f^2*x^2 + 6*a*b^2*c*d^2*f + b^3*d^3 + (3*a^2*b + b^3)*c^2*d*f^2 + 2*(3*a*b^2*d^3*f + (3*a^2*b + b^3)*c*d^2*f^2)*x)*\cosh(f*x + e))*\sinh(f*x + e)*\operatorname{dilog}(\cosh(f*x + e) + \sinh(f*x + e)) + 12*((3*a^2*b + b^3)*d^3*f^2*x^2 + 6*a*b^2*c*d^2*f + b^3*d^3 + (3*a^2*b + b^3)*c^2*d*f^2 + ((3*a^2*b + b^3)*d^3*f^2*x^2 + 6*a*b^2*c*d^2*f + b^3*d^3 + (3*a^2*b + b^3)*c^2*d*f^2 + 2*(3*a*b^2*d^3*f + (3*a^2*b + b^3)*c*d^2*f^2)*x)*\cosh(f*x + e)^4 + 4*((3*a^2*b + b^3)*d^3*f^2*x^2 + 6*a*b^2*c*d^2*f + b^3*d^3 + (3*a^2*b + b^3)*c^2*d*f^2 + 2*(3*a*b^2*d^3*f + (3*a^2*b + b^3)*c*d^2*f^2)*x)*\cosh(f*x + e)*\sinh(f*x + e)^3 + ((3*a^2*b + b^3)*d^3*f^2*x^2 + 6*a*b^2*c*d^2*f + b^3*d^3 + (3*a^2*b + b^3)*c^2*d*f^2 + 2*(3*a*b^2*d^3*f + (3*a^2*b + b^3)*c*d^2*f^2)*x)*\sinh(f*x + e)^4 - 2*((3*a^2*b + b^3)*d^3*f^2*x^2 + 6*a*b^2*c*d^2*f + b^3*d^3 + (3*a^2*b + b^3)*c^2*d*f^2 + 2*(3*a*b^2*d^3*f + (3*a^2*b + b^3)*c*d^2*f^2)*x)*\cosh(f*x + e)^2 - 2*((3*a^2*b + b^3)*d^3*f^2*x^2 + 6*a*b^2*c*d^2*f + b^3*d^3 + (3*a^2*b + b^3)*c^2*d*f^2 - 3*((3*a^2*b + b^3)*d^3*f^2*x^2 + 6*a*b^2*c*d^2*f + b^3*d^3 + (3*a^2*b + b^3)*c^2*d*f^2 + 2*(3*a*b^2*d^3*f + (3*a^2*b + b^3)*c*d^2*f^2)*x)*\cosh(f*x + e)^2 + 2*(3*a*b^2*d^3*f + (3*a^2*b + b^3)*c*d^2*f^2)*x)*\sinh(f*x + e)^2 + 2*(3*a*b^2*d^3*f + (3*a^2*b + b^3)*c*d^2*f^2)*x + 4*((3*a^2*b + b^3)*d^3*f^2*x^2 + 6*a*b^2*c*d^2*f + b^3*d^3 + (3*a^2*b + b^3)*c^2*d*f^2 + 2*(3*a*b^2*d^3*f + (3*a^2*b + b^3)*c*d^2*f^2)*x)*\cosh(f*x + e)^3 - ((3*a^2*b + b^3)*d^3*f^2*x^2 + 6*a*b^2*c*d^2*f + b^3*d^3 + (3*a^2*b + b^3)*c^2*d*f^2 + 2*(3*a*b^2*d^3*f + (3*a^2*b + b^3)*c*d^2*f^2)*x)*\cosh(f*x + e))*\sinh(f*x + e)*\operatorname{dilog}(-\cosh(f*x + e) - \sinh(f*x + e)) + 4*((3*a^2*b + b^3)*d^3*f^3*x^3 + 9*a*b^2*c^2*d*f^2 + 3*b^3*c*d^2*f + (3*a^2*b + b^3)*c^3*f^3 + ((3*a^2*b + b^3)*d^3*f^3*x^3 + 9*a*b^2*c^2*d*f^2 + 3*b^3*c*d^2*f + (3*a^2*b + b^3)*c^3*f^3 + 3*(3*a*b^2*d^3*f^2 + (3*a^2*b + b^3)*c*d^2*f^3)*x^2 + 3*(6*a*b^2*c*d^2*f^2 + b^3*d^3*f + (3*a^2*b + b^3)*c^2*d*f^3)*x)*\cosh(f*x + e)^4 + 4*((3*a^2*b + b^3)*d^3*f^3*x^3 + 9*a*b^2*c^2*d*f^2 + 3*b^3*c*d^2*f + (3*a^2*b + b^3)*c^3*f^3 + 3*(3*a*b^2*d^3*f^2 + (3*a^2*b + b^3)*c*d^2*f^3)*x^2 + 3*(6*a*b^2*c*d^2*f^2 + b^3*d^3*f + (3*a^2*b + b^3)*c^2*d*f^3)*x)*\cosh(f*x + e)*\sinh(f*x + e)^3 + ((3*a^2*b + b^3)*d^3*f^3*x^3 + 9*a*b^2*c^2*d*f^2 + 3*b^3*c*d^2*f + (3*a^2*b + b^3)*c^3*f^3 + 3*(3*a*b^2*d^3*f^2 + (3*a^2*b + b^3)*c*d^2*f^3)*x^2 + 3*(6*a*b^2*c*d^2*f^2 + b^3*d^3*f + (3*a^2*b
\end{aligned}$$

$$\begin{aligned}
& 1) + 4*((3*a^2*b + b^3)*d^3*f^3*x^3 - 9*a*b^2*d^3*e^2 + 3*b^3*d^3*e + (3*a^2*b + b^3)*d^3*e^3 + 3*(3*a^2*b + b^3)*c^2*d*e*f^2 + ((3*a^2*b + b^3)*d^3*f^3*x^3 - 9*a*b^2*d^3*e^2 + 3*b^3*d^3*e + (3*a^2*b + b^3)*d^3*e^3 + 3*(3*a^2*b + b^3)*c^2*d*e*f^2 + 3*(3*a*b^2*d^3*f^2 + (3*a^2*b + b^3)*c*d^2*f^3)*x^2 + 3*(6*a*b^2*c*d^2*e - (3*a^2*b + b^3)*c*d^2*e^2)*f + 3*(6*a*b^2*c*d^2*f^2 + b^3*d^3*f + (3*a^2*b + b^3)*c^2*d*f^3)*x)*cosh(f*x + e)^4 + 4*((3*a^2*b + b^3)*d^3*f^3*x^3 - 9*a*b^2*d^3*e^2 + 3*b^3*d^3*e + (3*a^2*b + b^3)*d^3*e^3 + 3*(3*a^2*b + b^3)*c^2*d*e*f^2 + 3*(3*a*b^2*d^3*f^2 + (3*a^2*b + b^3)*c*d^2*f^3)*x^2 + 3*(6*a*b^2*c*d^2*e - (3*a^2*b + b^3)*c*d^2*e^2)*f + 3*(6*a*b^2*c*d^2*f^2 + b^3*d^3*f + (3*a^2*b + b^3)*c^2*d*f^3)*x)*cosh(f*x + e)*sinh(f*x + e)^3 + ((3*a^2*b + b^3)*d^3*f^3*x^3 - 9*a*b^2*d^3*e^2 + 3*b^3*d^3*e + (3*a^2*b + b^3)*d^3*e^3 + 3*(3*a^2*b + b^3)*c^2*d*e*f^2 + 3*(3*a*b^2*d^3*f^2 + (3*a^2*b + b^3)*c*d^2*f^3)*x^2 + 3*(6*a*b^2*c*d^2*e - (3*a^2*b + b^3)*c*d^2*e^2)*f + 3*(6*a*b^2*c*d^2*f^2 + b^3*d^3*f + (3*a^2*b + b^3)*c^2*d*f^3)*x)*sinh(f*x + e)^4 + 3*(3*a*b^2*d^3*f^2 + (3*a^2*b + b^3)*c*d^2*f^3)*x^2 - 2*((3*a^2*b + b^3)*d^3*f^3*x^3 - 9*a*b^2*d^3*e^2 + 3*b^3*d^3*e + (3*a^2*b + b^3)*d^3*e^3 + 3*(3*a^2*b + b^3)*c^2*d*e*f^2 + 3*(3*a*b^2*d^3*f^2 + (3*a^2*b + b^3)*c*d^2*f^3)*x^2 + 3*(6*a*b^2*c*d^2*e - (3*a^2*b + b^3)*c*d^2*e^2)*f + 3*(6*a*b^2*c*d^2*f^2 + b^3*d^3*f + (3*a^2*b + b^3)*c^2*d*f^3)*x)*cosh(f*x + e)^2 - 2*((3*a^2*b + b^3)*d^3*f^3*x^3 - 9*a*b^2*d^3*e^2 + 3*b^3*d^3*e + (3*a^2*b + b^3)*d^3*e^3 + 3*(3*a^2*b + b^3)*c^2*d*e*f^2 + 3*(3*a*b^2*d^3*f^2 + (3*a^2*b + b^3)*c*d^2*f^3)*x^2 - 3*((3*a^2*b + b^3)*d^3*f^3*x^3 - 9*a*b^2*d^3*e^2 + 3*b^3*d^3*e + (3*a^2*b + b^3)*d^3*e^3 + 3*(3*a^2*b + b^3)*c^2*d*e*f^2 + 3*(3*a*b^2*d^3*f^2 + (3*a^2*b + b^3)*c*d^2*f^3)*x^2 + 3*(6*a*b^2*c*d^2*e - (3*a^2*b + b^3)*c*d^2*e^2)*f + 3*(6*a*b^2*c*d^2*f^2 + b^3*d^3*f + (3*a^2*b + b^3)*c^2*d*f^3)*x)*cosh(f*x + e)^2 + 3*(6*a*b^2*c*d^2*e - (3*a^2*b + b^3)*c*d^2*e^2)*f + 3*(6*a*b^2*c*d^2*f^2 + b^3*d^3*f + (3*a^2*b + b^3)*c^2*d*f^3)*x)*sinh(f*x + e)^2 + 3*(6*a*b^2*c*d^2*e - (3*a^2*b + b^3)*c*d^2*e^2)*f + 3*(6*a*b^2*c*d^2*f^2 + b^3*d^3*f + (3*a^2*b + b^3)*c^2*d*f^3)*x + 4*(((3*a^2*b + b^3)*d^3*f^3*x^3 - 9*a*b^2*d^3*e^2 + 3*b^3*d^3*e + (3*a^2*b + b^3)*d^3*e^3 + 3*(3*a^2*b + b^3)*c^2*d*e*f^2 + 3*(3*a*b^2*d^3*f^2 + (3*a^2*b + b^3)*c*d^2*f^3)*x^2 + 3*(6*a*b^2*c*d^2*e - (3*a^2*b + b^3)*c*d^2*e^2)*f + 3*(6*a*b^2*c*d^2*f^2 + b^3*d^3*f + (3*a^2*b + b^3)*c^2*d*f^3)*x)*cosh(f*x + e)^3 - ((3*a^2*b + b^3)*d^3*f^3*x^3 - 9*a*b^2*d^3*e^2 + 3*b^3*d^3*e + (3*a^2*b + b^3)*d^3*e^3 + 3*(3*a^2*b + b^3)*c^2*d*e*f^2 + 3*(3*a*b^2*d^3*f^2 + (3*a^2*b + b^3)*c*d^2*f^3)*x^2 + 3*(6*a*b^2*c*d^2*e - (3*a^2*b + b^3)*c*d^2*e^2)*f + 3*(6*a*b^2*c*d^2*f^2 + b^3*d^3*f + (3*a^2*b + b^3)*c^2*d*f^3)*x)*cosh(f*x + e))*sinh(f*x + e))*log(-cosh(f*x + e) - sinh(f*x + e) + 1) + 24*((3*a^2*b + b^3)*d^3*cosh(f*x + e)^4 + 4*(3*a^2*b + b^3)*d^3*cosh(f*x + e)*sinh(f*x + e)^3 + (3*a^2*b + b^3)*d^3*sinh(f*x + e)^4 - 2*(3*a^2*b + b^3)*d^3*cosh(f*x + e)^2 + (3*a^2*b + b^3)*d^3 + 2*(3*(3*a^2*b + b^3)*d^3*cosh(f*x + e)^2 - (3*a^2*b + b^3)*d^3)*sinh(f*x + e)^2 + 4*((3*a^2*b + b^3)*d^3*cosh(f*x + e)^3 - (3*a^2*b + b^3)*d^3*cosh(f*x + e))*sinh(f*x + e))*polylog(4, cosh(f*x + e) + sinh(f*x + e)) + 24*((3*a^2*b + b^3)*d^3*cosh(f*x + e)^4 + 4*(3*a^2*b + b^3)*d^3*cosh(f*x + e)*sinh(f*x + e)^3 + (3*a^2*b
\end{aligned}$$

$$\begin{aligned}
& b + b^3)d^3\sinh(f*x + e)^4 - 2*(3*a^2*b + b^3)d^3\cosh(f*x + e)^2 + (3*a^2*b + b^3)d^3 + 2*(3*(3*a^2*b + b^3)d^3\cosh(f*x + e)^2 - (3*a^2*b + b^3)d^3)\sinh(f*x + e)^2 + 4*((3*a^2*b + b^3)d^3\cosh(f*x + e)^3 - (3*a^2*b + b^3)d^3\cosh(f*x + e))\sinh(f*x + e))\text{polylog}(4, -\cosh(f*x + e) - \sinh(f*x + e)) - 24*(3*a*b^2*d^3 + (3*a^2*b + b^3)d^3*f*x + (3*a^2*b + b^3)*c*d^2*f + (3*a*b^2*d^3 + (3*a^2*b + b^3)d^3*f*x + (3*a^2*b + b^3)*c*d^2*f)\cosh(f*x + e)^4 + 4*(3*a*b^2*d^3 + (3*a^2*b + b^3)d^3*f*x + (3*a^2*b + b^3)*c*d^2*f)\cosh(f*x + e)\sinh(f*x + e)^3 + (3*a*b^2*d^3 + (3*a^2*b + b^3)d^3*f*x + (3*a^2*b + b^3)*c*d^2*f)\sinh(f*x + e)^4 - 2*(3*a*b^2*d^3 + (3*a^2*b + b^3)d^3*f*x + (3*a^2*b + b^3)*c*d^2*f)\cosh(f*x + e)^2 - 2*(3*a*b^2*d^3 + (3*a^2*b + b^3)d^3*f*x + (3*a^2*b + b^3)*c*d^2*f - 3*(3*a*b^2*d^3 + (3*a^2*b + b^3)d^3*f*x + (3*a^2*b + b^3)*c*d^2*f)\cosh(f*x + e)^2)\sinh(f*x + e)^2 + 4*((3*a*b^2*d^3 + (3*a^2*b + b^3)d^3*f*x + (3*a^2*b + b^3)*c*d^2*f)\cosh(f*x + e)^3 - (3*a*b^2*d^3 + (3*a^2*b + b^3)d^3*f*x + (3*a^2*b + b^3)*c*d^2*f)\cosh(f*x + e))\sinh(f*x + e))\text{polylog}(3, \cosh(f*x + e) + \sinh(f*x + e)) - 24*(3*a*b^2*d^3 + (3*a^2*b + b^3)d^3*f*x + (3*a^2*b + b^3)*c*d^2*f + (3*a*b^2*d^3 + (3*a^2*b + b^3)d^3*f*x + (3*a^2*b + b^3)*c*d^2*f)\cosh(f*x + e)^4 + 4*(3*a*b^2*d^3 + (3*a^2*b + b^3)d^3*f*x + (3*a^2*b + b^3)*c*d^2*f)\cosh(f*x + e)\sinh(f*x + e)^3 + (3*a*b^2*d^3 + (3*a^2*b + b^3)d^3*f*x + (3*a^2*b + b^3)*c*d^2*f)\sinh(f*x + e)^4 - 2*(3*a*b^2*d^3 + (3*a^2*b + b^3)d^3*f*x + (3*a^2*b + b^3)*c*d^2*f)\cosh(f*x + e)^2 - 2*(3*a*b^2*d^3 + (3*a^2*b + b^3)d^3*f*x + (3*a^2*b + b^3)*c*d^2*f - 3*(3*a*b^2*d^3 + (3*a^2*b + b^3)d^3*f*x + (3*a^2*b + b^3)*c*d^2*f)\cosh(f*x + e)^2)\sinh(f*x + e)^2 + 4*((3*a*b^2*d^3 + (3*a^2*b + b^3)d^3*f*x + (3*a^2*b + b^3)*c*d^2*f)\cosh(f*x + e)^3 - (3*a*b^2*d^3 + (3*a^2*b + b^3)d^3*f*x + (3*a^2*b + b^3)*c*d^2*f)\cosh(f*x + e))\sinh(f*x + e))\text{polylog}(3, -\cosh(f*x + e) - \sinh(f*x + e)) + 4*(((a^3 - 3*a^2*b + 3*a*b^2 - b^3)d^3*f^4*x^4 - 24*a*b^2*d^3*e^3 + 12*b^3*d^3*e^2 + 2*(3*a^2*b + b^3)d^3*e^4 - 8*(3*a^2*b + b^3)*c^3*e*f^3 - 4*(6*a*b^2*d^3*f^3 - (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*c*d^2*f^4)*x^3 - 12*(6*a*b^2*c^2*d*e - (3*a^2*b + b^3)*c^2*d*e^2)*f^2 - 6*(12*a*b^2*c*d^2*f^3 + 2*b^3*d^3*f^2 - (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*c^2*d*f^4)*x^2 + 8*(9*a*b^2*c*d^2*e^2 - 3*b^3*c*d^2*e - (3*a^2*b + b^3)*c*d^2*e^3)*f - 4*(18*a*b^2*c^2*d*f^3 + 6*b^3*c*d^2*f^2 - (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*c^3*f^4)*x)\cosh(f*x + e)^3 - ((a^3 - 3*a^2*b + 3*a*b^2 - b^3)d^3*f^4*x^4 - 24*a*b^2*d^3*e^3 + 12*b^3*d^3*e^2 + 2*(3*a^2*b + b^3)d^3*e^4 - 4*(2*(3*a^2*b + b^3)*c^3*e - (3*a*b^2 + b^3)*c^3)*f^3 + 4*((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*c*d^2*f^4 - (3*a*b^2 - b^3)d^3*f^3)*x^3 - 6*(12*a*b^2*c^2*d*e - b^3*c^2*d - 2*(3*a^2*b + b^3)*c^2*d*e^2)*f^2 - 6*(b^3*d^3*f^2 - (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*c^2*d*f^4 + 2*(3*a*b^2 - b^3)*c*d^2*f^3)*x^2 + 8*(9*a*b^2*c*d^2*e^2 - 3*b^3*c*d^2*e - (3*a^2*b + b^3)*c*d^2*e^3)*f - 4*(3*b^3*c*d^2*f^2 - (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*c^3*f^4 + 3*(3*a*b^2 - b^3)*c^2*d*f^3)*x)\cosh(f*x + e))\sinh(f*x + e))/(f^4\cosh(f*x + e)^4 + 4*f^4\cosh(f*x + e)\sinh(f*x + e)^3 + f^4\sinh(f*x + e)^4 - 2*f^4\cosh(f*x + e)^2 + f^4 + 2*(3*f^4\cosh(f*x + e)^2 - f^4)\sinh(f*x + e)^2 + 4*(f^4\cosh(f*x + e)^3 - f^4\cosh(f*x + e))\sinh(f*x + e))
\end{aligned}$$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b \coth(e + fx))^3 (c + dx)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**3*(a+b*coth(f*x+e))**3,x)

[Out] Integral((a + b*coth(e + f*x))**3*(c + d*x)**3, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^3 (b \coth(fx + e) + a)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*(a+b*coth(f*x+e))^3,x, algorithm="giac")

[Out] integrate((d*x + c)^3*(b*coth(f*x + e) + a)^3, x)

3.48 $\int (c + dx)^2 (a + b \coth(e + fx))^3 dx$

Optimal. Leaf size=401

$$\frac{3a^2bd(c + dx)\text{PolyLog}(2, e^{2(e+fx)})}{f^2} - \frac{3a^2bd^2\text{PolyLog}(3, e^{2(e+fx)})}{2f^3} + \frac{3ab^2d^2\text{PolyLog}(2, e^{2(e+fx)})}{f^3} + \frac{b^3d(c + dx)\text{PolyLog}(2, e^{2(e+fx)})}{f^2}$$

```
[Out] (b^3*c*d*x)/f + (b^3*d^2*x^2)/(2*f) - (3*a*b^2*(c + d*x)^2)/f + (a^3*(c + d
*x)^3)/(3*d) - (a^2*b*(c + d*x)^3)/d + (a*b^2*(c + d*x)^3)/d - (b^3*(c + d*
*x)^3)/(3*d) - (b^3*d*(c + d*x)*Coth[e + f*x])/f^2 - (3*a*b^2*(c + d*x)^2*Co
th[e + f*x])/f - (b^3*(c + d*x)^2*Coth[e + f*x]^2)/(2*f) + (6*a*b^2*d*(c +
d*x)*Log[1 - E^(2*(e + f*x))])/f^2 + (3*a^2*b*(c + d*x)^2*Log[1 - E^(2*(e +
f*x))])/f + (b^3*(c + d*x)^2*Log[1 - E^(2*(e + f*x))])/f + (b^3*d^2*Log[Si
nh[e + f*x]])/f^3 + (3*a*b^2*d^2*PolyLog[2, E^(2*(e + f*x))])/f^3 + (3*a^2*
b*d*(c + d*x)*PolyLog[2, E^(2*(e + f*x))])/f^2 + (b^3*d*(c + d*x)*PolyLog[2
, E^(2*(e + f*x))])/f^2 - (3*a^2*b*d^2*PolyLog[3, E^(2*(e + f*x))])/(2*f^3)
- (b^3*d^2*PolyLog[3, E^(2*(e + f*x))])/(2*f^3)
```

Rubi [A] time = 0.697871, antiderivative size = 401, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 11, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.55$, Rules used = {3722, 3716, 2190, 2531, 2282, 6589, 3720, 2279, 2391, 32, 3475}

$$\frac{3a^2bd(c + dx)\text{PolyLog}(2, e^{2(e+fx)})}{f^2} - \frac{3a^2bd^2\text{PolyLog}(3, e^{2(e+fx)})}{2f^3} + \frac{3ab^2d^2\text{PolyLog}(2, e^{2(e+fx)})}{f^3} + \frac{b^3d(c + dx)\text{PolyLog}(2, e^{2(e+fx)})}{f^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*(a + b*Coth[e + f*x])^3,x]
```

```
[Out] (b^3*c*d*x)/f + (b^3*d^2*x^2)/(2*f) - (3*a*b^2*(c + d*x)^2)/f + (a^3*(c + d
*x)^3)/(3*d) - (a^2*b*(c + d*x)^3)/d + (a*b^2*(c + d*x)^3)/d - (b^3*(c + d*
*x)^3)/(3*d) - (b^3*d*(c + d*x)*Coth[e + f*x])/f^2 - (3*a*b^2*(c + d*x)^2*Co
th[e + f*x])/f - (b^3*(c + d*x)^2*Coth[e + f*x]^2)/(2*f) + (6*a*b^2*d*(c +
d*x)*Log[1 - E^(2*(e + f*x))])/f^2 + (3*a^2*b*(c + d*x)^2*Log[1 - E^(2*(e +
f*x))])/f + (b^3*(c + d*x)^2*Log[1 - E^(2*(e + f*x))])/f + (b^3*d^2*Log[Si
nh[e + f*x]])/f^3 + (3*a*b^2*d^2*PolyLog[2, E^(2*(e + f*x))])/f^3 + (3*a^2*
b*d*(c + d*x)*PolyLog[2, E^(2*(e + f*x))])/f^2 + (b^3*d*(c + d*x)*PolyLog[2
, E^(2*(e + f*x))])/f^2 - (3*a^2*b*d^2*PolyLog[3, E^(2*(e + f*x))])/(2*f^3)
- (b^3*d^2*PolyLog[3, E^(2*(e + f*x))])/(2*f^3)
```

Rule 3722

```
Int[((c_.) + (d_.)*(x_)^(m_.))*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, (a + b*Tan[e + f*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_)^(m_.))*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] := -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-I*
e) + f*fz*x)))/E^(2*I*k*Pi)), x], x] /; FreeQ[{c, d, e, f, fz}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))*((c_.) + (d_.)*(x_)^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a]]/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_)^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_)^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol]
:> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Dist[
(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_)))]^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))
^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2,
-(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int (c + dx)^2 (a + b \coth(e + fx))^3 dx &= \int (a^3(c + dx)^2 + 3a^2b(c + dx)^2 \coth(e + fx) + 3ab^2(c + dx)^2 \coth^2(e + fx) + b^3(c + dx)^2 \coth^3(e + fx)) dx \\
&= \frac{a^3(c + dx)^3}{3d} + (3a^2b) \int (c + dx)^2 \coth(e + fx) dx + (3ab^2) \int (c + dx)^2 \coth^2(e + fx) dx + b^3 \int (c + dx)^2 \coth^3(e + fx) dx \\
&= \frac{a^3(c + dx)^3}{3d} - \frac{a^2b(c + dx)^3}{d} - \frac{3ab^2(c + dx)^2 \coth(e + fx)}{f} - \frac{b^3(c + dx)^2 \coth^2(e + fx)}{2f} \\
&= -\frac{3ab^2(c + dx)^2}{f} + \frac{a^3(c + dx)^3}{3d} - \frac{a^2b(c + dx)^3}{d} + \frac{ab^2(c + dx)^3}{d} - \frac{b^3(c + dx)^3}{3d} - \frac{b^3d}{3d} \\
&= \frac{b^3cdx}{f} + \frac{b^3d^2x^2}{2f} - \frac{3ab^2(c + dx)^2}{f} + \frac{a^3(c + dx)^3}{3d} - \frac{a^2b(c + dx)^3}{d} + \frac{ab^2(c + dx)^3}{d} - \frac{b^3d}{3d} \\
&= \frac{b^3cdx}{f} + \frac{b^3d^2x^2}{2f} - \frac{3ab^2(c + dx)^2}{f} + \frac{a^3(c + dx)^3}{3d} - \frac{a^2b(c + dx)^3}{d} + \frac{ab^2(c + dx)^3}{d} - \frac{b^3d}{3d} \\
&= \frac{b^3cdx}{f} + \frac{b^3d^2x^2}{2f} - \frac{3ab^2(c + dx)^2}{f} + \frac{a^3(c + dx)^3}{3d} - \frac{a^2b(c + dx)^3}{d} + \frac{ab^2(c + dx)^3}{d} - \frac{b^3d}{3d} \\
&= \frac{b^3cdx}{f} + \frac{b^3d^2x^2}{2f} - \frac{3ab^2(c + dx)^2}{f} + \frac{a^3(c + dx)^3}{3d} - \frac{a^2b(c + dx)^3}{d} + \frac{ab^2(c + dx)^3}{d} - \frac{b^3d}{3d}
\end{aligned}$$

Mathematica [C] time = 12.915, size = 2031, normalized size = 5.06

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(c + d*x)^2*(a + b*Coth[e + f*x])^3,x]

[Out] $-(a^2 b d^2 E^e \operatorname{Csch}[e] * ((2 f^3 x^3) / E^{(2 e)} - 3 (1 - E^{(-2 e)}) f^2 x^2 \operatorname{Log}[1 - E^{(-e - f x)}] - 3 (1 - E^{(-2 e)}) f^2 x^2 \operatorname{Log}[1 + E^{(-e - f x)}] + 6 (1 - E^{(-2 e)}) (f x \operatorname{PolyLog}[2, -E^{(-e - f x)}] + \operatorname{PolyLog}[3, -E^{(-e - f x)})] + 6 (1 - E^{(-2 e)}) (f x \operatorname{PolyLog}[2, E^{(-e - f x)}] + \operatorname{PolyLog}[3, E^{(-e - f x)})]) / (2 f^3) - (b^3 d^2 E^e \operatorname{Csch}[e] * ((2 f^3 x^3) / E^{(2 e)} - 3 (1 - E^{(-2 e)}) f^2 x^2 \operatorname{Log}[1 - E^{(-e - f x)}] - 3 (1 - E^{(-2 e)}) f^2 x^2 \operatorname{Log}[1 + E^{(-e - f x)}] + 6 (1 - E^{(-2 e)}) (f x \operatorname{PolyLog}[2, -E^{(-e - f x)}] + \operatorname{PolyLog}[3, -E^{(-e - f x)})] + 6 (1 - E^{(-2 e)}) (f x \operatorname{PolyLog}[2, E^{(-e - f x)}] + \operatorname{PolyLog}[3, E^{(-e - f x)})]) / (6 f^3) - (b^3 d^2 \operatorname{Csch}[e] * (-f x \operatorname{Cosh}[e]) + \operatorname{Log}[\operatorname{Cosh}[f x] \operatorname{Sinh}[e] + \operatorname{Cosh}[e] \operatorname{Sinh}[f x]] * \operatorname{Sinh}[e])) / (f^3 * (-\operatorname{Cosh}[e]^2 + \operatorname{Sinh}[e]^2)) - (6 a b^2 c d \operatorname{Csch}[e] * (-f x \operatorname{Cosh}[e]) + \operatorname{Log}[\operatorname{Cosh}[f x] \operatorname{Sinh}[e] + \operatorname{Cosh}[e] \operatorname{Sinh}[f x]] * \operatorname{Sinh}[e])) / (f^2 * (-\operatorname{Cosh}[e]^2 + \operatorname{Sinh}[e]^2)) - (3 a^2 b c^2 \operatorname{Csch}[e] * (-f x \operatorname{Cosh}[e]$

$$\begin{aligned}
&) + \text{Log}[\text{Cosh}[f*x]*\text{Sinh}[e] + \text{Cosh}[e]*\text{Sinh}[f*x]]*\text{Sinh}[e]) / (f*(-\text{Cosh}[e]^2 + \text{Sinh}[e]^2)) - (b^3*c^2*\text{Csch}[e]*(-(f*x*\text{Cosh}[e]) + \text{Log}[\text{Cosh}[f*x]*\text{Sinh}[e] + \text{Cosh}[e]*\text{Sinh}[f*x]]*\text{Sinh}[e])) / (f*(-\text{Cosh}[e]^2 + \text{Sinh}[e]^2)) + (\text{Csch}[e]*\text{Csch}[e + f*x]^2*(-6*b^3*c*d*\text{Cosh}[e] - 18*a*b^2*c^2*f*\text{Cosh}[e] - 6*b^3*d^2*x*\text{Cosh}[e] - 36*a*b^2*c*d*f*x*\text{Cosh}[e] - 18*a^2*b*c^2*f^2*x*\text{Cosh}[e] - 6*b^3*c^2*f^2*x*\text{Cosh}[e] - 18*a*b^2*d^2*f*x^2*\text{Cosh}[e] - 18*a^2*b*c*d*f^2*x^2*\text{Cosh}[e] - 6*b^3*c*d*f^2*x^2*\text{Cosh}[e] - 6*a^2*b*d^2*f^2*x^3*\text{Cosh}[e] - 2*b^3*d^2*f^2*x^3*\text{Cosh}[e] + 6*b^3*c*d*\text{Cosh}[e + 2*f*x] + 18*a*b^2*c^2*f*\text{Cosh}[e + 2*f*x] + 6*b^3*d^2*x*\text{Cosh}[e + 2*f*x] + 36*a*b^2*c*d*f*x*\text{Cosh}[e + 2*f*x] + 9*a^2*b*c^2*f^2*x*\text{Cosh}[e + 2*f*x] + 3*b^3*c^2*f^2*x*\text{Cosh}[e + 2*f*x] + 18*a*b^2*d^2*f*x^2*\text{Cosh}[e + 2*f*x] + 9*a^2*b*c*d*f^2*x^2*\text{Cosh}[e + 2*f*x] + 3*b^3*c*d*f^2*x^2*\text{Cosh}[e + 2*f*x] + 3*a^2*b*d^2*f^2*x^3*\text{Cosh}[e + 2*f*x] + b^3*d^2*f^2*x^3*\text{Cosh}[e + 2*f*x] + 9*a^2*b*c^2*f^2*x*\text{Cosh}[3*e + 2*f*x] + 3*b^3*c^2*f^2*x*\text{Cosh}[3*e + 2*f*x] + 9*a^2*b*c*d*f^2*x^2*\text{Cosh}[3*e + 2*f*x] + 3*b^3*c*d*f^2*x^2*\text{Cosh}[3*e + 2*f*x] + 3*a^2*b*d^2*f^2*x^3*\text{Cosh}[3*e + 2*f*x] + b^3*d^2*f^2*x^3*\text{Cosh}[3*e + 2*f*x] - 6*b^3*c^2*f*\text{Sinh}[e] - 12*b^3*c*d*f*x*\text{Sinh}[e] - 6*a^3*c^2*f^2*x*\text{Sinh}[e] - 18*a*b^2*c^2*f^2*x*\text{Sinh}[e] - 6*b^3*d^2*f*x^2*\text{Sinh}[e] - 6*a^3*c*d*f^2*x^2*\text{Sinh}[e] - 18*a*b^2*c*d*f^2*x^2*\text{Sinh}[e] - 2*a^3*d^2*f^2*x^3*\text{Sinh}[e] - 6*a*b^2*d^2*f^2*x^3*\text{Sinh}[e] - 3*a^3*c^2*f^2*x*\text{Sinh}[e + 2*f*x] - 9*a*b^2*c^2*f^2*x*\text{Sinh}[e + 2*f*x] - 3*a^3*c*d*f^2*x^2*\text{Sinh}[e + 2*f*x] - 9*a*b^2*c*d*f^2*x^2*\text{Sinh}[e + 2*f*x] - a^3*d^2*f^2*x^3*\text{Sinh}[e + 2*f*x] - 3*a*b^2*d^2*f^2*x^3*\text{Sinh}[e + 2*f*x] + 3*a^3*c^2*f^2*x*\text{Sinh}[3*e + 2*f*x] + 9*a*b^2*c^2*f^2*x*\text{Sinh}[3*e + 2*f*x] + 3*a^3*c*d*f^2*x^2*\text{Sinh}[3*e + 2*f*x] + 9*a*b^2*c*d*f^2*x^2*\text{Sinh}[3*e + 2*f*x] + a^3*d^2*f^2*x^3*\text{Sinh}[3*e + 2*f*x] + 3*a*b^2*d^2*f^2*x^3*\text{Sinh}[3*e + 2*f*x])) / (12*f^2) + (3*a*b^2*d^2*\text{Csch}[e]*\text{Sech}[e]*(-(f^2*x^2)/E^{\text{ArcTanh}[\text{Tanh}[e]]) + (I*(-(f*x*(-\text{Pi} + (2*I)*\text{ArcTanh}[\text{Tanh}[e])))) - \text{Pi}*\text{Log}[1 + E^{(2*f*x)}] - 2*(I*f*x + I*\text{ArcTanh}[\text{Tanh}[e]])*\text{Log}[1 - E^{((2*I)*(I*f*x + I*\text{ArcTanh}[\text{Tanh}[e]))}]]) + \text{Pi}*\text{Log}[\text{Cosh}[f*x]] + (2*I)*\text{ArcTanh}[\text{Tanh}[e]]*\text{Log}[I*\text{Sinh}[f*x + \text{ArcTanh}[\text{Tanh}[e]]]]) + I*\text{PolyLog}[2, E^{((2*I)*(I*f*x + I*\text{ArcTanh}[\text{Tanh}[e]))}))*\text{Tanh}[e])/ \text{Sqrt}[1 - \text{Tanh}[e]^2])) / (f^3*\text{Sqrt}[\text{Sech}[e]^2*(\text{Cosh}[e]^2 - \text{Sinh}[e]^2)]) + (3*a^2*b*c*d*\text{Csch}[e]*\text{Sech}[e]*(-(f^2*x^2)/E^{\text{ArcTanh}[\text{Tanh}[e]]) + (I*(-(f*x*(-\text{Pi} + (2*I)*\text{ArcTanh}[\text{Tanh}[e])))) - \text{Pi}*\text{Log}[1 + E^{(2*f*x)}] - 2*(I*f*x + I*\text{ArcTanh}[\text{Tanh}[e]])*\text{Log}[1 - E^{((2*I)*(I*f*x + I*\text{ArcTanh}[\text{Tanh}[e]))}]]) + \text{Pi}*\text{Log}[\text{Cosh}[f*x]] + (2*I)*\text{ArcTanh}[\text{Tanh}[e]]*\text{Log}[I*\text{Sinh}[f*x + \text{ArcTanh}[\text{Tanh}[e]]]]) + I*\text{PolyLog}[2, E^{((2*I)*(I*f*x + I*\text{ArcTanh}[\text{Tanh}[e]))}))*\text{Tanh}[e])/ \text{Sqrt}[1 - \text{Tanh}[e]^2])) / (f^2*\text{Sqrt}[\text{Sech}[e]^2*(\text{Cosh}[e]^2 - \text{Sinh}[e]^2)]) + (b^3*c*d*\text{Csch}[e]*\text{Sech}[e]*(-(f^2*x^2)/E^{\text{ArcTanh}[\text{Tanh}[e]]) + (I*(-(f*x*(-\text{Pi} + (2*I)*\text{ArcTanh}[\text{Tanh}[e])))) - \text{Pi}*\text{Log}[1 + E^{(2*f*x)}] - 2*(I*f*x + I*\text{ArcTanh}[\text{Tanh}[e]])*\text{Log}[1 - E^{((2*I)*(I*f*x + I*\text{ArcTanh}[\text{Tanh}[e]))}]]) + \text{Pi}*\text{Log}[\text{Cosh}[f*x]] + (2*I)*\text{ArcTanh}[\text{Tanh}[e]]*\text{Log}[I*\text{Sinh}[f*x + \text{ArcTanh}[\text{Tanh}[e]]]]) + I*\text{PolyLog}[2, E^{((2*I)*(I*f*x + I*\text{ArcTanh}[\text{Tanh}[e]))}))*\text{Tanh}[e])/ \text{Sqrt}[1 - \text{Tanh}[e]^2])) / (f^2*\text{Sqrt}[\text{Sech}[e]^2*(\text{Cosh}[e]^2 - \text{Sinh}[e]^2)])
\end{aligned}$$

Maple [B] time = 0.166, size = 1542, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((d*x+c)^2*(a+b*\text{coth}(f*x+e))^3,x)$

[Out] $2*b^3/f^2*c*d*\text{polylog}(2,\exp(f*x+e))+2*b^3/f^2*c*d*\text{polylog}(2,-\exp(f*x+e))+4/3*b^3/f^3*d^2*e^3-2*b^3/f^3*d^2*\text{polylog}(3,\exp(f*x+e))-2*b^3/f^3*d^2*\text{polylog}(3,-\exp(f*x+e))+b^3/f^3*d^2*\ln(\exp(f*x+e)-1)+b^3/f^3*d^2*\ln(\exp(f*x+e)+1)-2*b^3/f^3*d^2*\ln(\exp(f*x+e))+b^3/f^3*d^2*\ln(\exp(f*x+e)-1)+b^3/f^3*d^2*\ln(\exp(f*x+e)+1)-2*b^3/f^3*d^2*\ln(\exp(f*x+e))+6*b^2/f^2*\ln(1-\exp(f*x+e))*a*d^2*x+2*b^3/f*\ln(\exp(f*x+e)+1)*c*d*x+3*b/f*\ln(1-\exp(f*x+e))*a^2*d^2*x^2+6*b/f^2*\text{polylog}(2,\exp(f*x+e))*a^2*d^2*x+3*b/f^3*a^2*d^2*e^2*\ln(\exp(f*x+e)-1)-6*b/f^3*a^2*d^2*e^2*\ln(\exp(f*x+e))+3*b/f*\ln(\exp(f*x+e)+1)*a^2*d^2*x^2+6*b/f^2*\text{polylog}(2,-\exp(f*x+e))*a^2*d^2*x+6*b^2/f^3*\ln(1-\exp(f*x+e))*a*d^2*e-6*b^2/f^3*a^2*d^2*e*\ln(\exp(f*x+e)-1)+12*b^2/f^3*a^2*d^2*e*\ln(\exp(f*x+e))+6*b/f^2*a^2*c*d*\text{polylog}(2,\exp(f*x+e))+6*b/f^2*a^2*c*d*\text{polylog}(2,-\exp(f*x+e))+2*b^3/f*\ln(1-\exp(f*x+e))*c*d*x+2*b^3/f^2*\ln(1-\exp(f*x+e))*c*d*e+4*b^3/f^2*c*d*e*\ln(\exp(f*x+e))-12*b^2/f^2*a^2*c*d*\ln(\exp(f*x+e))+6*b^2/f^2*a^2*c*d*\ln(\exp(f*x+e)-1)+6*b^2/f^2*a^2*c*d*\ln(\exp(f*x+e)+1)-2*b^3/f^2*c*d*e*\ln(\exp(f*x+e)-1)-a^2*b*d^2*x^3-6*b/f^2*a^2*c*d*e*\ln(\exp(f*x+e)-1)+6*b/f*\ln(\exp(f*x+e)+1)*a^2*c*d*x+6*b/f*\ln(1-\exp(f*x+e))*a^2*c*d*x+6*b/f^2*\ln(1-\exp(f*x+e))*a^2*c*d*e+12*b/f^2*a^2*c*d*e*\ln(\exp(f*x+e))+1/3*a^3*d^2*x^3-1/3*b^3*d^2*x^3+c^2*a^3*x+b^3*c^2*x-6*b^2/f^3*a^2*d^2*e^2+2*b^3/f^2*d^2*e^2*x+4*b/f^3*a^2*d^2*e^3-2*b^3/f^2*c*d*e^2-6*b^2/f*a^2*d^2*x^2-b^3/f^3*\ln(1-\exp(f*x+e))*d^2*e^2-6*b/f^3*a^2*d^2*\text{polylog}(3,\exp(f*x+e))-6*b/f^3*a^2*d^2*\text{polylog}(3,-\exp(f*x+e))+6*b^2/f^3*a^2*d^2*\text{polylog}(2,\exp(f*x+e))+6*b^2/f^3*a^2*d^2*\text{polylog}(2,-\exp(f*x+e))+3*b/f*a^2*c^2*\ln(\exp(f*x+e)-1)+3*b/f*a^2*c^2*\ln(\exp(f*x+e)+1)+b^3/f^3*d^2*e^2*\ln(\exp(f*x+e)-1)-2*b^3/f^3*d^2*e^2*\ln(\exp(f*x+e))-6*b/f*a^2*c^2*\ln(\exp(f*x+e))+b^3/f*\ln(1-\exp(f*x+e))*d^2*x^2+2*b^3/f^2*\text{polylog}(2,\exp(f*x+e))*d^2*x+b^3/f*\ln(\exp(f*x+e)+1)*d^2*x^2+2*b^3/f^2*\text{polylog}(2,-\exp(f*x+e))*d^2*x+a*b^2*d^2*x^3+a^3*c*d*x^2-b^3*c*d*x^2-2*b^2*(3*a^2*d^2*f*x^2*\exp(2*f*x+2*e)+b*d^2*f*x^2*\exp(2*f*x+2*e)+6*a*c*d*f*x*\exp(2*f*x+2*e)+2*b*c*d*f*x*\exp(2*f*x+2*e)+3*a*c^2*f*\exp(2*f*x+2*e)-3*a*d^2*f*x^2+b*c^2*f*\exp(2*f*x+2*e)+b*d^2*x*\exp(2*f*x+2*e)-6*a*c*d*f*x+b*c*d*\exp(2*f*x+2*e)-3*a*c^2*f-b*d^2*x-c*b*d)/f^2/(\exp(2*f*x+2*e)-1)^2+3*a^2*b*c^2*x+3*a*b^2*c^2*x-12*b/f*a^2*c*d*e*x-6*b/f^2*a^2*c*d*e^2+6*b/f^2*a^2*d^2*e^2*x-4*b^3/f^3*c*d*e*x-12*b^2/f^2*a^2*d^2*e*x-3*b/f^3*\ln(1-\exp(f*x+e))*a^2*d^2*e^2+6*b^2/f^2*\ln(\exp(f*x+e)+1)*a^2*d^2*x-3*a^2*b*c*d*x^2+3*a*b^2*c*d*x^2$

Maxima [B] time = 1.65868, size = 1346, normalized size = 3.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2*(a+b*coth(f*x+e))^3,x, algorithm="maxima")

[Out] $\frac{1}{3}a^3d^2x^3 + a^3cdx^2 + a^3c^2x + 3a^2b^2c^2\log(\sinh(fx + e))/f + \frac{1}{3}(18ab^2c^2f + 6b^3cd + (3a^2bd^2f^2 + 3ab^2d^2f^2 + b^3d^2f^2)x^3 + 3(3a^2b^2cd^2f^2 + b^3cd^2f^2 + 3(c^2d^2f^2 + 2d^2f)ab^2)x^2 + 3(3(c^2f^2 + 4cd^2f)ab^2 + (c^2f^2 + 2d^2)b^3)x + ((3a^2bd^2f^2e^{4e} + 3ab^2d^2f^2e^{4e} + b^3d^2f^2e^{4e})x^3 + 3(3a^2b^2cd^2f^2e^{4e} + 3ab^2c^2d^2f^2e^{4e} + b^3cd^2f^2e^{4e})x^2 + 3(3ab^2c^2f^2e^{4e} + b^3c^2f^2e^{4e})x)e^{4fx} - 2(9ab^2c^2fe^{2e} + 3(c^2fe^{2e} + cde^{2e})b^3 + (3a^2bd^2f^2e^{2e} + 3ab^2d^2f^2e^{2e} + b^3d^2f^2e^{2e})x^3 + 3(3a^2b^2cd^2f^2e^{2e} + 3(c^2d^2f^2e^{2e} + d^2fe^{2e})ab^2 + (c^2d^2f^2e^{2e} + d^2fe^{2e})b^3)x^2 + 3(3(c^2f^2e^{2e} + 2cd^2fe^{2e})ab^2 + (c^2f^2e^{2e} + 2cd^2fe^{2e} + d^2e^{2e})b^3)x)e^{2fx})/(f^2e^{4fx + 4e} - 2f^2e^{(2fx + 2e)} + f^2) - 2(6ab^2cd^2f + (c^2f^2 + d^2)b^3)x/f^2 + (3a^2bd^2 + b^3d^2)(f^2x^2\log(e^{fx + e} + 1) + 2fx\text{dilog}(-e^{fx + e})) - 2\text{polylog}(3, -e^{fx + e}))/f^3 + (3a^2bd^2 + b^3d^2)(f^2x^2\log(-e^{fx + e} + 1) + 2fx\text{dilog}(e^{fx + e})) - 2\text{polylog}(3, e^{fx + e}))/f^3 + 2(3a^2b^2cd^2f + b^3cd^2f + 3ab^2d^2)(fx\log(e^{fx + e} + 1) + \text{dilog}(-e^{fx + e}))/f^3 + 2(3a^2b^2cd^2f + b^3cd^2f + 3ab^2d^2)(fx\log(e^{fx + e} + 1) + \text{dilog}(e^{fx + e}))/f^3 + (6ab^2cd^2f + (c^2f^2 + d^2)b^3)\log(e^{fx + e} + 1)/f^3 + (6ab^2cd^2f + (c^2f^2 + d^2)b^3)\log(e^{fx + e} - 1)/f^3 - 2/3((3a^2bd^2 + b^3d^2)f^3x^3 + 3(3a^2b^2cd^2f + b^3cd^2f + 3ab^2d^2)f^2x^2)/f^3$

Fricas [C] time = 3.19641, size = 13835, normalized size = 34.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2*(a+b*coth(f*x+e))^3,x, algorithm="fricas")

[Out] $\frac{1}{3}((a^3 - 3a^2b + 3ab^2 - b^3)d^2f^3x^3 + 3(a^3 - 3a^2b + 3ab^2 - b^3)cd^2f^3x^2 + 18ab^2d^2e^2 + 3(a^3 - 3a^2b + 3ab^2 - b^3$

$$\begin{aligned}
& *b + b^3)*c*d*f)*\cosh(f*x + e)*\sinh(f*x + e)^3 + (3*a*b^2*d^2 + (3*a^2*b + \\
& b^3)*d^2*f*x + (3*a^2*b + b^3)*c*d*f)*\sinh(f*x + e)^4 + (3*a^2*b + b^3)*c*d \\
& *f - 2*(3*a*b^2*d^2 + (3*a^2*b + b^3)*d^2*f*x + (3*a^2*b + b^3)*c*d*f)*\cosh \\
& (f*x + e)^2 - 2*(3*a*b^2*d^2 + (3*a^2*b + b^3)*d^2*f*x + (3*a^2*b + b^3)*c \\
& *d*f - 3*(3*a*b^2*d^2 + (3*a^2*b + b^3)*d^2*f*x + (3*a^2*b + b^3)*c*d*f)*\cos \\
& h(f*x + e)^2)*\sinh(f*x + e)^2 + 4*((3*a*b^2*d^2 + (3*a^2*b + b^3)*d^2*f*x + \\
& (3*a^2*b + b^3)*c*d*f)*\cosh(f*x + e)^3 - (3*a*b^2*d^2 + (3*a^2*b + b^3)*d^ \\
& 2*f*x + (3*a^2*b + b^3)*c*d*f)*\cosh(f*x + e))*\sinh(f*x + e))*\operatorname{dilog}(-\cosh(f* \\
& x + e) - \sinh(f*x + e)) + 3*((3*a^2*b + b^3)*d^2*f^2*x^2 + 6*a*b^2*c*d*f + \\
& b^3*d^2 + (3*a^2*b + b^3)*c^2*f^2 + ((3*a^2*b + b^3)*d^2*f^2*x^2 + 6*a*b^2*c \\
& *d*f + b^3*d^2 + (3*a^2*b + b^3)*c^2*f^2 + 2*(3*a*b^2*d^2*f + (3*a^2*b + b \\
& ^3)*c*d*f^2)*x)*\cosh(f*x + e)^4 + 4*((3*a^2*b + b^3)*d^2*f^2*x^2 + 6*a*b^2*c \\
& *d*f + b^3*d^2 + (3*a^2*b + b^3)*c^2*f^2 + 2*(3*a*b^2*d^2*f + (3*a^2*b + b \\
& ^3)*c*d*f^2)*x)*\cosh(f*x + e)*\sinh(f*x + e)^3 + ((3*a^2*b + b^3)*d^2*f^2*x^ \\
& 2 + 6*a*b^2*c*d*f + b^3*d^2 + (3*a^2*b + b^3)*c^2*f^2 + 2*(3*a*b^2*d^2*f + \\
& (3*a^2*b + b^3)*c*d*f^2)*x)*\sinh(f*x + e)^4 - 2*((3*a^2*b + b^3)*d^2*f^2*x^ \\
& 2 + 6*a*b^2*c*d*f + b^3*d^2 + (3*a^2*b + b^3)*c^2*f^2 + 2*(3*a*b^2*d^2*f + \\
& (3*a^2*b + b^3)*c*d*f^2)*x)*\cosh(f*x + e)^2 - 2*((3*a^2*b + b^3)*d^2*f^2*x^ \\
& 2 + 6*a*b^2*c*d*f + b^3*d^2 + (3*a^2*b + b^3)*c^2*f^2 - 3*((3*a^2*b + b^3)* \\
& d^2*f^2*x^2 + 6*a*b^2*c*d*f + b^3*d^2 + (3*a^2*b + b^3)*c^2*f^2 + 2*(3*a*b^ \\
& 2*d^2*f + (3*a^2*b + b^3)*c*d*f^2)*x)*\cosh(f*x + e)^2 + 2*(3*a*b^2*d^2*f + \\
& (3*a^2*b + b^3)*c*d*f^2)*x)*\sinh(f*x + e)^2 + 2*(3*a*b^2*d^2*f + (3*a^2*b + \\
& b^3)*c*d*f^2)*x + 4*((3*a^2*b + b^3)*d^2*f^2*x^2 + 6*a*b^2*c*d*f + b^3*d^ \\
& 2 + (3*a^2*b + b^3)*c^2*f^2 + 2*(3*a*b^2*d^2*f + (3*a^2*b + b^3)*c*d*f^2)*x \\
&)*\cosh(f*x + e)^3 - ((3*a^2*b + b^3)*d^2*f^2*x^2 + 6*a*b^2*c*d*f + b^3*d^2 \\
& + (3*a^2*b + b^3)*c^2*f^2 + 2*(3*a*b^2*d^2*f + (3*a^2*b + b^3)*c*d*f^2)*x)* \\
& \cosh(f*x + e))*\sinh(f*x + e))*\log(\cosh(f*x + e) + \sinh(f*x + e) + 1) - 3*(6 \\
& *a*b^2*d^2*e - b^3*d^2 - (3*a^2*b + b^3)*d^2*e^2 - (3*a^2*b + b^3)*c^2*f^2 \\
& + (6*a*b^2*d^2*e - b^3*d^2 - (3*a^2*b + b^3)*d^2*e^2 - (3*a^2*b + b^3)*c^2* \\
& f^2 - 2*(3*a*b^2*c*d - (3*a^2*b + b^3)*c*d*e)*f)*\cosh(f*x + e)^4 + 4*(6*a*b \\
& ^2*d^2*e - b^3*d^2 - (3*a^2*b + b^3)*d^2*e^2 - (3*a^2*b + b^3)*c^2*f^2 - 2* \\
& (3*a*b^2*c*d - (3*a^2*b + b^3)*c*d*e)*f)*\cosh(f*x + e)*\sinh(f*x + e)^3 + (6 \\
& *a*b^2*d^2*e - b^3*d^2 - (3*a^2*b + b^3)*d^2*e^2 - (3*a^2*b + b^3)*c^2*f^2 \\
& - 2*(3*a*b^2*c*d - (3*a^2*b + b^3)*c*d*e)*f)*\sinh(f*x + e)^4 - 2*(6*a*b^2*d \\
& ^2*e - b^3*d^2 - (3*a^2*b + b^3)*d^2*e^2 - (3*a^2*b + b^3)*c^2*f^2 - 2*(3*a \\
& *b^2*c*d - (3*a^2*b + b^3)*c*d*e)*f)*\cosh(f*x + e)^2 - 2*(6*a*b^2*d^2*e - b \\
& ^3*d^2 - (3*a^2*b + b^3)*d^2*e^2 - (3*a^2*b + b^3)*c^2*f^2 - 3*(6*a*b^2*d^2 \\
& *e - b^3*d^2 - (3*a^2*b + b^3)*d^2*e^2 - (3*a^2*b + b^3)*c^2*f^2 - 2*(3*a*b \\
& ^2*c*d - (3*a^2*b + b^3)*c*d*e)*f)*\cosh(f*x + e)^2 - 2*(3*a*b^2*c*d - (3*a^ \\
& 2*b + b^3)*c*d*e)*f)*\sinh(f*x + e)^2 - 2*(3*a*b^2*c*d - (3*a^2*b + b^3)*c*d \\
& *e)*f + 4*((6*a*b^2*d^2*e - b^3*d^2 - (3*a^2*b + b^3)*d^2*e^2 - (3*a^2*b + \\
& b^3)*c^2*f^2 - 2*(3*a*b^2*c*d - (3*a^2*b + b^3)*c*d*e)*f)*\cosh(f*x + e)^3 - \\
& (6*a*b^2*d^2*e - b^3*d^2 - (3*a^2*b + b^3)*d^2*e^2 - (3*a^2*b + b^3)*c^2*f \\
& ^2 - 2*(3*a*b^2*c*d - (3*a^2*b + b^3)*c*d*e)*f)*\cosh(f*x + e))*\sinh(f*x + e \\
&))*\log(\cosh(f*x + e) + \sinh(f*x + e) - 1) + 3*((3*a^2*b + b^3)*d^2*f^2*x^2
\end{aligned}$$

$$\begin{aligned}
& + 6*a*b^2*d^2*e - (3*a^2*b + b^3)*d^2*e^2 + 2*(3*a^2*b + b^3)*c*d*e*f + ((3 \\
& *a^2*b + b^3)*d^2*f^2*x^2 + 6*a*b^2*d^2*e - (3*a^2*b + b^3)*d^2*e^2 + 2*(3* \\
& a^2*b + b^3)*c*d*e*f + 2*(3*a*b^2*d^2*f + (3*a^2*b + b^3)*c*d*f^2)*x)*\cosh(\\
& f*x + e)^4 + 4*((3*a^2*b + b^3)*d^2*f^2*x^2 + 6*a*b^2*d^2*e - (3*a^2*b + b^ \\
& 3)*d^2*e^2 + 2*(3*a^2*b + b^3)*c*d*e*f + 2*(3*a*b^2*d^2*f + (3*a^2*b + b^3) \\
& *c*d*f^2)*x)*\cosh(f*x + e)*\sinh(f*x + e)^3 + ((3*a^2*b + b^3)*d^2*f^2*x^2 + \\
& 6*a*b^2*d^2*e - (3*a^2*b + b^3)*d^2*e^2 + 2*(3*a^2*b + b^3)*c*d*e*f + 2*(3 \\
& *a*b^2*d^2*f + (3*a^2*b + b^3)*c*d*f^2)*x)*\sinh(f*x + e)^4 - 2*((3*a^2*b + \\
& b^3)*d^2*f^2*x^2 + 6*a*b^2*d^2*e - (3*a^2*b + b^3)*d^2*e^2 + 2*(3*a^2*b + b \\
& ^3)*c*d*e*f + 2*(3*a*b^2*d^2*f + (3*a^2*b + b^3)*c*d*f^2)*x)*\cosh(f*x + e)^ \\
& 2 - 2*((3*a^2*b + b^3)*d^2*f^2*x^2 + 6*a*b^2*d^2*e - (3*a^2*b + b^3)*d^2*e^ \\
& 2 + 2*(3*a^2*b + b^3)*c*d*e*f - 3*((3*a^2*b + b^3)*d^2*f^2*x^2 + 6*a*b^2*d^ \\
& 2*e - (3*a^2*b + b^3)*d^2*e^2 + 2*(3*a^2*b + b^3)*c*d*e*f + 2*(3*a*b^2*d^2* \\
& f + (3*a^2*b + b^3)*c*d*f^2)*x)*\cosh(f*x + e)^2 + 2*(3*a*b^2*d^2*f + (3*a^2 \\
& *b + b^3)*c*d*f^2)*x)*\sinh(f*x + e)^2 + 2*(3*a*b^2*d^2*f + (3*a^2*b + b^3)* \\
& c*d*f^2)*x + 4*((3*a^2*b + b^3)*d^2*f^2*x^2 + 6*a*b^2*d^2*e - (3*a^2*b + b \\
& ^3)*d^2*e^2 + 2*(3*a^2*b + b^3)*c*d*e*f + 2*(3*a*b^2*d^2*f + (3*a^2*b + b^3) \\
&)*c*d*f^2)*x)*\cosh(f*x + e)^3 - ((3*a^2*b + b^3)*d^2*f^2*x^2 + 6*a*b^2*d^2* \\
& e - (3*a^2*b + b^3)*d^2*e^2 + 2*(3*a^2*b + b^3)*c*d*e*f + 2*(3*a*b^2*d^2*f \\
& + (3*a^2*b + b^3)*c*d*f^2)*x)*\cosh(f*x + e))*\sinh(f*x + e))*\log(-\cosh(f*x + \\
& e) - \sinh(f*x + e) + 1) - 6*((3*a^2*b + b^3)*d^2*\cosh(f*x + e)^4 + 4*(3*a^ \\
& 2*b + b^3)*d^2*\cosh(f*x + e)*\sinh(f*x + e)^3 + (3*a^2*b + b^3)*d^2*\sinh(f*x \\
& + e)^4 - 2*(3*a^2*b + b^3)*d^2*\cosh(f*x + e)^2 + (3*a^2*b + b^3)*d^2 + 2*(\\
& 3*(3*a^2*b + b^3)*d^2*\cosh(f*x + e)^2 - (3*a^2*b + b^3)*d^2)*\sinh(f*x + e)^ \\
& 2 + 4*((3*a^2*b + b^3)*d^2*\cosh(f*x + e)^3 - (3*a^2*b + b^3)*d^2*\cosh(f*x + \\
& e))*\sinh(f*x + e))*\text{polylog}(3, \cosh(f*x + e) + \sinh(f*x + e)) - 6*((3*a^2*b \\
& + b^3)*d^2*\cosh(f*x + e)^4 + 4*(3*a^2*b + b^3)*d^2*\cosh(f*x + e)*\sinh(f*x \\
& + e)^3 + (3*a^2*b + b^3)*d^2*\sinh(f*x + e)^4 - 2*(3*a^2*b + b^3)*d^2*\cosh(f \\
& *x + e)^2 + (3*a^2*b + b^3)*d^2 + 2*(3*(3*a^2*b + b^3)*d^2*\cosh(f*x + e)^2 \\
& - (3*a^2*b + b^3)*d^2)*\sinh(f*x + e)^2 + 4*((3*a^2*b + b^3)*d^2*\cosh(f*x + \\
& e)^3 - (3*a^2*b + b^3)*d^2*\cosh(f*x + e))*\sinh(f*x + e))*\text{polylog}(3, -\cosh(f \\
& *x + e) - \sinh(f*x + e)) + 4*((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d^2*f^3*x^3 \\
& + 18*a*b^2*d^2*e^2 - 6*b^3*d^2*e - 2*(3*a^2*b + b^3)*d^2*e^3 - 6*(3*a^2*b + \\
& b^3)*c^2*e*f^2 - 3*(6*a*b^2*d^2*f^2 - (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*c*d* \\
& f^3)*x^2 - 6*(6*a*b^2*c*d*e - (3*a^2*b + b^3)*c*d*e^2)*f - 3*(12*a*b^2*c*d* \\
& f^2 + 2*b^3*d^2*f - (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*c^2*f^3)*x)*\cosh(f*x + \\
& e)^3 - ((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d^2*f^3*x^3 + 18*a*b^2*d^2*e^2 - 6* \\
& b^3*d^2*e - 2*(3*a^2*b + b^3)*d^2*e^3 - 3*(2*(3*a^2*b + b^3)*c^2*e - (3*a*b \\
& ^2 + b^3)*c^2)*f^2 + 3*((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*c*d*f^3 - (3*a*b^2 \\
& - b^3)*d^2*f^2)*x^2 - 3*(12*a*b^2*c*d*e - b^3*c*d - 2*(3*a^2*b + b^3)*c*d*e \\
& ^2)*f - 3*(b^3*d^2*f - (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*c^2*f^3 + 2*(3*a*b^2 \\
& - b^3)*c*d*f^2)*x)*\cosh(f*x + e))*\sinh(f*x + e))/(f^3*\cosh(f*x + e)^4 + 4* \\
& f^3*\cosh(f*x + e)*\sinh(f*x + e)^3 + f^3*\sinh(f*x + e)^4 - 2*f^3*\cosh(f*x + \\
& e)^2 + f^3 + 2*(3*f^3*\cosh(f*x + e)^2 - f^3)*\sinh(f*x + e)^2 + 4*(f^3*\cosh(\\
& f*x + e)^3 - f^3*\cosh(f*x + e))*\sinh(f*x + e))
\end{aligned}$$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b \coth(e + fx))^3 (c + dx)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**2*(a+b*coth(f*x+e))**3,x)

[Out] Integral((a + b*coth(e + f*x))**3*(c + d*x)**2, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^2 (b \coth(fx + e) + a)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2*(a+b*coth(f*x+e))^3,x, algorithm="giac")

[Out] integrate((d*x + c)^2*(b*coth(f*x + e) + a)^3, x)

3.49 $\int (c + dx)(a + b \coth(e + fx))^3 dx$

Optimal. Leaf size=261

$$\frac{3a^2bd \operatorname{PolyLog}\left(2, e^{2(e+fx)}\right)}{2f^2} + \frac{b^3d \operatorname{PolyLog}\left(2, e^{2(e+fx)}\right)}{2f^2} + \frac{3a^2b(c+dx) \log\left(1 - e^{2(e+fx)}\right)}{f} - \frac{3a^2b(c+dx)^2}{2d} + \frac{a^3(c+dx)^2}{2d}$$

```
[Out] 3*a*b^2*c*x + (b^3*d*x)/(2*f) + (3*a*b^2*d*x^2)/2 + (a^3*(c + d*x)^2)/(2*d)
- (3*a^2*b*(c + d*x)^2)/(2*d) - (b^3*(c + d*x)^2)/(2*d) - (b^3*d*Coth[e +
f*x])/(2*f^2) - (3*a*b^2*(c + d*x)*Coth[e + f*x])/f - (b^3*(c + d*x)*Coth[e
+ f*x]^2)/(2*f) + (3*a^2*b*(c + d*x)*Log[1 - E^(2*(e + f*x))])/f + (b^3*(c
+ d*x)*Log[1 - E^(2*(e + f*x))])/f + (3*a*b^2*d*Log[Sinh[e + f*x]])/f^2 +
(3*a^2*b*d*PolyLog[2, E^(2*(e + f*x))])/(2*f^2) + (b^3*d*PolyLog[2, E^(2*(e
+ f*x))])/(2*f^2)
```

Rubi [A] time = 0.350042, antiderivative size = 261, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 9, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {3722, 3716, 2190, 2279, 2391, 3720, 3475, 3473, 8}

$$\frac{3a^2bd \operatorname{PolyLog}\left(2, e^{2(e+fx)}\right)}{2f^2} + \frac{b^3d \operatorname{PolyLog}\left(2, e^{2(e+fx)}\right)}{2f^2} + \frac{3a^2b(c+dx) \log\left(1 - e^{2(e+fx)}\right)}{f} - \frac{3a^2b(c+dx)^2}{2d} + \frac{a^3(c+dx)^2}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*(a + b*Coth[e + f*x])^3, x]
```

```
[Out] 3*a*b^2*c*x + (b^3*d*x)/(2*f) + (3*a*b^2*d*x^2)/2 + (a^3*(c + d*x)^2)/(2*d)
- (3*a^2*b*(c + d*x)^2)/(2*d) - (b^3*(c + d*x)^2)/(2*d) - (b^3*d*Coth[e +
f*x])/(2*f^2) - (3*a*b^2*(c + d*x)*Coth[e + f*x])/f - (b^3*(c + d*x)*Coth[e
+ f*x]^2)/(2*f) + (3*a^2*b*(c + d*x)*Log[1 - E^(2*(e + f*x))])/f + (b^3*(c
+ d*x)*Log[1 - E^(2*(e + f*x))])/f + (3*a*b^2*d*Log[Sinh[e + f*x]])/f^2 +
(3*a^2*b*d*PolyLog[2, E^(2*(e + f*x))])/(2*f^2) + (b^3*d*PolyLog[2, E^(2*(e
+ f*x))])/(2*f^2)
```

Rule 3722

```
Int[((c_.) + (d_.)*(x_.))^(m_.)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_.)])^(n_.)
, x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, (a + b*Tan[e + f*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 3716


```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] := -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-I*
e) + f*fz*x))/E^(2*I*k*Pi))], x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:= Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] := Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rubi steps

$$\begin{aligned}
 \int (c + dx)(a + b \coth(e + fx))^3 dx &= \int (a^3(c + dx) + 3a^2b(c + dx) \coth(e + fx) + 3ab^2(c + dx) \coth^2(e + fx) + b^3(c + dx) \coth^3(e + fx)) dx \\
 &= \frac{a^3(c + dx)^2}{2d} + (3a^2b) \int (c + dx) \coth(e + fx) dx + (3ab^2) \int (c + dx) \coth^2(e + fx) dx + b^3 \int (c + dx) \coth^3(e + fx) dx \\
 &= \frac{a^3(c + dx)^2}{2d} - \frac{3a^2b(c + dx)^2}{2d} - \frac{3ab^2(c + dx) \coth(e + fx)}{f} - \frac{b^3(c + dx) \coth^2(e + fx)}{2f} \\
 &= 3ab^2cx + \frac{3}{2}ab^2dx^2 + \frac{a^3(c + dx)^2}{2d} - \frac{3a^2b(c + dx)^2}{2d} - \frac{b^3(c + dx)^2}{2d} - \frac{b^3d \coth(e + fx)}{2f^2} \\
 &= 3ab^2cx + \frac{b^3dx}{2f} + \frac{3}{2}ab^2dx^2 + \frac{a^3(c + dx)^2}{2d} - \frac{3a^2b(c + dx)^2}{2d} - \frac{b^3(c + dx)^2}{2d} - \frac{b^3d \coth(e + fx)}{2f} \\
 &= 3ab^2cx + \frac{b^3dx}{2f} + \frac{3}{2}ab^2dx^2 + \frac{a^3(c + dx)^2}{2d} - \frac{3a^2b(c + dx)^2}{2d} - \frac{b^3(c + dx)^2}{2d} - \frac{b^3d \coth(e + fx)}{2f} \\
 &= 3ab^2cx + \frac{b^3dx}{2f} + \frac{3}{2}ab^2dx^2 + \frac{a^3(c + dx)^2}{2d} - \frac{3a^2b(c + dx)^2}{2d} - \frac{b^3(c + dx)^2}{2d} - \frac{b^3d \coth(e + fx)}{2f}
 \end{aligned}$$

Mathematica [A] time = 3.45699, size = 266, normalized size = 1.02

$$\sinh(e + fx)(a + b \coth(e + fx))^3 \left(-bd(3a^2 + b^2) \sinh^2(e + fx) \text{PolyLog}\left(2, e^{-2(e+fx)}\right) + \sinh^2(e + fx)(-(e + fx)(-3a^2 + b^2) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)*(a + b*Coth[e + f*x])^3, x]

[Out] ((a + b*Coth[e + f*x])^3*Sinh[e + f*x]*((-(e + f*x)*(-3*a^2*b*d*(e + f*x) - b^3*d*(e + f*x) + a^3*(-2*c*f + d*(e - f*x)) + 3*a*b^2*(-2*c*f + d*(e - f*x)))) + 2*b*(3*a^2 + b^2)*d*(e + f*x)*Log[1 - E^(-2*(e + f*x))] - 2*b*(-3*a*b*d + 3*a^2*(d*e - c*f) + b^2*(d*e - c*f))*Log[Sinh[e + f*x]]*Sinh[e + f*x]^2 - b*(3*a^2 + b^2)*d*PolyLog[2, E^(-2*(e + f*x))]*Sinh[e + f*x]^2 - (b^2*(2*b*f*(c + d*x) + (b*d + 6*a*f*(c + d*x))*Sinh[2*(e + f*x)]))/2)/(2*f^2*(b*Cosh[e + f*x] + a*Sinh[e + f*x])^3)

Maple [B] time = 0.121, size = 651, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x+c)*(a+b*coth(f*x+e))^3,x)`

[Out] $3*a*b^2*c*x+3/2*a*b^2*d*x^2-b^2*(6*a*d*f*x*\exp(2*f*x+2*e)+2*b*d*f*x*\exp(2*f*x+2*e)+6*a*c*f*\exp(2*f*x+2*e)+2*b*c*f*\exp(2*f*x+2*e)-6*a*d*f*x+b*d*\exp(2*f*x+2*e)-6*a*c*f-b*d)/f^2/(\exp(2*f*x+2*e)-1)^2-2*b^3/f*d*e*x+b^3/f*\ln(1-\exp(f*x+e))*d*x+b^3/f*\ln(\exp(f*x+e)+1)*d*x+3*b^2/f^2*d*a*\ln(\exp(f*x+e)-1)+3*b^2/f^2*d*a*\ln(\exp(f*x+e)+1)-b^3/f^2*d*e*\ln(\exp(f*x+e)-1)+3*b/f*a^2*c*\ln(\exp(f*x+e)-1)+3*b/f*a^2*c*\ln(\exp(f*x+e)+1)-6*b^2/f^2*d*a*\ln(\exp(f*x+e))+2*b^3/f^2*d*e*\ln(\exp(f*x+e))+3*b/f^2*a^2*d*polylog(2,\exp(f*x+e))+3*b/f^2*a^2*d*polylog(2,-\exp(f*x+e))+b^3/f^2*\ln(1-\exp(f*x+e))*d*e-6*b/f*a^2*c*\ln(\exp(f*x+e))-b^3/f^2*d*e^2+b^3/f*c*\ln(\exp(f*x+e)-1)+b^3/f*c*\ln(\exp(f*x+e)+1)+b^3/f^2*d*polylog(2,\exp(f*x+e))+b^3/f^2*d*polylog(2,-\exp(f*x+e))-2*b^3/f*c*\ln(\exp(f*x+e))-3/2*a^2*b*d*x^2+3*a^2*b*c*x+1/2*a^3*d*x^2-1/2*b^3*d*x^2+c*a^3*x+b^3*c*x-6*b/f*a^2*d*e*x+3*b/f^2*\ln(1-\exp(f*x+e))*a^2*d*e+6*b/f^2*d*a^2*e*\ln(\exp(f*x+e))-3*b/f^2*a^2*d*e*\ln(\exp(f*x+e)-1)+3*b/f*\ln(1-\exp(f*x+e))*a^2*d*x+3*b/f*\ln(\exp(f*x+e)+1)*a^2*d*x-3*b/f^2*a^2*d*e^2$

Maxima [B] time = 1.50365, size = 713, normalized size = 2.73

$$\frac{1}{2}a^3dx^2 + a^3cx + \frac{3a^2bc \log(\sinh(fx + e))}{f} - (3a^2bd + b^3d)x^2 - \frac{2(b^3cf + 3ab^2d)x}{f} + \frac{12ab^2cf + 2b^3d + (3a^2bdf^2 +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)*(a+b*coth(f*x+e))^3,x, algorithm="maxima")`

[Out] $1/2*a^3*d*x^2 + a^3*c*x + 3*a^2*b*c*\log(\sinh(f*x + e))/f - (3*a^2*b*d + b^3*d)*x^2 - 2*(b^3*c*f + 3*a*b^2*d)*x/f + 1/2*(12*a*b^2*c*f + 2*b^3*d + (3*a^2*b*d*f^2 + 3*a*b^2*d*f^2 + b^3*d*f^2)*x^2 + 2*(b^3*c*f^2 + 3*(c*f^2 + 2*d*f)*a*b^2)*x + ((3*a^2*b*d*f^2*e^(4*e) + 3*a*b^2*d*f^2*e^(4*e) + b^3*d*f^2*e^(4*e))*x^2 + 2*(3*a*b^2*c*f^2*e^(4*e) + b^3*c*f^2*e^(4*e))*x)*e^(4*f*x) - 2*(6*a*b^2*c*f*e^(2*e) + (2*c*f*e^(2*e) + d*e^(2*e))*b^3 + (3*a^2*b*d*f^2*e^(2*e) + 3*a*b^2*d*f^2*e^(2*e) + b^3*d*f^2*e^(2*e))*x^2 + 2*(3*(c*f^2*e^(2*$

$$e) + d*f*e^{(2*e)}*a*b^2 + (c*f^2*e^{(2*e)} + d*f*e^{(2*e)})*b^3*x)*e^{(2*f*x)}) / (f^2*e^{(4*f*x + 4*e)} - 2*f^2*e^{(2*f*x + 2*e)} + f^2) + (3*a^2*b*d + b^3*d)*(f*x*\log(e^{(f*x + e)} + 1) + \operatorname{dilog}(-e^{(f*x + e)})) / f^2 + (3*a^2*b*d + b^3*d)*(f*x*\log(-e^{(f*x + e)} + 1) + \operatorname{dilog}(e^{(f*x + e)})) / f^2 + (b^3*c*f + 3*a*b^2*d)*\log(e^{(f*x + e)} + 1) / f^2 + (b^3*c*f + 3*a*b^2*d)*\log(e^{(f*x + e)} - 1) / f^2$$

Fricas [B] time = 2.6298, size = 6711, normalized size = 25.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*(a+b*coth(f*x+e))^3,x, algorithm="fricas")

[Out] $1/2*((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*f^2*x^2 - 12*a*b^2*d*e + 2*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*c*f^2*x + ((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*f^2*x^2 - 12*a*b^2*d*e + 2*(3*a^2*b + b^3)*d*e^2 - 4*(3*a^2*b + b^3)*c*e*f - 2*(6*a*b^2*d*f - (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*c*f^2)*x)*\cosh(f*x + e)^4 + 4*((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*f^2*x^2 - 12*a*b^2*d*e + 2*(3*a^2*b + b^3)*d*e^2 - 4*(3*a^2*b + b^3)*c*e*f - 2*(6*a*b^2*d*f - (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*c*f^2)*x)*\cosh(f*x + e)*\sinh(f*x + e)^3 + ((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*f^2*x^2 - 12*a*b^2*d*e + 2*(3*a^2*b + b^3)*d*e^2 - 4*(3*a^2*b + b^3)*c*e*f - 2*(6*a*b^2*d*f - (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*c*f^2)*x)*\sinh(f*x + e)^4 + 2*b^3*d + 2*(3*a^2*b + b^3)*d*e^2 - 2*((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*f^2*x^2 - 12*a*b^2*d*e + b^3*d + 2*(3*a^2*b + b^3)*d*e^2 - 2*(2*(3*a^2*b + b^3)*c*e - (3*a*b^2 + b^3)*c)*f + 2*((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*c*f^2 - (3*a*b^2 - b^3)*d*f)*x)*\cosh(f*x + e)^2 - 2*((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*f^2*x^2 - 12*a*b^2*d*e + b^3*d + 2*(3*a^2*b + b^3)*d*e^2 - 3*((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*f^2*x^2 - 12*a*b^2*d*e + 2*(3*a^2*b + b^3)*d*e^2 - 4*(3*a^2*b + b^3)*c*e*f - 2*(6*a*b^2*d*f - (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*c*f^2)*x)*\cosh(f*x + e)^2 - 2*(2*(3*a^2*b + b^3)*c*e - (3*a*b^2 + b^3)*c)*f + 2*((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*c*f^2 - (3*a*b^2 - b^3)*d*f)*x)*\sinh(f*x + e)^2 + 4*(3*a*b^2*c - (3*a^2*b + b^3)*c*e)*f + 2*((3*a^2*b + b^3)*d*\cosh(f*x + e)^4 + 4*(3*a^2*b + b^3)*d*\cosh(f*x + e)*\sinh(f*x + e)^3 + (3*a^2*b + b^3)*d*\sinh(f*x + e)^4 - 2*(3*a^2*b + b^3)*d*\cosh(f*x + e)^2 + 2*(3*(3*a^2*b + b^3)*d*\cosh(f*x + e)^2 - (3*a^2*b + b^3)*d)*\sinh(f*x + e)^2 + (3*a^2*b + b^3)*d + 4*((3*a^2*b + b^3)*d*\cosh(f*x + e)^3 - (3*a^2*b + b^3)*d*\cosh(f*x + e))*\sinh(f*x + e)*\operatorname{dilog}(\cosh(f*x + e) + \sinh(f*x + e)) + 2*((3*a^2*b + b^3)*d*\cosh(f*x + e)^4 + 4*(3*a^2*b + b^3)*d*\cosh(f*x + e)*\sinh(f*x + e)^3 + (3*a^2*b + b^3)*d*\sinh(f*x + e)^4 - 2*(3*a^2*b + b^3)*d*\cosh(f*x + e)^2 + 2*(3*(3*a^2*b + b^3)*d*\cosh(f*x + e)^2 - (3*a^2*b + b^3)*d)*\sinh(f*x + e)^2 + (3*a^2*b + b^3)*d + 4*((3*a^2*b + b^3)*d*\cosh(f*x + e)^3 - (3*a^2*b + b^3)*d*\cosh(f*x + e))*\sinh(f*x + e)*\operatorname{dilog}(\cosh(f*x + e) - \sinh(f*x + e)) + 2*((3*a^2*b + b^3)*d*\cosh(f*x + e)^4 + 4*(3*a^2*b + b^3)*d*\cosh(f*x + e)*\sinh(f*x + e)^3 + (3*a^2*b + b^3)*d*\sinh(f*x + e)^4 - 2*(3*a^2*b + b^3)*d*\cosh(f*x + e)^2 + 2*(3*(3*a^2*b + b^3)*d*\cosh(f*x + e)^2 - (3*a^2*b + b^3)*d)*\sinh(f*x + e)^2 + (3*a^2*b + b^3)*d + 4*((3*a^2*b + b^3)*d*\cosh(f*x + e)^3 - (3*a^2*b + b^3)*d*\cosh(f*x + e))*\sinh(f*x + e)*\operatorname{dilog}(\cosh(f*x + e) - \sinh(f*x + e))$

$$\begin{aligned}
& x + e)^3 - (3a^2b + b^3)*d*\cosh(f*x + e))*\sinh(f*x + e))*\operatorname{dilog}(-\cosh(f*x \\
& + e) - \sinh(f*x + e)) + 2*((3a*b^2*d + (3a^2*b + b^3)*d*f*x + (3a^2*b + \\
& b^3)*c*f)*\cosh(f*x + e)^4 + 4*(3a*b^2*d + (3a^2*b + b^3)*d*f*x + (3a^2*b \\
& + b^3)*c*f)*\cosh(f*x + e)*\sinh(f*x + e)^3 + (3a*b^2*d + (3a^2*b + b^3)*d \\
& *f*x + (3a^2*b + b^3)*c*f)*\sinh(f*x + e)^4 + 3a*b^2*d + (3a^2*b + b^3)*d \\
& *f*x + (3a^2*b + b^3)*c*f - 2*(3a*b^2*d + (3a^2*b + b^3)*d*f*x + (3a^2* \\
& b + b^3)*c*f)*\cosh(f*x + e)^2 - 2*(3a*b^2*d + (3a^2*b + b^3)*d*f*x + (3a \\
& ^2*b + b^3)*c*f - 3*(3a*b^2*d + (3a^2*b + b^3)*d*f*x + (3a^2*b + b^3)*c* \\
& f)*\cosh(f*x + e)^2)*\sinh(f*x + e)^2 + 4*((3a*b^2*d + (3a^2*b + b^3)*d*f*x \\
& + (3a^2*b + b^3)*c*f)*\cosh(f*x + e)^3 - (3a*b^2*d + (3a^2*b + b^3)*d*f*x \\
& x + (3a^2*b + b^3)*c*f)*\cosh(f*x + e))*\sinh(f*x + e))*\log(\cosh(f*x + e) + \\
& \sinh(f*x + e) + 1) + 2*((3a*b^2*d - (3a^2*b + b^3)*d*e + (3a^2*b + b^3)* \\
& c*f)*\cosh(f*x + e)^4 + 4*(3a*b^2*d - (3a^2*b + b^3)*d*e + (3a^2*b + b^3) \\
& *c*f)*\cosh(f*x + e)*\sinh(f*x + e)^3 + (3a*b^2*d - (3a^2*b + b^3)*d*e + (3 \\
& a^2*b + b^3)*c*f)*\sinh(f*x + e)^4 + 3a*b^2*d - (3a^2*b + b^3)*d*e + (3a \\
& ^2*b + b^3)*c*f - 2*(3a*b^2*d - (3a^2*b + b^3)*d*e + (3a^2*b + b^3)*c*f) \\
& *\cosh(f*x + e)^2 - 2*(3a*b^2*d - (3a^2*b + b^3)*d*e + (3a^2*b + b^3)*c*f \\
& - 3*(3a*b^2*d - (3a^2*b + b^3)*d*e + (3a^2*b + b^3)*c*f)*\cosh(f*x + e)^ \\
& 2)*\sinh(f*x + e)^2 + 4*((3a*b^2*d - (3a^2*b + b^3)*d*e + (3a^2*b + b^3)* \\
& c*f)*\cosh(f*x + e)^3 - (3a*b^2*d - (3a^2*b + b^3)*d*e + (3a^2*b + b^3)*c \\
& *f)*\cosh(f*x + e))*\sinh(f*x + e))*\log(\cosh(f*x + e) + \sinh(f*x + e) - 1) + \\
& 2*(((3a^2*b + b^3)*d*f*x + (3a^2*b + b^3)*d*e)*\cosh(f*x + e)^4 + 4*((3a^ \\
& 2*b + b^3)*d*f*x + (3a^2*b + b^3)*d*e)*\cosh(f*x + e)*\sinh(f*x + e)^3 + ((3 \\
& a^2*b + b^3)*d*f*x + (3a^2*b + b^3)*d*e)*\sinh(f*x + e)^4 + (3a^2*b + b^3 \\
&)*d*f*x + (3a^2*b + b^3)*d*e - 2*((3a^2*b + b^3)*d*f*x + (3a^2*b + b^3)* \\
& d*e)*\cosh(f*x + e)^2 - 2*((3a^2*b + b^3)*d*f*x + (3a^2*b + b^3)*d*e - 3*(\\
& (3a^2*b + b^3)*d*f*x + (3a^2*b + b^3)*d*e)*\cosh(f*x + e)^2)*\sinh(f*x + e) \\
& ^2 + 4*(((3a^2*b + b^3)*d*f*x + (3a^2*b + b^3)*d*e)*\cosh(f*x + e)^3 - ((3 \\
& a^2*b + b^3)*d*f*x + (3a^2*b + b^3)*d*e)*\cosh(f*x + e))*\sinh(f*x + e))*\log \\
& (-\cosh(f*x + e) - \sinh(f*x + e) + 1) + 4*(((a^3 - 3a^2*b + 3a*b^2 - b^3) \\
& *d*f^2*x^2 - 12a*b^2*d*e + 2*(3a^2*b + b^3)*d*e^2 - 4*(3a^2*b + b^3)*c*e \\
& *f - 2*(6a*b^2*d*f - (a^3 - 3a^2*b + 3a*b^2 - b^3)*c*f^2)*x)*\cosh(f*x + \\
& e)^3 - ((a^3 - 3a^2*b + 3a*b^2 - b^3)*d*f^2*x^2 - 12a*b^2*d*e + b^3*d + \\
& 2*(3a^2*b + b^3)*d*e^2 - 2*(2*(3a^2*b + b^3)*c*e - (3a*b^2 + b^3)*c)*f + \\
& 2*((a^3 - 3a^2*b + 3a*b^2 - b^3)*c*f^2 - (3a*b^2 - b^3)*d*f)*x)*\cosh(f*x \\
& + e))*\sinh(f*x + e))/(f^2*\cosh(f*x + e)^4 + 4*f^2*\cosh(f*x + e)*\sinh(f*x \\
& + e)^3 + f^2*\sinh(f*x + e)^4 - 2*f^2*\cosh(f*x + e)^2 + 2*(3*f^2*\cosh(f*x + \\
& e)^2 - f^2)*\sinh(f*x + e)^2 + f^2 + 4*(f^2*\cosh(f*x + e)^3 - f^2*\cosh(f*x + \\
& e))*\sinh(f*x + e))
\end{aligned}$$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b \coth(e + fx))^3 (c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*(a+b*coth(f*x+e))**3,x)

[Out] Integral((a + b*coth(e + f*x))**3*(c + d*x), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)(b \coth(fx + e) + a)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*(a+b*coth(f*x+e))^3,x, algorithm="giac")

[Out] integrate((d*x + c)*(b*coth(f*x + e) + a)^3, x)

$$3.50 \quad \int \frac{(a+b \coth(e+fx))^3}{c+dx} dx$$

Optimal. Leaf size=22

$$\text{Unintegrable} \left(\frac{(a+b \coth(e+fx))^3}{c+dx}, x \right)$$

[Out] Unintegrable[(a + b*Coth[e + f*x])^3/(c + d*x), x]

Rubi [A] time = 0.054938, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{(a+b \coth(e+fx))^3}{c+dx} dx$$

Verification is Not applicable to the result.

[In] Int[(a + b*Coth[e + f*x])^3/(c + d*x), x]

[Out] Defer[Int] [(a + b*Coth[e + f*x])^3/(c + d*x), x]

Rubi steps

$$\int \frac{(a+b \coth(e+fx))^3}{c+dx} dx = \int \frac{(a+b \coth(e+fx))^3}{c+dx} dx$$

Mathematica [A] time = 62.5076, size = 0, normalized size = 0.

$$\int \frac{(a+b \coth(e+fx))^3}{c+dx} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b*Coth[e + f*x])^3/(c + d*x), x]

[Out] Integrate[(a + b*Coth[e + f*x])^3/(c + d*x), x]

Maple [A] time = 0.599, size = 0, normalized size = 0.

$$\int \frac{(a + b \coth(fx + e))^3}{dx + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*coth(f*x+e))^3/(d*x+c),x)

[Out] int((a+b*coth(f*x+e))^3/(d*x+c),x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{a^3 \log(dx + c)}{d} + \frac{(3a^2b + 3ab^2 + b^3) \log(dx + c)}{d} + \frac{6ab^2dfx + 6ab^2cf - b^3d - (6ab^2cfe^{(2e)} + (2cfe^{(2e)} - de^{(2e)}))b^3}{d^2f^2x^2 + 2cdf^2x + c^2f^2 + (d^2f^2x^2e^{(4e)} + 2cdf^2xe^{(4e)} + c^2f^2e^{(4e)})e^{(2e)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*coth(f*x+e))^3/(d*x+c),x, algorithm="maxima")

[Out] a^3*log(d*x + c)/d + (3*a^2*b + 3*a*b^2 + b^3)*log(d*x + c)/d + (6*a*b^2*d*f*x + 6*a*b^2*c*f - b^3*d - (6*a*b^2*c*f*e^(2*e) + (2*c*f*e^(2*e) - d*e^(2*e))*b^3 + 2*(3*a*b^2*d*f*e^(2*e) + b^3*d*f*e^(2*e))*x)*e^(2*f*x))/(d^2*f^2*x^2 + 2*c*d*f^2*x + c^2*f^2 + (d^2*f^2*x^2*e^(4*e) + 2*c*d*f^2*x*e^(4*e) + c^2*f^2*e^(4*e))*e^(4*f*x) - 2*(d^2*f^2*x^2*e^(2*e) + 2*c*d*f^2*x*e^(2*e) + c^2*f^2*e^(2*e))*e^(2*f*x)) - integrate((3*a^2*b*c^2*f^2 - 3*a*b^2*c*d*f + (c^2*f^2 + d^2)*b^3 + (3*a^2*b*d^2*f^2 + b^3*d^2*f^2)*x^2 + (6*a^2*b*c*d*f^2 + 2*b^3*c*d*f^2 - 3*a*b^2*d^2*f)*x)/(d^3*f^2*x^3 + 3*c*d^2*f^2*x^2 + 3*c^2*d*f^2*x + c^3*f^2 + (d^3*f^2*x^3*e^e + 3*c*d^2*f^2*x^2*e^e + 3*c^2*d*f^2*x*e^e + c^3*f^2*e^e)*e^(f*x)), x) + integrate(-(3*a^2*b*c^2*f^2 - 3*a*b^2*c*d*f + (c^2*f^2 + d^2)*b^3 + (3*a^2*b*d^2*f^2 + b^3*d^2*f^2)*x^2 + (6*a^2*b*c*d*f^2 + 2*b^3*c*d*f^2 - 3*a*b^2*d^2*f)*x)/(d^3*f^2*x^3 + 3*c*d^2*f^2*x^2 + 3*c^2*d*f^2*x + c^3*f^2 - (d^3*f^2*x^3*e^e + 3*c*d^2*f^2*x^2*e^e + 3*c^2*d*f^2*x*e^e + c^3*f^2*e^e)*e^(f*x)), x)

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^3 \coth(fx + e)^3 + 3ab^2 \coth(fx + e)^2 + 3a^2b \coth(fx + e) + a^3}{dx + c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*coth(f*x+e))^3/(d*x+c),x, algorithm="fricas")

[Out] integral((b^3*coth(f*x + e)^3 + 3*a*b^2*coth(f*x + e)^2 + 3*a^2*b*coth(f*x + e) + a^3)/(d*x + c), x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \coth(e + fx))^3}{c + dx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*coth(f*x+e))**3/(d*x+c),x)

[Out] Integral((a + b*coth(e + f*x))**3/(c + d*x), x)

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \coth(fx + e) + a)^3}{dx + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*coth(f*x+e))^3/(d*x+c),x, algorithm="giac")

[Out] integrate((b*coth(f*x + e) + a)^3/(d*x + c), x)

$$3.51 \quad \int \frac{(a+b \coth(e+fx))^3}{(c+dx)^2} dx$$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{(a+b \coth(e+fx))^3}{(c+dx)^2}, x\right)$$

[Out] Unintegrable[(a + b*Coth[e + f*x])^3/(c + d*x)^2, x]

Rubi [A] time = 0.0529811, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{(a+b \coth(e+fx))^3}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

[In] Int[(a + b*Coth[e + f*x])^3/(c + d*x)^2, x]

[Out] Defer[Int] [(a + b*Coth[e + f*x])^3/(c + d*x)^2, x]

Rubi steps

$$\int \frac{(a+b \coth(e+fx))^3}{(c+dx)^2} dx = \int \frac{(a+b \coth(e+fx))^3}{(c+dx)^2} dx$$

Mathematica [A] time = 59.5268, size = 0, normalized size = 0.

$$\int \frac{(a+b \coth(e+fx))^3}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b*Coth[e + f*x])^3/(c + d*x)^2, x]

[Out] Integrate[(a + b*Coth[e + f*x])^3/(c + d*x)^2, x]

Maple [A] time = 0.924, size = 0, normalized size = 0.

$$\int \frac{(a + b \coth(fx + e))^3}{(dx + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*coth(f*x+e))^3/(d*x+c)^2,x)

[Out] int((a+b*coth(f*x+e))^3/(d*x+c)^2,x)

Maxima [A] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*coth(f*x+e))^3/(d*x+c)^2,x, algorithm="maxima")

[Out]
$$-a^3/(d^2x + cd) - (3a^2b^2c^2f^2 + 3(c^2f^2 - 2cd^2f)ab^2 + (c^2f^2 + 2d^2)b^3 + (3a^2b^2d^2f^2 + 3ab^2d^2f^2 + b^3d^2f^2)x^2 + 2(3a^2b^2cd^2f^2 + b^3cd^2f^2 + 3(c^2d^2f^2 - d^2f)ab^2)x + (3a^2b^2c^2f^2e^{4e} + 3ab^2c^2f^2e^{4e} + b^3c^2f^2e^{4e} + (3a^2b^2d^2f^2e^{4e} + 3ab^2d^2f^2e^{4e} + b^3d^2f^2e^{4e}))x^2 + 2(3a^2b^2cd^2f^2e^{4e} + 3ab^2cd^2f^2e^{4e} + b^3cd^2f^2e^{4e}))x)e^{4fx} - 2(3a^2b^2c^2f^2e^{2e} + 3(c^2f^2e^{2e} - cd^2f^2e^{2e}))ab^2 + (c^2f^2e^{2e} - cd^2f^2e^{2e} + d^2e^{2e})b^3 + (3a^2b^2d^2f^2e^{2e} + 3ab^2d^2f^2e^{2e} + b^3d^2f^2e^{2e}))x^2 + (6a^2b^2cd^2f^2e^{2e} + 3(2cd^2f^2e^{2e} - d^2f^2e^{2e}))ab^2 + (2cd^2f^2e^{2e} - d^2f^2e^{2e})b^3)x)e^{2fx})/(d^4f^2x^3 + 3cd^3f^2x^2 + 3c^2d^2f^2x + c^3d^2f^2 + (d^4f^2x^3e^{4e} + 3cd^3f^2x^2e^{4e} + 3c^2d^2f^2xe^{4e} + c^3d^2f^2e^{4e}))e^{4fx} - 2(d^4f^2x^3e^{2e} + 3cd^3f^2x^2e^{2e} + 3c^2d^2f^2xe^{2e} + c^3d^2f^2e^{2e}))e^{2fx}) - \int ((3a^2b^2c^2f^2 - 6ab^2cd^2f + (c^2f^2 + 3d^2)b^3 + (3a^2b^2d^2f^2 + b^3d^2f^2)x^2 + 2(3a^2b^2cd^2f^2 + b^3cd^2f^2 - 3ab^2d^2f)x)/(d^4f^2x^4 + 4cd^3f^2x^3 + 6c^2d^2f^2x^2 + 4c^3d^2f^2x + c^4f^2 + (d^4f^2x^4e^e + 4cd^3f^2x^3e^e + 6c^2d^2f^2x^2e^e + 4c^3d^2f^2xe^e + c^4f^2e^e))e^{fx}), x) + \int (- (3a^2b^2c^2f^2 - 6ab^2cd^2f + (c^2f^2 + 3d^2)b^3 + (3a^2b^2d^2f^2 + b^3d^2f^2)x^2 + 2(3a^2b^2cd^2f^2 + b^3cd^2f^2 - 3ab^2d^2f)x)/(d^4f^2x^4 + 4cd^3f^2x^3 + 6c^2d^2f^2x^2 + 4c^3d^2f^2x + c^4f^2 + (d^4f^2x^4e^e + 4cd^3f^2x^3e^e + 6c^2d^2f^2x^2e^e + 4c^3d^2f^2xe^e + c^4f^2e^e))e^{fx}), x)$$

$$\frac{(d^2 d^2 f) x}{(d^4 f^2 x^4 + 4 c d^3 f^2 x^3 + 6 c^2 d^2 f^2 x^2 + 4 c^3 d f^2 x + c^4 f^2 - (d^4 f^2 x^4 e^e + 4 c d^3 f^2 x^3 e^e + 6 c^2 d^2 f^2 x^2 e^e + 4 c^3 d f^2 x e^e + c^4 f^2 e^e) e^{(f x)})}, x$$

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{b^3 \coth(fx + e)^3 + 3ab^2 \coth(fx + e)^2 + 3a^2b \coth(fx + e) + a^3}{d^2 x^2 + 2cdx + c^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*coth(f*x+e))^3/(d*x+c)^2,x, algorithm="fricas")

[Out] integral((b^3*coth(f*x + e)^3 + 3*a*b^2*coth(f*x + e)^2 + 3*a^2*b*coth(f*x + e) + a^3)/(d^2*x^2 + 2*c*d*x + c^2), x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \coth(e + fx))^3}{(c + dx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*coth(f*x+e))**3/(d*x+c)**2,x)

[Out] Integral((a + b*coth(e + f*x))**3/(c + d*x)**2, x)

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \coth(fx + e) + a)^3}{(dx + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*coth(f*x+e))^3/(d*x+c)^2,x, algorithm="giac")

```
[Out] integrate((b*coth(f*x + e) + a)^3/(d*x + c)^2, x)
```

$$3.52 \quad \int \frac{(c+dx)^3}{a+b \coth(e+fx)} dx$$

Optimal. Leaf size=210

$$\frac{3bd^2(c+dx)\text{PolyLog}\left(3, \frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{2f^3(a^2-b^2)} + \frac{3bd(c+dx)^2\text{PolyLog}\left(2, \frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{2f^2(a^2-b^2)} + \frac{3bd^3\text{PolyLog}\left(4, \frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{4f^4(a^2-b^2)} - \frac{b(c+dx)^4}{4(a+b)d} - \frac{b^2(c+dx)^3 \text{Log}\left[1 - \frac{a-b}{(a+b)E^{2(e+fx)}}\right]}{(a^2-b^2)f} + \frac{3bd^2(c+dx)^2 \text{PolyLog}\left[2, \frac{a-b}{(a+b)E^{2(e+fx)}}\right]}{2(a^2-b^2)f^2} + \frac{3bd^3(c+dx) \text{PolyLog}\left[3, \frac{a-b}{(a+b)E^{2(e+fx)}}\right]}{2(a^2-b^2)f^3} + \frac{3bd^4 \text{PolyLog}\left[4, \frac{a-b}{(a+b)E^{2(e+fx)}}\right]}{4(a^2-b^2)f^4}$$

[Out] (c + d*x)^4/(4*(a + b)*d) - (b*(c + d*x)^3*Log[1 - (a - b)/((a + b)*E^(2*(e + f*x)))]/((a^2 - b^2)*f) + (3*b*d*(c + d*x)^2*PolyLog[2, (a - b)/((a + b)*E^(2*(e + f*x)))]/(2*(a^2 - b^2)*f^2) + (3*b*d^2*(c + d*x)*PolyLog[3, (a - b)/((a + b)*E^(2*(e + f*x)))]/(2*(a^2 - b^2)*f^3) + (3*b*d^3*PolyLog[4, (a - b)/((a + b)*E^(2*(e + f*x)))]/(4*(a^2 - b^2)*f^4)

Rubi [A] time = 0.344883, antiderivative size = 210, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$, Rules used = {3731, 2190, 2531, 6609, 2282, 6589}

$$\frac{3bd^2(c+dx)\text{PolyLog}\left(3, \frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{2f^3(a^2-b^2)} + \frac{3bd(c+dx)^2\text{PolyLog}\left(2, \frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{2f^2(a^2-b^2)} + \frac{3bd^3\text{PolyLog}\left(4, \frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{4f^4(a^2-b^2)} - \frac{b(c+dx)^4}{4(a+b)d} - \frac{b^2(c+dx)^3 \text{Log}\left[1 - \frac{a-b}{(a+b)E^{2(e+fx)}}\right]}{(a^2-b^2)f} + \frac{3bd^2(c+dx)^2 \text{PolyLog}\left[2, \frac{a-b}{(a+b)E^{2(e+fx)}}\right]}{2(a^2-b^2)f^2} + \frac{3bd^3(c+dx) \text{PolyLog}\left[3, \frac{a-b}{(a+b)E^{2(e+fx)}}\right]}{2(a^2-b^2)f^3} + \frac{3bd^4 \text{PolyLog}\left[4, \frac{a-b}{(a+b)E^{2(e+fx)}}\right]}{4(a^2-b^2)f^4}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)^3/(a + b*Coth[e + f*x]),x]

[Out] (c + d*x)^4/(4*(a + b)*d) - (b*(c + d*x)^3*Log[1 - (a - b)/((a + b)*E^(2*(e + f*x)))]/((a^2 - b^2)*f) + (3*b*d*(c + d*x)^2*PolyLog[2, (a - b)/((a + b)*E^(2*(e + f*x)))]/(2*(a^2 - b^2)*f^2) + (3*b*d^2*(c + d*x)*PolyLog[3, (a - b)/((a + b)*E^(2*(e + f*x)))]/(2*(a^2 - b^2)*f^3) + (3*b*d^3*PolyLog[4, (a - b)/((a + b)*E^(2*(e + f*x)))]/(4*(a^2 - b^2)*f^4)

Rule 3731

Int[((c_.) + (d_.)*(x_))^(m_.)/((a_.) + (b_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(d*(m + 1)*(a + I*b)), x] + Dist[2*I*b, Int[((c + d*x)^m*E^(2*I*k*Pi)*E^Simp[2*I*(e + f*x), x])/((a + I*b)^2 + (a^2 + b^2)*E^(2*I*k*Pi)*E^Simp[2*I*(e + f*x), x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[4*k] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]

Rule 2190

```
Int[(((F_)^((g_)*(e_) + (f_)*(x_)))^(n_))*((c_) + (d_)*(x_))^(m_)]/
((a_) + (b_)*(F_)^((g_)*(e_) + (f_)*(x_)))^(n_), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_)*(F_)^((c_)*(a_) + (b_)*(x_))]^(n_)]*(f_) + (g_)
*(x_)^(m_), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
))^n]])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_) + (f_)*(x_))^(m_)*PolyLog[n_, (d_)*(F_)^((c_)*(a_) + (b_
)*(x_))]^(p_)], x_Symbol] := Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(
m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_)*(a_) + (b_)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_)*(a_) + (b_)*(x_)]^(p_)]/((d_) + (e_)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{(c+dx)^3}{a+b \coth(e+fx)} dx &= \frac{(c+dx)^4}{4(a+b)d} - (2b) \int \frac{e^{-2(e+fx)}(c+dx)^3}{(a+b)^2 + (-a^2+b^2)e^{-2(e+fx)}} dx \\
&= \frac{(c+dx)^4}{4(a+b)d} - \frac{b(c+dx)^3 \log\left(1 - \frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{(a^2-b^2)f} + \frac{(3bd) \int (c+dx)^2 \log\left(1 + \frac{(-a^2+b^2)e^{-2(e+fx)}}{(a+b)^2}\right) dx}{(a^2-b^2)f} \\
&= \frac{(c+dx)^4}{4(a+b)d} - \frac{b(c+dx)^3 \log\left(1 - \frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{(a^2-b^2)f} + \frac{3bd(c+dx)^2 \text{Li}_2\left(\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{2(a^2-b^2)f^2} - \frac{(3bd^2) \int (c+dx)}{2(a^2-b^2)f^2} \\
&= \frac{(c+dx)^4}{4(a+b)d} - \frac{b(c+dx)^3 \log\left(1 - \frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{(a^2-b^2)f} + \frac{3bd(c+dx)^2 \text{Li}_2\left(\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{2(a^2-b^2)f^2} + \frac{3bd^2(c+dx)}{2(a^2-b^2)f^2} \\
&= \frac{(c+dx)^4}{4(a+b)d} - \frac{b(c+dx)^3 \log\left(1 - \frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{(a^2-b^2)f} + \frac{3bd(c+dx)^2 \text{Li}_2\left(\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{2(a^2-b^2)f^2} + \frac{3bd^2(c+dx)}{2(a^2-b^2)f^2} \\
&= \frac{(c+dx)^4}{4(a+b)d} - \frac{b(c+dx)^3 \log\left(1 - \frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{(a^2-b^2)f} + \frac{3bd(c+dx)^2 \text{Li}_2\left(\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{2(a^2-b^2)f^2} + \frac{3bd^2(c+dx)}{2(a^2-b^2)f^2}
\end{aligned}$$

Mathematica [A] time = 3.36607, size = 239, normalized size = 1.14

$$\frac{b \left(\frac{3d \left(2f^2(c+dx)^2 \text{PolyLog}\left(2, \frac{(a-b)e^{-2(e+fx)}}{a+b}\right) + d \left(2f(c+dx) \text{PolyLog}\left(3, \frac{(a-b)e^{-2(e+fx)}}{a+b}\right) + d \text{PolyLog}\left(4, \frac{(a-b)e^{-2(e+fx)}}{a+b}\right) \right) \right)}{f^4(a-b)} - \frac{4(c+dx)^3 \log\left(\frac{(b-a)e^{-2(e+fx)}}{a+b} + 1\right)}{f(a-b)} + \frac{d}{4(a+b)} \right)}{4(a+b)}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)^3/(a + b*Coth[e + f*x]),x]

[Out] (b*((2*(c + d*x)^4)/(d*(a*(-1 + E^(2*e)) + b*(1 + E^(2*e)))) - (4*(c + d*x)^3*Log[1 + (-a + b)/((a + b)*E^(2*(e + f*x))])/(a - b)*f) + (3*d*(2*f^2*(c + d*x)^2*PolyLog[2, (a - b)/((a + b)*E^(2*(e + f*x))]) + d*(2*f*(c + d*x)*PolyLog[3, (a - b)/((a + b)*E^(2*(e + f*x))]) + d*PolyLog[4, (a - b)/((a + b)*E^(2*(e + f*x))]))/(a - b)*f^4))/(4*(a + b)) + (x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3)*Sinh[e])/(4*(b*Cosh[e] + a*Sinh[e]))

Maple [B] time = 0.203, size = 1145, normalized size = 5.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x+c)^3/(a+b*coth(f*x+e)),x)`

[Out]
$$\begin{aligned} & 3*b/(a+b)/f^2*c^2*d/(a-b)*e^{2+3*b/(a+b)*c^2*d/(a-b)*x^2+2*b/(a+b)/f^3*d^3*e} \\ & ^3/(a-b)*x+3/2*b/(a+b)/f^3*c*d^2/(a-b)*\text{polylog}(3,(a+b)*\exp(2*f*x+2*e)/(a-b) \\ &)-3/2*b/(a+b)/f^2*c^2*d/(a-b)*\text{polylog}(2,(a+b)*\exp(2*f*x+2*e)/(a-b))-b/(a+b) \\ & /f*d^3/(a-b)*\ln(1-(a+b)*\exp(2*f*x+2*e)/(a-b))*x^3-b/(a+b)/f^4*d^3/(a-b)*\ln(\\ & 1-(a+b)*\exp(2*f*x+2*e)/(a-b))*e^{-3-3/2*b/(a+b)/f^2*d^3/(a-b)*\text{polylog}(2,(a+b) \\ & *\exp(2*f*x+2*e)/(a-b))*x^2+3/2*b/(a+b)/f^3*d^3/(a-b)*\text{polylog}(3,(a+b)*\exp(2* \\ & f*x+2*e)/(a-b))*x-2*b/(a+b)/f^4*d^3*e^3/(a-b)*\ln(\exp(f*x+e))+b/(a+b)/f^4*d^ \\ & 3*e^3/(a-b)*\ln(a*\exp(2*f*x+2*e)+b*\exp(2*f*x+2*e)-a+b)+2*b/(a+b)*c*d^2/(a-b) \\ & *x^3-4*b/(a+b)/f^3*c*d^2/(a-b)*e^{3+1/(a+b)*c*d^2*x^3+3/2/(a+b)*c^2*d*x^2+1/ \\ & 4/(a+b)*d^3*x^4+1/(a+b)*c^3*x-b/(a+b)/f*c^3/(a-b)*\ln(a*\exp(2*f*x+2*e)+b*\exp \\ & (2*f*x+2*e)-a+b)+3*b/(a+b)/f^2*c^2*d*e/(a-b)*\ln(a*\exp(2*f*x+2*e)+b*\exp(2*f* \\ & x+2*e)-a+b)-6*b/(a+b)/f^2*c^2*d*e/(a-b)*\ln(\exp(f*x+e))+3*b/(a+b)/f^3*c*d^2*e \\ & ^2/(a-b)*\ln(1-(a+b)*\exp(2*f*x+2*e)/(a-b))-3*b/(a+b)/f^2*c^2*d/(a-b)*\ln(1-(\\ & a+b)*\exp(2*f*x+2*e)/(a-b))*e^{-3*b/(a+b)/f*c*d^2/(a-b)*\ln(1-(a+b)*\exp(2*f*x+2 \\ & *e)/(a-b))*x^2-3*b/(a+b)/f^2*c*d^2/(a-b)*\text{polylog}(2,(a+b)*\exp(2*f*x+2*e)/(a- \\ & b))*x-3*b/(a+b)/f^3*c*d^2*e^2/(a-b)*\ln(a*\exp(2*f*x+2*e)+b*\exp(2*f*x+2*e)-a+ \\ & b)+3/2*b/(a+b)/f^4*d^3*e^4/(a-b)+1/2*b/(a+b)*d^3/(a-b)*x^4-3/4*b/(a+b)/f^4* \\ & d^3/(a-b)*\text{polylog}(4,(a+b)*\exp(2*f*x+2*e)/(a-b))+2*b/(a+b)/f*c^3/(a-b)*\ln(\exp \\ & (f*x+e))+6*b/(a+b)/f*c^2*d/(a-b)*e*x-6*b/(a+b)/f^2*c*d^2/(a-b)*e^2*x+6*b/(\\ & a+b)/f^3*c*d^2*e^2/(a-b)*\ln(\exp(f*x+e))-3*b/(a+b)/f*c^2*d/(a-b)*\ln(1-(a+b)* \\ & \exp(2*f*x+2*e)/(a-b))*x \end{aligned}$$

Maxima [B] time = 1.80275, size = 703, normalized size = 3.35

$$\frac{3 \left(2 f x \log \left(-\frac{(a e^{(2 e)}+b e^{(2 e)}) e^{(2 f x)}}{a-b} + 1 \right) + \text{Li}_2 \left(\frac{(a e^{(2 e)}+b e^{(2 e)}) e^{(2 f x)}}{a-b} \right) \right) b c^2 d}{2 (a^2 f^2 - b^2 f^2)} - \frac{3 \left(2 f^2 x^2 \log \left(-\frac{(a e^{(2 e)}+b e^{(2 e)}) e^{(2 f x)}}{a-b} + 1 \right) + 2 f x \text{Li}_2 \left(\frac{(a e^{(2 e)}+b e^{(2 e)}) e^{(2 f x)}}{a-b} \right) \right)}{2 (a^2 f^3 - b^2 f^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)^3/(a+b*coth(f*x+e)),x, algorithm="maxima")`

```
[Out] -3/2*(2*f*x*log(-(a*e^(2*e) + b*e^(2*e))*e^(2*f*x)/(a - b) + 1) + dilog((a*
e^(2*e) + b*e^(2*e))*e^(2*f*x)/(a - b)))*b*c^2*d/(a^2*f^2 - b^2*f^2) - 3/2*
(2*f^2*x^2*log(-(a*e^(2*e) + b*e^(2*e))*e^(2*f*x)/(a - b) + 1) + 2*f*x*dilo
g((a*e^(2*e) + b*e^(2*e))*e^(2*f*x)/(a - b)) - polylog(3, (a*e^(2*e) + b*e^
(2*e))*e^(2*f*x)/(a - b)))*b*c*d^2/(a^2*f^3 - b^2*f^3) - 1/3*(4*f^3*x^3*log
(-(a*e^(2*e) + b*e^(2*e))*e^(2*f*x)/(a - b) + 1) + 6*f^2*x^2*dilog((a*e^(2*
e) + b*e^(2*e))*e^(2*f*x)/(a - b)) - 6*f*x*polylog(3, (a*e^(2*e) + b*e^(2*e
))*e^(2*f*x)/(a - b)) + 3*polylog(4, (a*e^(2*e) + b*e^(2*e))*e^(2*f*x)/(a -
b)))*b*d^3/(a^2*f^4 - b^2*f^4) - c^3*(b*log(-(a - b)*e^(-2*f*x - 2*e) + a
+ b)/((a^2 - b^2)*f) - (f*x + e)/((a + b)*f)) + 1/2*(b*d^3*f^4*x^4 + 4*b*c*
d^2*f^4*x^3 + 6*b*c^2*d*f^4*x^2)/(a^2*f^4 - b^2*f^4) + 1/4*(d^3*x^4 + 4*c*d
^2*x^3 + 6*c^2*d*x^2)/(a + b)
```

Fricas [C] time = 2.3728, size = 1808, normalized size = 8.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+b*coth(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/4*((a + b)*d^3*f^4*x^4 + 4*(a + b)*c*d^2*f^4*x^3 + 6*(a + b)*c^2*d*f^4*x^
2 + 4*(a + b)*c^3*f^4*x - 24*b*d^3*polylog(4, sqrt((a + b)/(a - b))*(cosh(f
*x + e) + sinh(f*x + e))) - 24*b*d^3*polylog(4, -sqrt((a + b)/(a - b))*(cos
h(f*x + e) + sinh(f*x + e))) - 12*(b*d^3*f^2*x^2 + 2*b*c*d^2*f^2*x + b*c^2*
d*f^2)*dilog(sqrt((a + b)/(a - b))*(cosh(f*x + e) + sinh(f*x + e))) - 12*(b
*d^3*f^2*x^2 + 2*b*c*d^2*f^2*x + b*c^2*d*f^2)*dilog(-sqrt((a + b)/(a - b))*
(cosh(f*x + e) + sinh(f*x + e))) + 4*(b*d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b*c^2
*d*e*f^2 - b*c^3*f^3)*log(2*(a + b)*cosh(f*x + e) + 2*(a + b)*sinh(f*x + e)
+ 2*(a - b)*sqrt((a + b)/(a - b))) + 4*(b*d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b*
c^2*d*e*f^2 - b*c^3*f^3)*log(2*(a + b)*cosh(f*x + e) + 2*(a + b)*sinh(f*x +
e) - 2*(a - b)*sqrt((a + b)/(a - b))) - 4*(b*d^3*f^3*x^3 + 3*b*c*d^2*f^3*x
^2 + 3*b*c^2*d*f^3*x + b*d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2)*log(s
qrt((a + b)/(a - b))*(cosh(f*x + e) + sinh(f*x + e)) + 1) - 4*(b*d^3*f^3*x^
3 + 3*b*c*d^2*f^3*x^2 + 3*b*c^2*d*f^3*x + b*d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b
*c^2*d*e*f^2)*log(-sqrt((a + b)/(a - b))*(cosh(f*x + e) + sinh(f*x + e)) +
1) + 24*(b*d^3*f*x + b*c*d^2*f)*polylog(3, sqrt((a + b)/(a - b))*(cosh(f*x
+ e) + sinh(f*x + e))) + 24*(b*d^3*f*x + b*c*d^2*f)*polylog(3, -sqrt((a + b
)/(a - b))*(cosh(f*x + e) + sinh(f*x + e))))/(a^2 - b^2)*f^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(c + dx)^3}{a + b \coth(e + fx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**3/(a+b*coth(f*x+e)),x)

[Out] Integral((c + d*x)**3/(a + b*coth(e + f*x)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx + c)^3}{b \coth(fx + e) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3/(a+b*coth(f*x+e)),x, algorithm="giac")

[Out] integrate((d*x + c)^3/(b*coth(f*x + e) + a), x)

3.53 $\int \frac{(c+dx)^2}{a+b \coth(e+fx)} dx$

Optimal. Leaf size=156

$$\frac{bd(c+dx)\text{PolyLog}\left(2, \frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{f^2(a^2-b^2)} + \frac{bd^2\text{PolyLog}\left(3, \frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{2f^3(a^2-b^2)} - \frac{b(c+dx)^2 \log\left(1 - \frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{f(a^2-b^2)} + \frac{(c+dx)^3}{3d(a+b)}$$

[Out] (c + d*x)^3/(3*(a + b)*d) - (b*(c + d*x)^2*Log[1 - (a - b)/((a + b)*E^(2*(e + f*x)))]/((a^2 - b^2)*f) + (b*d*(c + d*x)*PolyLog[2, (a - b)/((a + b)*E^(2*(e + f*x)))]/((a^2 - b^2)*f^2) + (b*d^2*PolyLog[3, (a - b)/((a + b)*E^(2*(e + f*x)))]/(2*(a^2 - b^2)*f^3)

Rubi [A] time = 0.284352, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {3731, 2190, 2531, 2282, 6589}

$$\frac{bd(c+dx)\text{PolyLog}\left(2, \frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{f^2(a^2-b^2)} + \frac{bd^2\text{PolyLog}\left(3, \frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{2f^3(a^2-b^2)} - \frac{b(c+dx)^2 \log\left(1 - \frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{f(a^2-b^2)} + \frac{(c+dx)^3}{3d(a+b)}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)^2/(a + b*Coth[e + f*x]),x]

[Out] (c + d*x)^3/(3*(a + b)*d) - (b*(c + d*x)^2*Log[1 - (a - b)/((a + b)*E^(2*(e + f*x)))]/((a^2 - b^2)*f) + (b*d*(c + d*x)*PolyLog[2, (a - b)/((a + b)*E^(2*(e + f*x)))]/((a^2 - b^2)*f^2) + (b*d^2*PolyLog[3, (a - b)/((a + b)*E^(2*(e + f*x)))]/(2*(a^2 - b^2)*f^3)

Rule 3731

Int[((c_.) + (d_.)*(x_))^(m_.)/((a_.) + (b_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(d*(m + 1)*(a + I*b)), x] + Dist[2*I*b, Int[((c + d*x)^m*E^(2*I*k*Pi)*E^Simp[2*I*(e + f*x), x])/((a + I*b)^2 + (a^2 + b^2)*E^(2*I*k*Pi)*E^Simp[2*I*(e + f*x), x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[4*k] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]

Rule 2190

Int[(((F_)^(g_.)*((e_.) + (f_.)*(x_)))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/((a_.) + (b_.)*((F_)^(g_.)*((e_.) + (f_.)*(x_)))^(n_.)), x_Symbol] :> Simp

```

[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

```

Rule 2531

```

Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_)))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
))))^n])]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x))))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]

```

Rule 2282

```

Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

```

Rule 6589

```

Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]

```

Rubi steps

$$\begin{aligned}
\int \frac{(c+dx)^2}{a+b \coth(e+fx)} dx &= \frac{(c+dx)^3}{3(a+b)d} - (2b) \int \frac{e^{-2(e+fx)}(c+dx)^2}{(a+b)^2 + (-a^2+b^2)e^{-2(e+fx)}} dx \\
&= \frac{(c+dx)^3}{3(a+b)d} - \frac{b(c+dx)^2 \log\left(1 - \frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{(a^2-b^2)f} + \frac{(2bd) \int (c+dx) \log\left(1 + \frac{(-a^2+b^2)e^{-2(e+fx)}}{(a+b)^2}\right) dx}{(a^2-b^2)f} \\
&= \frac{(c+dx)^3}{3(a+b)d} - \frac{b(c+dx)^2 \log\left(1 - \frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{(a^2-b^2)f} + \frac{bd(c+dx)\text{Li}_2\left(\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{(a^2-b^2)f^2} - \frac{(bd^2) \int \text{Li}_2\left(-\frac{(-a^2+b^2)e^{-2(e+fx)}}{(a+b)^2}\right) dx}{(a^2-b^2)f^2} \\
&= \frac{(c+dx)^3}{3(a+b)d} - \frac{b(c+dx)^2 \log\left(1 - \frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{(a^2-b^2)f} + \frac{bd(c+dx)\text{Li}_2\left(\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{(a^2-b^2)f^2} + \frac{(bd^2) \text{Subst}\left(\int \text{Li}_2\left(-\frac{(-a^2+b^2)e^{-2(e+fx)}}{(a+b)^2}\right) dx\right)}{2(a^2-b^2)f^2} \\
&= \frac{(c+dx)^3}{3(a+b)d} - \frac{b(c+dx)^2 \log\left(1 - \frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{(a^2-b^2)f} + \frac{bd(c+dx)\text{Li}_2\left(\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{(a^2-b^2)f^2} + \frac{bd^2\text{Li}_3\left(\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{2(a^2-b^2)f^2}
\end{aligned}$$

Mathematica [A] time = 3.59568, size = 191, normalized size = 1.22

$$\frac{b \left(\frac{3d \left(2f(c+dx) \text{PolyLog}\left(2, \frac{(a-b)e^{-2(e+fx)}}{a+b}\right) + d \text{PolyLog}\left(3, \frac{(a-b)e^{-2(e+fx)}}{a+b}\right)\right)}{f^3(a-b)} - \frac{6(c+dx)^2 \log\left(\frac{(b-a)e^{-2(e+fx)}}{a+b} + 1\right)}{f(a-b)} + \frac{4(c+dx)^3}{d(a(e^{2e}-1)+b(e^{2e}+1))} \right)}{6(a+b)} + \frac{x \sinh(e) (3c^2 + 3cd + d^2)}{3(a \sinh(e) + b \cosh(e))}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)^2/(a + b*Coth[e + f*x]),x]

[Out] (b*((4*(c + d*x)^3)/(d*(a*(-1 + E^(2*e)) + b*(1 + E^(2*e)))) - (6*(c + d*x)^2*Log[1 + (-a + b)/((a + b)*E^(2*(e + f*x))]))/((a - b)*f) + (3*d*(2*f*(c + d*x)*PolyLog[2, (a - b)/((a + b)*E^(2*(e + f*x)))] + d*PolyLog[3, (a - b)/((a + b)*E^(2*(e + f*x))]))/((a - b)*f^3))/((6*(a + b)) + (x*(3*c^2 + 3*c*d*x + d^2*x^2)*Sinh[e])/(3*(b*Cosh[e] + a*Sinh[e])))

Maple [B] time = 0.147, size = 722, normalized size = 4.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((d*x+c)^2/(a+b*\text{coth}(f*x+e)), x)$

[Out] $\frac{1}{3} \frac{d^2 x^3 + 1}{(a+b)} + \frac{c d x^2 + 1}{(a+b)} + \frac{c^2 x + 2b}{(a+b)} + \frac{f c^2}{(a-b)} \ln(\exp(f x + e)) - \frac{b}{(a+b)} \frac{f c^2}{(a-b)} \ln(a \exp(2 f x + 2 e) + b \exp(2 f x + 2 e) - a + b) + \frac{2 b}{(a+b)} \frac{f^3 d^2 e^2}{(a-b)} \ln(\exp(f x + e)) - \frac{b}{(a+b)} \frac{f^3 d^2 e^2}{(a-b)} \ln(a \exp(2 f x + 2 e) + b \exp(2 f x + 2 e) - a + b) + \frac{2}{3} \frac{b}{(a+b)} \frac{d^2}{(a-b)} x^3 - \frac{2 b}{(a+b)} \frac{f^2 d^2}{(a-b)} e^2 x - \frac{4}{3} \frac{b}{(a+b)} \frac{f^3 d^2}{(a-b)} e^3 - \frac{b}{(a+b)} \frac{f d^2}{(a-b)} \ln(1 - (a+b) \exp(2 f x + 2 e) / (a-b)) x^2 + \frac{b}{(a+b)} \frac{f^3 d^2}{(a-b)} \ln(1 - (a+b) \exp(2 f x + 2 e) / (a-b)) e^2 - \frac{b}{(a+b)} \frac{f^2 d^2}{(a-b)} \text{polylog}(2, (a+b) \exp(2 f x + 2 e) / (a-b)) x + \frac{1}{2} \frac{b}{(a+b)} \frac{f^3 d^2}{(a-b)} \text{polylog}(3, (a+b) \exp(2 f x + 2 e) / (a-b)) - \frac{4 b}{(a+b)} \frac{f^2 c d e}{(a-b)} \ln(\exp(f x + e)) + \frac{2 b}{(a+b)} \frac{f^2 c d e}{(a-b)} \ln(a \exp(2 f x + 2 e) + b \exp(2 f x + 2 e) - a + b) - \frac{2 b}{(a+b)} \frac{f c d}{(a-b)} \ln(1 - (a+b) \exp(2 f x + 2 e) / (a-b)) x - \frac{2 b}{(a+b)} \frac{f^2 c d}{(a-b)} \ln(1 - (a+b) \exp(2 f x + 2 e) / (a-b)) e + \frac{2 b}{(a+b)} \frac{c d}{(a-b)} x^2 + \frac{4 b}{(a+b)} \frac{f c d}{(a-b)} e x + \frac{2 b}{(a+b)} \frac{f^2 c d}{(a-b)} e^2 - \frac{b}{(a+b)} \frac{f^2 c d}{(a-b)} \text{polylog}(2, (a+b) \exp(2 f x + 2 e) / (a-b))$

Maxima [B] time = 1.63411, size = 448, normalized size = 2.87

$$\frac{\left(2 f x \log\left(-\frac{(a e^{(2 e)} + b e^{(2 e)}) e^{(2 f x)}}{a-b} + 1\right) + \text{Li}_2\left(\frac{(a e^{(2 e)} + b e^{(2 e)}) e^{(2 f x)}}{a-b}\right)\right) b c d}{a^2 f^2 - b^2 f^2} - \frac{\left(2 f^2 x^2 \log\left(-\frac{(a e^{(2 e)} + b e^{(2 e)}) e^{(2 f x)}}{a-b} + 1\right) + 2 f x \text{Li}_2\left(\frac{(a e^{(2 e)} + b e^{(2 e)}) e^{(2 f x)}}{a-b}\right)\right)}{2(a^2 f^3 - b^2 f^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((d*x+c)^2/(a+b*\text{coth}(f*x+e)), x, \text{algorithm}="maxima")$

[Out] $-(2 f x \log(-(a e^{(2 e)} + b e^{(2 e)}) e^{(2 f x)} / (a - b) + 1) + \text{dilog}((a e^{(2 e)} + b e^{(2 e)}) e^{(2 f x)} / (a - b))) b c d / (a^2 f^2 - b^2 f^2) - 1/2 (2 f^2 x^2 \log(-(a e^{(2 e)} + b e^{(2 e)}) e^{(2 f x)} / (a - b) + 1) + 2 f x \text{dilog}((a e^{(2 e)} + b e^{(2 e)}) e^{(2 f x)} / (a - b)) - \text{polylog}(3, (a e^{(2 e)} + b e^{(2 e)}) e^{(2 f x)} / (a - b))) b d^2 / (a^2 f^3 - b^2 f^3) - c^2 (b \log(-(a - b) e^{(-2 f x - 2 e)} + a + b) / ((a^2 - b^2) f) - (f x + e) / ((a + b) f)) + 2/3 (b d^2 f^3 x^3 + 3 b c d f^3 x^2) / (a^2 f^3 - b^2 f^3) + 1/3 (d^2 x^3 + 3 c d x^2) / (a + b)$

Fricas [C] time = 2.3046, size = 1258, normalized size = 8.06

$$(a + b) d^2 f^3 x^3 + 3 (a + b) c d f^3 x^2 + 3 (a + b) c^2 f^3 x + 6 b d^2 \text{polylog}\left(3, \sqrt{\frac{a+b}{a-b}} (\cosh(f x + e) + \sinh(f x + e))\right) + 6 b d^2 \text{polylog}\left(2, \sqrt{\frac{a+b}{a-b}} (\cosh(f x + e) + \sinh(f x + e))\right) + 6 b d^2 \text{polylog}\left(1, \sqrt{\frac{a+b}{a-b}} (\cosh(f x + e) + \sinh(f x + e))\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+b*coth(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/3*((a + b)*d^2*f^3*x^3 + 3*(a + b)*c*d*f^3*x^2 + 3*(a + b)*c^2*f^3*x + 6*
b*d^2*polylog(3, sqrt((a + b)/(a - b))*(cosh(f*x + e) + sinh(f*x + e))) + 6
*b*d^2*polylog(3, -sqrt((a + b)/(a - b))*(cosh(f*x + e) + sinh(f*x + e))) -
6*(b*d^2*f*x + b*c*d*f)*dilog(sqrt((a + b)/(a - b))*(cosh(f*x + e) + sinh(
f*x + e))) - 6*(b*d^2*f*x + b*c*d*f)*dilog(-sqrt((a + b)/(a - b))*(cosh(f*x
+ e) + sinh(f*x + e))) - 3*(b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*log(2*(a
+ b)*cosh(f*x + e) + 2*(a + b)*sinh(f*x + e) + 2*(a - b)*sqrt((a + b)/(a -
b))) - 3*(b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*log(2*(a + b)*cosh(f*x + e)
+ 2*(a + b)*sinh(f*x + e) - 2*(a - b)*sqrt((a + b)/(a - b))) - 3*(b*d^2*f^2
*x^2 + 2*b*c*d*f^2*x - b*d^2*e^2 + 2*b*c*d*e*f)*log(sqrt((a + b)/(a - b))*(
cosh(f*x + e) + sinh(f*x + e)) + 1) - 3*(b*d^2*f^2*x^2 + 2*b*c*d*f^2*x - b*
d^2*e^2 + 2*b*c*d*e*f)*log(-sqrt((a + b)/(a - b))*(cosh(f*x + e) + sinh(f*x
+ e)) + 1))/((a^2 - b^2)*f^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(c + dx)^2}{a + b \coth(e + fx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2/(a+b*coth(f*x+e)),x)
```

```
[Out] Integral((c + d*x)**2/(a + b*coth(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx + c)^2}{b \coth(fx + e) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+b*coth(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2/(b*coth(f*x + e) + a), x)
```


$$3.54 \quad \int \frac{c+dx}{a+b \coth(e+fx)} dx$$

Optimal. Leaf size=108

$$\frac{bd \operatorname{PolyLog}\left(2, \frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{2f^2(a^2-b^2)} - \frac{b(c+dx) \log\left(1 - \frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{f(a^2-b^2)} + \frac{(c+dx)^2}{2d(a+b)}$$

[Out] (c + d*x)^2/(2*(a + b)*d) - (b*(c + d*x)*Log[1 - (a - b)/((a + b)*E^(2*(e + f*x)))]/((a^2 - b^2)*f) + (b*d*PolyLog[2, (a - b)/((a + b)*E^(2*(e + f*x)))])/(2*(a^2 - b^2)*f^2)

Rubi [A] time = 0.162962, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3731, 2190, 2279, 2391}

$$\frac{bd \operatorname{PolyLog}\left(2, \frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{2f^2(a^2-b^2)} - \frac{b(c+dx) \log\left(1 - \frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{f(a^2-b^2)} + \frac{(c+dx)^2}{2d(a+b)}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)/(a + b*Coth[e + f*x]), x]

[Out] (c + d*x)^2/(2*(a + b)*d) - (b*(c + d*x)*Log[1 - (a - b)/((a + b)*E^(2*(e + f*x)))]/((a^2 - b^2)*f) + (b*d*PolyLog[2, (a - b)/((a + b)*E^(2*(e + f*x)))])/(2*(a^2 - b^2)*f^2)

Rule 3731

Int[(((c_) + (d_)*(x_))^(m_))/((a_) + (b_)*tan[(e_) + Pi*(k_) + (f_)*(x_)]), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(d*(m + 1)*(a + I*b)), x] + Dist[2*I*b, Int[((c + d*x)^m * E^(2*I*k*Pi) * E^Simp[2*I*(e + f*x), x])]/((a + I*b)^2 + (a^2 + b^2) * E^(2*I*k*Pi) * E^Simp[2*I*(e + f*x), x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[4*k] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]

Rule 2190

Int[(((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)*((c_) + (d_)*(x_))^(m_))/((a_) + (b_)*((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)), x_Symbol] :> Simp[(((c + d*x)^m * Log[1 + (b*(F^(g*(e + f*x))))^n]/a)]/(b*f*g*n*Log[F]), x] - Di

```
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x))
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_))))^(n_)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned} \int \frac{c + dx}{a + b \coth(e + fx)} dx &= \frac{(c + dx)^2}{2(a + b)d} - (2b) \int \frac{e^{-2(e+fx)}(c + dx)}{(a + b)^2 + (-a^2 + b^2)e^{-2(e+fx)}} dx \\ &= \frac{(c + dx)^2}{2(a + b)d} - \frac{b(c + dx) \log\left(1 - \frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{(a^2 - b^2)f} + \frac{(bd) \int \log\left(1 + \frac{(-a^2+b^2)e^{-2(e+fx)}}{(a+b)^2}\right) dx}{(a^2 - b^2)f} \\ &= \frac{(c + dx)^2}{2(a + b)d} - \frac{b(c + dx) \log\left(1 - \frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{(a^2 - b^2)f} - \frac{(bd) \operatorname{Subst}\left(\int \frac{\log\left(1 + \frac{(-a^2+b^2)x}{(a+b)^2}\right)}{x} dx, x, e^{-2(e+fx)}\right)}{2(a^2 - b^2)f^2} \\ &= \frac{(c + dx)^2}{2(a + b)d} - \frac{b(c + dx) \log\left(1 - \frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{(a^2 - b^2)f} + \frac{bd \operatorname{Li}_2\left(\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{2(a^2 - b^2)f^2} \end{aligned}$$

Mathematica [A] time = 2.92234, size = 144, normalized size = 1.33

$$\frac{b \left(\frac{d \operatorname{PolyLog}\left(2, \frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{f^2(a-b)} - \frac{2(c+dx) \log\left(\frac{(b-a)e^{-2(e+fx)}}{a+b} + 1\right)}{f(a-b)} + \frac{2(c+dx)^2}{d(a(e^{2e}-1)+b(e^{2e}+1))} \right)}{2(a+b)} + \frac{x \sinh(e)(2c + dx)}{2(a \sinh(e) + b \cosh(e))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a + b*Coth[e + f*x]),x]
```

```
[Out] (b*((2*(c + d*x)^2)/(d*(a*(-1 + E^(2*e)) + b*(1 + E^(2*e)))) - (2*(c + d*x)
*Log[1 + (-a + b)/((a + b)*E^(2*(e + f*x))]))/((a - b)*f) + (d*PolyLog[2, (
a - b)/((a + b)*E^(2*(e + f*x))])/((a - b)*f^2))/((2*(a + b)) + (x*(2*c +
d*x)*Sinh[e])/(2*(b*Cosh[e] + a*Sinh[e])))
```

Maple [B] time = 0.131, size = 357, normalized size = 3.3

$$\frac{dx^2}{2b+2a} + \frac{cx}{a+b} + 2 \frac{cb \ln(e^{fx+e})}{(a+b)f(a-b)} - \frac{cb \ln(ae^{2fx+2e} + be^{2fx+2e} - a + b)}{(a+b)f(a-b)} - \frac{bdx}{(a+b)f(a-b)} \ln\left(1 - \frac{(a+b)e^{2fx+2e}}{a-b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(a+b*coth(f*x+e)),x)
```

```
[Out] 1/2/(a+b)*d*x^2+1/(a+b)*c*x+2*b/(a+b)/f*c/(a-b)*ln(exp(f*x+e))-b/(a+b)/f*c/
(a-b)*ln(a*exp(2*f*x+2*e)+b*exp(2*f*x+2*e)-a+b)-b/(a+b)/f*d/(a-b)*ln(1-(a+b)
)*exp(2*f*x+2*e)/(a-b)*x-b/(a+b)/f^2*d/(a-b)*ln(1-(a+b)*exp(2*f*x+2*e)/(a-
b))*e+b/(a+b)*d/(a-b)*x^2+2*b/(a+b)/f*d/(a-b)*e*x+b/(a+b)/f^2*d/(a-b)*e^2-1
/2*b/(a+b)/f^2*d/(a-b)*polylog(2,(a+b)*exp(2*f*x+2*e)/(a-b))-2*b/(a+b)/f^2*
d*e/(a-b)*ln(exp(f*x+e))+b/(a+b)/f^2*d*e/(a-b)*ln(a*exp(2*f*x+2*e)+b*exp(2*
f*x+2*e)-a+b)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2} \left(4b \int -\frac{x}{a^2 - b^2 - (a^2 e^{2e} + 2abe^{2e} + b^2 e^{2e}) e^{2fx}} dx - \frac{x^2}{a+b} \right) d - c \left(\frac{b \log(-(a-b)e^{(-2fx-2e)} + a + b)}{(a^2 - b^2)f} - \frac{fx + e}{(a+b)f} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+b*coth(f*x+e)),x, algorithm="maxima")
```

```
[Out] -1/2*(4*b*integrate(-x/(a^2 - b^2 - (a^2*e^(2*e) + 2*a*b*e^(2*e) + b^2*e^(2
*e))*e^(2*f*x)), x) - x^2/(a + b))*d - c*(b*log(-(a - b)*e^(-2*f*x - 2*e) +
a + b)/((a^2 - b^2)*f) - (f*x + e)/((a + b)*f))
```

Fricas [B] time = 2.29172, size = 798, normalized size = 7.39

$$(a+b)df^2x^2 + 2(a+b)cf^2x - 2bd\text{Li}_2\left(\sqrt{\frac{a+b}{a-b}}(\cosh(fx+e) + \sinh(fx+e))\right) - 2bd\text{Li}_2\left(-\sqrt{\frac{a+b}{a-b}}(\cosh(fx+e) + \sinh(fx+e))\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)/(a+b*coth(f*x+e)),x, algorithm="fricas")

[Out] $\frac{1}{2}((a+b)d^2x^2 + 2(a+b)cf^2x - 2bd\text{dilog}(\sqrt{\frac{a+b}{a-b}}(\cosh(fx+e) + \sinh(fx+e))) - 2bd\text{dilog}(-\sqrt{\frac{a+b}{a-b}}(\cosh(fx+e) + \sinh(fx+e)))) + 2(bde - bcf)\log(2(a+b)\cosh(fx+e) + 2(a+b)\sinh(fx+e) + 2(a-b)\sqrt{\frac{a+b}{a-b}}) + 2(bde - bcf)\log(2(a+b)\cosh(fx+e) + 2(a+b)\sinh(fx+e) - 2(a-b)\sqrt{\frac{a+b}{a-b}}) - 2(bdfx + bde)\log(\sqrt{\frac{a+b}{a-b}}(\cosh(fx+e) + \sinh(fx+e)) + 1) - 2(bdfx + bde)\log(-\sqrt{\frac{a+b}{a-b}}(\cosh(fx+e) + \sinh(fx+e)) + 1))/((a^2 - b^2)f^2)$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{c + dx}{a + b \coth(e + fx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)/(a+b*coth(f*x+e)),x)

[Out] Integral((c + d*x)/(a + b*coth(e + f*x)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx + c}{b \coth(fx + e) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)/(a+b*coth(f*x+e)),x, algorithm="giac")

[Out] integrate((d*x + c)/(b*coth(f*x + e) + a), x)

$$3.55 \quad \int \frac{1}{(c+dx)(a+b \coth(e+fx))} dx$$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{1}{(c+dx)(a+b \coth(e+fx))}, x\right)$$

[Out] Unintegrable[1/((c + d*x)*(a + b*Coth[e + f*x])), x]

Rubi [A] time = 0.0627868, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{1}{(c+dx)(a+b \coth(e+fx))} dx$$

Verification is Not applicable to the result.

[In] Int[1/((c + d*x)*(a + b*Coth[e + f*x])), x]

[Out] Defer[Int][1/((c + d*x)*(a + b*Coth[e + f*x])), x]

Rubi steps

$$\int \frac{1}{(c+dx)(a+b \coth(e+fx))} dx = \int \frac{1}{(c+dx)(a+b \coth(e+fx))} dx$$

Mathematica [A] time = 14.0718, size = 0, normalized size = 0.

$$\int \frac{1}{(c+dx)(a+b \coth(e+fx))} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/((c + d*x)*(a + b*Coth[e + f*x])), x]

[Out] Integrate[1/((c + d*x)*(a + b*Coth[e + f*x])), x]

Maple [A] time = 0.155, size = 0, normalized size = 0.

$$\int \frac{1}{(dx + c)(a + b \coth(fx + e))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(d*x+c)/(a+b*coth(f*x+e)),x)

[Out] int(1/(d*x+c)/(a+b*coth(f*x+e)),x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-2b \int -\frac{1}{a^2c - b^2c + (a^2d - b^2d)x - (a^2ce^{(2e)} + 2abce^{(2e)} + b^2ce^{(2e)} + (a^2de^{(2e)} + 2abde^{(2e)} + b^2de^{(2e)})x} e^{(2fx)} dx + \frac{\log(\dots)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d*x+c)/(a+b*coth(f*x+e)),x, algorithm="maxima")

[Out] -2*b*integrate(-1/(a^2*c - b^2*c + (a^2*d - b^2*d)*x - (a^2*c*e^(2*e) + 2*a*b*c*e^(2*e) + b^2*c*e^(2*e) + (a^2*d*e^(2*e) + 2*a*b*d*e^(2*e) + b^2*d*e^(2*e))*x)*e^(2*f*x)), x) + log(d*x + c)/(a*d + b*d)

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{adx + ac + (bdx + bc) \coth(fx + e)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d*x+c)/(a+b*coth(f*x+e)),x, algorithm="fricas")

[Out] integral(1/(a*d*x + a*c + (b*d*x + b*c)*coth(f*x + e)), x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a + b \coth(e + fx))(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d*x+c)/(a+b*coth(f*x+e)),x)

[Out] Integral(1/((a + b*coth(e + f*x))*(c + d*x)), x)

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dx + c)(b \coth(fx + e) + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d*x+c)/(a+b*coth(f*x+e)),x, algorithm="giac")

[Out] integrate(1/((d*x + c)*(b*coth(f*x + e) + a)), x)

$$3.56 \quad \int \frac{1}{(c+dx)^2(a+b \coth(e+fx))} dx$$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{1}{(c+dx)^2(a+b \coth(e+fx))}, x\right)$$

[Out] Unintegrable[1/((c + d*x)^2*(a + b*Coth[e + f*x])), x]

Rubi [A] time = 0.0599392, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{1}{(c+dx)^2(a+b \coth(e+fx))} dx$$

Verification is Not applicable to the result.

[In] Int[1/((c + d*x)^2*(a + b*Coth[e + f*x])),x]

[Out] Defer[Int][1/((c + d*x)^2*(a + b*Coth[e + f*x])), x]

Rubi steps

$$\int \frac{1}{(c+dx)^2(a+b \coth(e+fx))} dx = \int \frac{1}{(c+dx)^2(a+b \coth(e+fx))} dx$$

Mathematica [A] time = 22.4706, size = 0, normalized size = 0.

$$\int \frac{1}{(c+dx)^2(a+b \coth(e+fx))} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/((c + d*x)^2*(a + b*Coth[e + f*x])),x]

[Out] Integrate[1/((c + d*x)^2*(a + b*Coth[e + f*x])), x]

Maple [A] time = 0.156, size = 0, normalized size = 0.

$$\int \frac{1}{(dx+c)^2 (a+b\coth(fx+e))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(d*x+c)^2/(a+b*coth(f*x+e)),x)

[Out] int(1/(d*x+c)^2/(a+b*coth(f*x+e)),x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-2b \int -\frac{1}{a^2c^2 - b^2c^2 + (a^2d^2 - b^2d^2)x^2 + 2(a^2cd - b^2cd)x - (a^2c^2e^{2e} + 2abc^2e^{2e} + b^2c^2e^{2e}) + (a^2d^2e^{2e} + 2abd^2e^{2e})} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d*x+c)^2/(a+b*coth(f*x+e)),x, algorithm="maxima")

[Out] -2*b*integrate(-1/(a^2*c^2 - b^2*c^2 + (a^2*d^2 - b^2*d^2)*x^2 + 2*(a^2*c*d - b^2*c*d)*x - (a^2*c^2*e^(2*e) + 2*a*b*c^2*e^(2*e) + b^2*c^2*e^(2*e) + (a^2*d^2*e^(2*e) + 2*a*b*d^2*e^(2*e) + b^2*d^2*e^(2*e))*x^2 + 2*(a^2*c*d*e^(2*e) + 2*a*b*c*d*e^(2*e) + b^2*c*d*e^(2*e))*x)*e^(2*f*x)), x) - 1/(a*c*d + b*c*d + (a*d^2 + b*d^2)*x)

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{ad^2x^2 + 2acdx + ac^2 + (bd^2x^2 + 2bcdx + bc^2) \coth(fx+e)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d*x+c)^2/(a+b*coth(f*x+e)),x, algorithm="fricas")

[Out] integral(1/(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*coth(f*x + e)), x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a + b \coth(e + fx))(c + dx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d*x+c)**2/(a+b*coth(f*x+e)),x)

[Out] Integral(1/((a + b*coth(e + f*x))*(c + d*x)**2), x)

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dx + c)^2(b \coth(fx + e) + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d*x+c)^2/(a+b*coth(f*x+e)),x, algorithm="giac")

[Out] integrate(1/((d*x + c)^2*(b*coth(f*x + e) + a)), x)

$$3.57 \quad \int \frac{(c+dx)^3}{(a+b \coth(e+fx))^2} dx$$

Optimal. Leaf size=638

$$\frac{3b^2d^2(c+dx)\text{PolyLog}\left(2, \frac{(a+b)e^{2e+2fx}}{a-b}\right)}{f^3(a^2-b^2)^2} - \frac{3b^2d^2(c+dx)\text{PolyLog}\left(3, \frac{(a+b)e^{2e+2fx}}{a-b}\right)}{f^3(a^2-b^2)^2} + \frac{3b^2d(c+dx)^2\text{PolyLog}\left(2, \frac{(a+b)e^{2e+2fx}}{a-b}\right)}{f^2(a^2-b^2)^2}$$

[Out] $(-2*b^2*(c+d*x)^3)/((a^2-b^2)^2*f) + (2*b^2*(c+d*x)^3)/((a-b)*(a+b)^2*(a-b-(a+b)*E^(2*e+2*f*x))*f) + (c+d*x)^4/(4*(a-b)^2*d) + (3*b^2*d*(c+d*x)^2*\text{Log}[1-((a+b)*E^(2*e+2*f*x))/(a-b)])/((a^2-b^2)^2*f^2) - (2*b*(c+d*x)^3*\text{Log}[1-((a+b)*E^(2*e+2*f*x))/(a-b)])/((a-b)^2*(a+b)*f) + (2*b^2*(c+d*x)^3*\text{Log}[1-((a+b)*E^(2*e+2*f*x))/(a-b)])/((a^2-b^2)^2*f) + (3*b^2*d^2*(c+d*x)*\text{PolyLog}[2, ((a+b)*E^(2*e+2*f*x))/(a-b)])/((a^2-b^2)^2*f^3) - (3*b*d*(c+d*x)^2*\text{PolyLog}[2, ((a+b)*E^(2*e+2*f*x))/(a-b)])/((a-b)^2*(a+b)*f^2) + (3*b^2*d*(c+d*x)^2*\text{PolyLog}[2, ((a+b)*E^(2*e+2*f*x))/(a-b)])/((a^2-b^2)^2*f^2) - (3*b^2*d^3*\text{PolyLog}[3, ((a+b)*E^(2*e+2*f*x))/(a-b)])/(2*(a^2-b^2)^2*f^4) + (3*b*d^2*(c+d*x)*\text{PolyLog}[3, ((a+b)*E^(2*e+2*f*x))/(a-b)])/((a-b)^2*(a+b)*f^3) - (3*b^2*d^2*(c+d*x)*\text{PolyLog}[3, ((a+b)*E^(2*e+2*f*x))/(a-b)])/((a^2-b^2)^2*f^3) - (3*b*d^3*\text{PolyLog}[4, ((a+b)*E^(2*e+2*f*x))/(a-b)])/(2*(a-b)^2*(a+b)*f^4) + (3*b^2*d^3*\text{PolyLog}[4, ((a+b)*E^(2*e+2*f*x))/(a-b)])/(2*(a^2-b^2)^2*f^4)$

Rubi [A] time = 2.27365, antiderivative size = 638, normalized size of antiderivative = 1., number of steps used = 28, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {3734, 2254, 2185, 2184, 2190, 2531, 6609, 2282, 6589, 2191}

$$\frac{3b^2d^2(c+dx)\text{PolyLog}\left(2, \frac{(a+b)e^{2e+2fx}}{a-b}\right)}{f^3(a^2-b^2)^2} - \frac{3b^2d^2(c+dx)\text{PolyLog}\left(3, \frac{(a+b)e^{2e+2fx}}{a-b}\right)}{f^3(a^2-b^2)^2} + \frac{3b^2d(c+dx)^2\text{PolyLog}\left(2, \frac{(a+b)e^{2e+2fx}}{a-b}\right)}{f^2(a^2-b^2)^2}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)^3/(a + b*Coth[e + f*x])^2, x]

[Out] $(-2*b^2*(c+d*x)^3)/((a^2-b^2)^2*f) + (2*b^2*(c+d*x)^3)/((a-b)*(a+b)^2*(a-b-(a+b)*E^(2*e+2*f*x))*f) + (c+d*x)^4/(4*(a-b)^2*d) + (3*b^2*d*(c+d*x)^2*\text{Log}[1-((a+b)*E^(2*e+2*f*x))/(a-b)])/((a^2-b^2)^2*f^2) - (2*b*(c+d*x)^3*\text{Log}[1-((a+b)*E^(2*e+2*f*x))/(a-b)])/((a-b)^2*(a+b)*f) + (2*b^2*(c+d*x)^3*\text{Log}[1-((a+b)*E^(2*e+2*f*x))/(a-b)])/((a^2-b^2)^2*f) + (3*b^2*d^2*(c+d*x)*\text{PolyLog}[2, ((a+b)*E^(2*e+2*f*x))/(a-b)])/((a^2-b^2)^2*f^3) - (3*b*d*(c+d*x)^2*\text{PolyLog}[2, ((a+b)*E^(2*e+2*f*x))/(a-b)])/((a-b)^2*(a+b)*f^2) + (3*b^2*d*(c+d*x)^2*\text{PolyLog}[2, ((a+b)*E^(2*e+2*f*x))/(a-b)])/((a^2-b^2)^2*f^2) - (3*b^2*d^3*\text{PolyLog}[3, ((a+b)*E^(2*e+2*f*x))/(a-b)])/(2*(a^2-b^2)^2*f^4) + (3*b*d^2*(c+d*x)*\text{PolyLog}[3, ((a+b)*E^(2*e+2*f*x))/(a-b)])/((a-b)^2*(a+b)*f^3) - (3*b^2*d^2*(c+d*x)*\text{PolyLog}[3, ((a+b)*E^(2*e+2*f*x))/(a-b)])/((a^2-b^2)^2*f^3) - (3*b*d^3*\text{PolyLog}[4, ((a+b)*E^(2*e+2*f*x))/(a-b)])/(2*(a-b)^2*(a+b)*f^4) + (3*b^2*d^3*\text{PolyLog}[4, ((a+b)*E^(2*e+2*f*x))/(a-b)])/(2*(a^2-b^2)^2*f^4)$

$$\frac{a-b}{(a^2-b^2)^2 f} + \frac{(3b^2 d^2 (c+dx) \text{PolyLog}[2, (a+b)E^{2e+2fx}]) / (a-b)}{(a^2-b^2)^2 f^3} - \frac{(3b d^2 (c+dx) \text{PolyLog}[2, (a+b)E^{2e+2fx}]) / (a-b)}{(a-b)^2 (a+b) f^2} + \frac{(3b^2 d^2 (c+dx) \text{PolyLog}[2, (a+b)E^{2e+2fx}]) / (a-b)}{(a^2-b^2)^2 f^2} - \frac{(3b^2 d^3 \text{PolyLog}[3, (a+b)E^{2e+2fx}]) / (a-b)}{(2(a^2-b^2)^2 f^4)} + \frac{(3b d^2 (c+dx) \text{PolyLog}[3, (a+b)E^{2e+2fx}]) / (a-b)}{(a-b)^2 (a+b) f^3} - \frac{(3b^2 d^2 (c+dx) \text{PolyLog}[3, (a+b)E^{2e+2fx}]) / (a-b)}{(a^2-b^2)^2 f^3} - \frac{(3b d^3 \text{PolyLog}[4, (a+b)E^{2e+2fx}]) / (a-b)}{(2(a-b)^2 (a+b) f^4)} + \frac{(3b^2 d^3 \text{PolyLog}[4, (a+b)E^{2e+2fx}]) / (a-b)}{(2(a^2-b^2)^2 f^4)}$$

Rule 3734

```
Int[((c_) + (d_)*(x_))^(m_)*((a_) + (b_)*tan[(e_) + (f_)*(x_)])^(n_),
  x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, (1/(a - I*b) - (2*I*b)/(a^2
+ b^2 + (a - I*b)^2*E^(2*I*(e + f*x)))^(-n), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[a^2 + b^2, 0] && ILtQ[n, 0] && IGtQ[m, 0]
```

Rule 2254

```
Int[((a_) + (b_)*(F_)^(u_))^(p_)*((c_) + (d_)*(F_)^(v_))^(q_)*((e_)
+ (f_)*(x_))^(m_), x_Symbol] := With[{w = ExpandIntegrand[(e + f*x)^m, (a
+ b*F^u)^p*(c + d*F^v)^q, x]}, Int[w, x] /; SumQ[w]] /; FreeQ[{F, a, b, c,
d, e, f, m}, x] && IntegersQ[p, q] && LinearQ[{u, v}, x] && RationalQ[Simp
lify[u/v]]
```

Rule 2185

```
Int[((a_) + (b_)*((F_)^(g_)*((e_) + (f_)*(x_))))^(n_))^(p_)*((c_) +
(d_)*(x_))^(m_), x_Symbol] := Dist[1/a, Int[(c + d*x)^m*(a + b*(F^(g*(e +
f*x))))^n]^(p + 1), x], x] - Dist[b/a, Int[(c + d*x)^m*(F^(g*(e + f*x)))^n*
(a + b*(F^(g*(e + f*x))))^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n},
x] && ILtQ[p, 0] && IGtQ[m, 0]
```

Rule 2184

```
Int[((c_) + (d_)*(x_))^(m_)/((a_) + (b_)*((F_)^(g_)*((e_) + (f_)*(x_
))))^(n_)), x_Symbol] := Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x))))^n]/(a + b*(F^(g*(e + f*x))))^n, x],
x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^(g_)*((e_) + (f_)*(x_))))^(n_)*((c_) + (d_)*(x_))^(m_))/
((a_) + (b_)*((F_)^(g_)*((e_) + (f_)*(x_))))^(n_)), x_Symbol] := Simp
```

```

[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Dist
[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

```

Rule 2531

```

Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_)))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
))^n]])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]

```

Rule 6609

```

Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.)
*(x_)))^(p_.)], x_Symbol] := Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(
m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
d, e, f, n, p}, x] && GtQ[m, 0]

```

Rule 2282

```

Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

```

Rule 6589

```

Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]

```

Rule 2191

```

Int[((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((a_.) + (b_.)*((F_)^((g_.)*
(e_.) + (f_.)*(x_))))^(n_.))^((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :=
Simp[((c + d*x)^m*(a + b*(F^(g*(e + f*x)))^n)^(p + 1))/(b*f*g*n*(p + 1)*Lo
g[F]), x] - Dist[(d*m)/(b*f*g*n*(p + 1)*Log[F]), Int[(c + d*x)^(m - 1)*(a +
b*(F^(g*(e + f*x)))^n)^(p + 1), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m
, n, p}, x] && NeQ[p, -1]

```

Rubi steps

$$\begin{aligned}
\int \frac{(c+dx)^3}{(a+b \coth(e+fx))^2} dx &= \int \left[\frac{(c+dx)^3}{(a-b)^2} + \frac{4b^2 e^{4e+4fx} (c+dx)^3}{(a-b)^2 \left(a \left(1 - \frac{b}{a} \right) - a \left(1 + \frac{b}{a} \right) e^{2e+2fx} \right)^2} + \frac{4b e^{2e+2fx} (c+dx)^3}{(a-b)^2 \left(a \left(1 - \frac{b}{a} \right) - a \left(1 + \frac{b}{a} \right) e^{2e+2fx} \right)} \right] dx \\
&= \frac{(c+dx)^4}{4(a-b)^2 d} + \frac{(4b) \int \frac{e^{2e+2fx} (c+dx)^3}{a \left(1 - \frac{b}{a} \right) - a \left(1 + \frac{b}{a} \right) e^{2e+2fx}} dx}{(a-b)^2} + \frac{(4b^2) \int \frac{e^{4e+4fx} (c+dx)^3}{\left(a \left(1 - \frac{b}{a} \right) - a \left(1 + \frac{b}{a} \right) e^{2e+2fx} \right)^2} dx}{(a-b)^2} \\
&= \frac{(c+dx)^4}{4(a-b)^2 d} - \frac{2b(c+dx)^3 \log \left(1 - \frac{(a+b)e^{2e+2fx}}{a-b} \right)}{(a-b)^2 (a+b)f} + \frac{(4b^2) \int \left(\frac{(c+dx)^3}{(a+b)^2} + \frac{(a-b)^2 (c+dx)^3}{(a+b)^2 (a-b - (a+b)e^{2e+2fx})^2} \right) dx}{(a-b)^2} \\
&= \frac{(c+dx)^4}{4(a-b)^2 d} + \frac{b^2 (c+dx)^4}{(a^2 - b^2)^2 d} - \frac{2b(c+dx)^3 \log \left(1 - \frac{(a+b)e^{2e+2fx}}{a-b} \right)}{(a-b)^2 (a+b)f} - \frac{3bd(c+dx)^2 \text{Li}_2 \left(\frac{(a+b)e^{2e+2fx}}{a-b} \right)}{(a-b)^2 (a+b)f^2} \\
&= \frac{(c+dx)^4}{4(a-b)^2 d} - \frac{b^2 (c+dx)^4}{(a^2 - b^2)^2 d} - \frac{2b(c+dx)^3 \log \left(1 - \frac{(a+b)e^{2e+2fx}}{a-b} \right)}{(a-b)^2 (a+b)f} - \frac{3bd(c+dx)^2 \text{Li}_2 \left(\frac{(a+b)e^{2e+2fx}}{a-b} \right)}{(a-b)^2 (a+b)f^2} \\
&= \frac{2b^2 (c+dx)^3}{(a-b)(a+b)^2 (a-b - (a+b)e^{2e+2fx}) f} + \frac{(c+dx)^4}{4(a-b)^2 d} - \frac{2b(c+dx)^3 \log \left(1 - \frac{(a+b)e^{2e+2fx}}{a-b} \right)}{(a-b)^2 (a+b)f} + \\
&= -\frac{2b^2 (c+dx)^3}{(a^2 - b^2)^2 f} + \frac{2b^2 (c+dx)^3}{(a-b)(a+b)^2 (a-b - (a+b)e^{2e+2fx}) f} + \frac{(c+dx)^4}{4(a-b)^2 d} - \frac{2b(c+dx)^3 \log \left(1 - \frac{(a+b)e^{2e+2fx}}{a-b} \right)}{(a-b)^2 (a+b)f} \\
&= -\frac{2b^2 (c+dx)^3}{(a^2 - b^2)^2 f} + \frac{2b^2 (c+dx)^3}{(a-b)(a+b)^2 (a-b - (a+b)e^{2e+2fx}) f} + \frac{(c+dx)^4}{4(a-b)^2 d} + \frac{3b^2 d (c+dx)^2 \log \left(1 - \frac{(a+b)e^{2e+2fx}}{a-b} \right)}{(a^2 - b^2)^2} \\
&= -\frac{2b^2 (c+dx)^3}{(a^2 - b^2)^2 f} + \frac{2b^2 (c+dx)^3}{(a-b)(a+b)^2 (a-b - (a+b)e^{2e+2fx}) f} + \frac{(c+dx)^4}{4(a-b)^2 d} + \frac{3b^2 d (c+dx)^2 \log \left(1 - \frac{(a+b)e^{2e+2fx}}{a-b} \right)}{(a^2 - b^2)^2} \\
&= -\frac{2b^2 (c+dx)^3}{(a^2 - b^2)^2 f} + \frac{2b^2 (c+dx)^3}{(a-b)(a+b)^2 (a-b - (a+b)e^{2e+2fx}) f} + \frac{(c+dx)^4}{4(a-b)^2 d} + \frac{3b^2 d (c+dx)^2 \log \left(1 - \frac{(a+b)e^{2e+2fx}}{a-b} \right)}{(a^2 - b^2)^2} \\
&= -\frac{2b^2 (c+dx)^3}{(a^2 - b^2)^2 f} + \frac{2b^2 (c+dx)^3}{(a-b)(a+b)^2 (a-b - (a+b)e^{2e+2fx}) f} + \frac{(c+dx)^4}{4(a-b)^2 d} + \frac{3b^2 d (c+dx)^2 \log \left(1 - \frac{(a+b)e^{2e+2fx}}{a-b} \right)}{(a^2 - b^2)^2}
\end{aligned}$$

Mathematica [A] time = 12.2993, size = 646, normalized size = 1.01

$$-12bd^2(bd - 2acf) \left(2fx \operatorname{PolyLog} \left(2, \frac{(a-b)e^{-2(e+fx)}}{a+b} \right) + \operatorname{PolyLog} \left(3, \frac{(a-b)e^{-2(e+fx)}}{a+b} \right) \right) + 24bcd f(acf - bd) \operatorname{PolyLog} \left(2, \frac{(a-b)e^{-2(e+fx)}}{a+b} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)^3/(a + b*Coth[e + f*x])^2,x]

[Out] ((-16*(a - b)*b^2*f^3*(c + d*x)^3)/(a*(-1 + E^(2*e)) + b*(1 + E^(2*e))) + (8*a*(a - b)*b*f^4*(c + d*x)^4)/(d*(a*(-1 + E^(2*e)) + b*(1 + E^(2*e)))) + 4*8*b*c*d*f^2*(b*d - a*c*f)*x*Log[1 + (-a + b)/((a + b)*E^(2*(e + f*x)))] + 2*4*b*d^2*f^2*(b*d - 2*a*c*f)*x^2*Log[1 + (-a + b)/((a + b)*E^(2*(e + f*x)))] - 16*a*b*d^3*f^3*x^3*Log[1 + (-a + b)/((a + b)*E^(2*(e + f*x)))] + 8*b*c^2*f^2*(-3*b*d + 2*a*c*f)*(2*f*x - Log[a - b - (a + b)*E^(2*(e + f*x))]) + 24*b*c*d*f*(-(b*d) + a*c*f)*PolyLog[2, (a - b)/((a + b)*E^(2*(e + f*x)))] - 1*2*b*d^2*(b*d - 2*a*c*f)*(2*f*x*PolyLog[2, (a - b)/((a + b)*E^(2*(e + f*x)))] + PolyLog[3, (a - b)/((a + b)*E^(2*(e + f*x)))] + 12*a*b*d^3*(2*f^2*x^2*PolyLog[2, (a - b)/((a + b)*E^(2*(e + f*x)))] + 2*f*x*PolyLog[3, (a - b)/((a + b)*E^(2*(e + f*x)))] + PolyLog[4, (a - b)/((a + b)*E^(2*(e + f*x)))] - ((a - b)*(a + b)*f^3*((a^2 + b^2)*f*x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3)*Cosh[f*x] - (a^2 - b^2)*f*x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3)*Cosh[2*e + f*x] + 2*b*(-4*b*(c + d*x)^3 + a*f*x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3))*Sinh[f*x]))/((b*Cosh[e] + a*Sinh[e])*(b*Cosh[e + f*x] + a*Sinh[e + f*x])))/(8*(a - b)^2*(a + b)^2*f^4)

Maple [B] time = 0.301, size = 2039, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^3/(a+b*coth(f*x+e))^2,x)

[Out] 6*b/(a+b)^2/(a-b)^2*a*c^2*d*x^2-12*b^2/(a+b)^2/(a-b)^2/f^2*c*d^2*e*x+6*b/(a+b)^2/(a-b)^2/f^2*a*c^2*d*e^2+4*b/(a+b)^2/(a-b)^2*a*c*d^2*x^3-8*b/(a+b)^2/(a-b)^2/f^3*a*c*d^2*e^3+4*b/(a+b)^2/(a-b)^2/f^3*a*d^3*e^3*x-4*b/(a+b)^2/(a-b)^2/f^4*a*d^3*e^3*ln(exp(f*x+e))+3*b/(a+b)^2/(a-b)^2/f^3*a*c*d^2*polylog(3, (a+b)*exp(2*f*x+2*e)/(a-b))-6*b^2/(a+b)^2/(a-b)^2/f^3*c*d^2*e*ln(a*exp(2*f*x+2*e)+b*exp(2*f*x+2*e)-a+b)+12*b^2/(a+b)^2/(a-b)^2/f^3*c*d^2*e*ln(exp(f*x+e))+3*b/(a+b)^2/(a-b)^2/f^3*a*d^3*polylog(3, (a+b)*exp(2*f*x+2*e)/(a-b))*x-2

$$\begin{aligned} & *b/(a+b)^2/(a-b)^2/f*a*d^3*\ln(1-(a+b)*\exp(2*f*x+2*e)/(a-b))*x^3-2*b/(a+b)^2 \\ & / (a-b)^2/f^4*a*d^3*\ln(1-(a+b)*\exp(2*f*x+2*e)/(a-b))*e^3-3*b/(a+b)^2/(a-b)^2 \\ & /f^2*a*d^3*\text{polylog}(2, (a+b)*\exp(2*f*x+2*e)/(a-b))*x^2+1/4/(a^2+2*a*b+b^2)*d^3 \\ & *x^4+1/(a^2+2*a*b+b^2)*c^3*x+b/(a+b)^2/(a-b)^2*a*d^3*x^4-6*b^2/(a+b)^2/(a-b) \\ & ^2/f*c*d^2*x^2-6*b^2/(a+b)^2/(a-b)^2/f^3*c*d^2*e^2+6*b^2/(a+b)^2/(a-b)^2/f \\ & ^3*d^3*e^2*x+12*b/(a+b)^2/(a-b)^2/f*a*c^2*d*e*x+1/(a^2+2*a*b+b^2)*c*d^2*x^3 \\ & +3/2/(a^2+2*a*b+b^2)*c^2*d*x^2-3*b/(a+b)^2/(a-b)^2/f^2*a*c^2*d*\text{polylog}(2, (a+b) \\ & *\exp(2*f*x+2*e)/(a-b))+6*b^2/(a+b)^2/(a-b)^2/f^2*c*d^2*\ln(1-(a+b)*\exp(2 \\ & *f*x+2*e)/(a-b))*x+6*b^2/(a+b)^2/(a-b)^2/f^3*c*d^2*\ln(1-(a+b)*\exp(2*f*x+2*e) \\ &)/(a-b))*e+2*b/(a+b)^2/(a-b)^2/f^4*a*d^3*e^3*\ln(a*\exp(2*f*x+2*e)+b*\exp(2*f*x \\ & +2*e)-a+b)-12*b/(a+b)^2/(a-b)^2/f^2*a*c*d^2*e^2*x+6*b/(a+b)^2/(a-b)^2/f^3* \\ & a*c*d^2*e^2*\ln(1-(a+b)*\exp(2*f*x+2*e)/(a-b))-6*b/(a+b)^2/(a-b)^2/f*a*c^2*d* \\ & \ln(1-(a+b)*\exp(2*f*x+2*e)/(a-b))*x-6*b/(a+b)^2/(a-b)^2/f*a*c*d^2*\ln(1-(a+b) \\ & *\exp(2*f*x+2*e)/(a-b))*x^2-6*b/(a+b)^2/(a-b)^2/f^2*a*c*d^2*\text{polylog}(2, (a+b)* \\ & \exp(2*f*x+2*e)/(a-b))*x-12*b/(a+b)^2/(a-b)^2/f^2*a*c^2*d*e*\ln(\exp(f*x+e))+6 \\ & *b/(a+b)^2/(a-b)^2/f^2*a*c^2*d*e*\ln(a*\exp(2*f*x+2*e)+b*\exp(2*f*x+2*e)-a+b)+ \\ & 12*b/(a+b)^2/(a-b)^2/f^3*a*c*d^2*e^2*\ln(\exp(f*x+e))-6*b/(a+b)^2/(a-b)^2/f^3 \\ & *a*c*d^2*e^2*\ln(a*\exp(2*f*x+2*e)+b*\exp(2*f*x+2*e)-a+b)-6*b/(a+b)^2/(a-b)^2/f \\ & ^2*a*c^2*d*\ln(1-(a+b)*\exp(2*f*x+2*e)/(a-b))*e-2/(a-b)/f/(a^2+2*a*b+b^2)*(d \\ & ^3*x^3+3*c*d^2*x^2+3*c^2*d*x+c^3)*b^2/(a*\exp(2*f*x+2*e)+b*\exp(2*f*x+2*e)-a+ \\ & b)+3*b/(a+b)^2/(a-b)^2/f^4*a*d^3*e^4-2*b/(a+b)^2/(a-b)^2/f*a*c^3*\ln(a*\exp(2 \\ & *f*x+2*e)+b*\exp(2*f*x+2*e)-a+b)+3*b^2/(a+b)^2/(a-b)^2/f^2*c^2*d*\ln(a*\exp(2* \\ & f*x+2*e)+b*\exp(2*f*x+2*e)-a+b)-6*b^2/(a+b)^2/(a-b)^2/f^2*c^2*d*\ln(\exp(f*x+e) \\ &))+4*b/(a+b)^2/(a-b)^2/f*a*c^3*\ln(\exp(f*x+e))-3/2*b/(a+b)^2/(a-b)^2/f^4*a*d \\ & ^3*\text{polylog}(4, (a+b)*\exp(2*f*x+2*e)/(a-b))+3*b^2/(a+b)^2/(a-b)^2/f^3*c*d^2*\text{po} \\ & \text{lylog}(2, (a+b)*\exp(2*f*x+2*e)/(a-b))+3*b^2/(a+b)^2/(a-b)^2/f^4*d^3*e^2*\ln(a* \\ & \exp(2*f*x+2*e)+b*\exp(2*f*x+2*e)-a+b)-6*b^2/(a+b)^2/(a-b)^2/f^4*d^3*e^2*\ln(e \\ & xp(f*x+e))+3*b^2/(a+b)^2/(a-b)^2/f^2*d^3*\ln(1-(a+b)*\exp(2*f*x+2*e)/(a-b))*x \\ & ^2-3*b^2/(a+b)^2/(a-b)^2/f^4*d^3*\ln(1-(a+b)*\exp(2*f*x+2*e)/(a-b))*e^2+3*b^2 \\ & / (a+b)^2/(a-b)^2/f^3*d^3*\text{polylog}(2, (a+b)*\exp(2*f*x+2*e)/(a-b))*x-2*b^2/(a+b) \\ &)^2/(a-b)^2/f*d^3*x^3+4*b^2/(a+b)^2/(a-b)^2/f^4*d^3*e^3-3/2*b^2/(a+b)^2/(a-b) \\ & ^2/f^4*d^3*\text{polylog}(3, (a+b)*\exp(2*f*x+2*e)/(a-b)) \end{aligned}$$

Maxima [A] time = 2.77539, size = 1426, normalized size = 2.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3/(a+b*coth(f*x+e))^2,x, algorithm="maxima")

[Out] $-6*b^2*c^2*d*f*x/(a^4*f^2 - 2*a^2*b^2*f^2 + b^4*f^2) - 2/3*(4*f^3*x^3*\log(-$
 $(a*e^{(2*e)} + b*e^{(2*e)})*e^{(2*f*x)/(a - b) + 1} + 6*f^2*x^2*\text{dilog}((a*e^{(2*e)}$

$$\begin{aligned}
& + b^2 e^{2e} e^{2fx} / (a - b) - 6 f x \operatorname{polylog}(3, (a e^{2e} + b e^{2e})) \\
& e^{2fx} / (a - b) + 3 \operatorname{polylog}(4, (a e^{2e} + b e^{2e})) e^{2fx} / (a - b \\
&)) * a b d^3 / (a^4 f^4 - 2 a^2 b^2 f^4 + b^4 f^4) + 3 b^2 c^2 d \log((a e^{2e} \\
&) + b e^{2e}) e^{2fx} - a + b) / (a^4 f^2 - 2 a^2 b^2 f^2 + b^4 f^2) - c^3 \\
& * (2 a b \log(-(a - b) e^{-2fx - 2e} + a + b) / ((a^4 - 2 a^2 b^2 + b^4) f) \\
& + 2 b^2 / ((a^4 - 2 a^2 b^2 + b^4 - (a^4 - 2 a^3 b + 2 a b^3 - b^4) e^{-2fx \\
& - 2e})) * f) - (f x + e) / ((a^2 + 2 a b + b^2) f)) - 3/2 * (2 a b c d^2 f - b^2 \\
& * d^3) * (2 f^2 x^2 \log(-(a e^{2e} + b e^{2e})) e^{2fx} / (a - b) + 1) + 2 f x \\
& * \operatorname{dilog}((a e^{2e} + b e^{2e})) e^{2fx} / (a - b) - \operatorname{polylog}(3, (a e^{2e} \\
& + b e^{2e})) e^{2fx} / (a - b)) / (a^4 f^4 - 2 a^2 b^2 f^4 + b^4 f^4) - 3 * (a \\
& * b c^2 d f - b^2 c d^2) * (2 f x \log(-(a e^{2e} + b e^{2e})) e^{2fx} / (a - \\
& b) + 1) + \operatorname{dilog}((a e^{2e} + b e^{2e})) e^{2fx} / (a - b)) / (a^4 f^3 - 2 a^2 \\
& b^2 f^3 + b^4 f^3) + (a b d^3 f^4 x^4 + 2 * (2 a b c d^2 f - b^2 d^3) f^3 x \\
& ^3 + 6 * (a b c^2 d f^2 - b^2 c d^2 f) f^2 x^2) / (a^4 f^4 - 2 a^2 b^2 f^4 + b^4 \\
& f^4) + 1/4 * (24 b^2 c^2 d x + (a^2 d^3 f - 2 a b d^3 f + b^2 d^3 f) x^4 + \\
& 4 * (a^2 c d^2 f - 2 a b c d^2 f + (c d^2 f + 2 d^3) b^2) x^3 + 6 * (a^2 c^2 d * \\
& f - 2 a b c^2 d f + (c^2 d f + 4 c d^2) b^2) x^2 - ((a^2 d^3 f e^{2e} - b^2 \\
& d^3 f e^{2e}) x^4 + 4 * (a^2 c d^2 f e^{2e} - b^2 c d^2 f e^{2e}) x^3 + \\
& 6 * (a^2 c^2 d f e^{2e} - b^2 c^2 d f e^{2e}) x^2) e^{2fx}) / (a^4 f - 2 a^2 \\
& b^2 f + b^4 f - (a^4 f e^{2e} + 2 a^3 b f e^{2e} - 2 a b^3 f e^{2e} - \\
& b^4 f e^{2e})) e^{2fx}
\end{aligned}$$

Fricas [C] time = 3.55548, size = 12840, normalized size = 20.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((dx+c)^3/(a+b*coth(f*x+e))^2,x, algorithm="fricas")

[Out]
$$\begin{aligned}
& -1/4 * ((a^3 + a^2 b - a b^2 - b^3) d^3 f^4 x^4 + 4 * (a^3 + a^2 b - a b^2 - b^3) \\
& * c d^2 f^4 x^3 + 6 * (a^3 + a^2 b - a b^2 - b^3) * c^2 d f^4 x^2 + 4 * (a^3 + a \\
& ^2 b - a b^2 - b^3) * c^3 f^4 x - 4 * (a^2 b - a b^2) d^3 e^4 - 8 * (a b^2 - b^3) \\
& * d^3 e^3 + 8 * (2 * (a^2 b - a b^2) * c^3 e + (a b^2 - b^3) * c^3) * f^3 - 24 * ((a^2 b \\
& - a b^2) * c^2 d e^2 + (a b^2 - b^3) * c^2 d e) * f^2 - ((a^3 + 3 a^2 b + 3 a b^2 \\
& + b^3) * d^3 f^4 x^4 - 4 * (a^2 b + a b^2) * d^3 e^4 + 16 * (a^2 b + a b^2) * c^3 e \\
& * f^3 - 8 * (a b^2 + b^3) * d^3 e^3 + 4 * ((a^3 + 3 a^2 b + 3 a b^2 + b^3) * c d^2 f \\
& ^4 - 2 * (a b^2 + b^3) * d^3 f^3) * x^3 - 24 * ((a^2 b + a b^2) * c^2 d e^2 + (a b^2 \\
& + b^3) * c^2 d e) * f^2 + 6 * ((a^3 + 3 a^2 b + 3 a b^2 + b^3) * c^2 d f^4 - 4 * (a b \\
& ^2 + b^3) * c d^2 f^3) * x^2 + 8 * (2 * (a^2 b + a b^2) * c d^2 e^3 + 3 * (a b^2 + b^3) \\
& * c d^2 e^2) * f + 4 * ((a^3 + 3 a^2 b + 3 a b^2 + b^3) * c^3 f^4 - 6 * (a b^2 + b^3) \\
&) * c^2 d f^3) * x) * \cosh(f x + e)^2 - 2 * ((a^3 + 3 a^2 b + 3 a b^2 + b^3) * d^3 f^
\end{aligned}$$

$$\begin{aligned}
& 4*x^4 - 4*(a^2*b + a*b^2)*d^3*e^4 + 16*(a^2*b + a*b^2)*c^3*e*f^3 - 8*(a*b^2 + b^3)*d^3*e^3 + 4*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*c*d^2*f^4 - 2*(a*b^2 + b^3)*d^3*f^3)*x^3 - 24*((a^2*b + a*b^2)*c^2*d*e^2 + (a*b^2 + b^3)*c^2*d*e)*f^2 + 6*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*c^2*d*f^4 - 4*(a*b^2 + b^3)*c*d^2*f^3)*x^2 + 8*(2*(a^2*b + a*b^2)*c*d^2*e^3 + 3*(a*b^2 + b^3)*c*d^2*e^2)*f + 4*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*c^3*f^4 - 6*(a*b^2 + b^3)*c^2*d*f^3)*x)*cosh(f*x + e)*sinh(f*x + e) - ((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d^3*f^4*x^4 - 4*(a^2*b + a*b^2)*d^3*e^4 + 16*(a^2*b + a*b^2)*c^3*e*f^3 - 8*(a*b^2 + b^3)*d^3*e^3 + 4*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*c*d^2*f^4 - 2*(a*b^2 + b^3)*d^3*f^3)*x^3 - 24*((a^2*b + a*b^2)*c^2*d*e^2 + (a*b^2 + b^3)*c^2*d*e)*f^2 + 6*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*c^2*d*f^4 - 4*(a*b^2 + b^3)*c*d^2*f^3)*x^2 + 8*(2*(a^2*b + a*b^2)*c*d^2*e^3 + 3*(a*b^2 + b^3)*c*d^2*e^2)*f + 4*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*c^3*f^4 - 6*(a*b^2 + b^3)*c^2*d*f^3)*x)*sinh(f*x + e)^2 + 8*(2*(a^2*b - a*b^2)*c*d^2*e^3 + 3*(a*b^2 - b^3)*c*d^2*e^2)*f - 24*((a^2*b - a*b^2)*d^3*f^2*x^2 + (a^2*b - a*b^2)*c^2*d*f^2 - (a*b^2 - b^3)*c*d^2*f - ((a^2*b + a*b^2)*d^3*f^2*x^2 + (a^2*b + a*b^2)*c^2*d*f^2 - (a*b^2 + b^3)*c*d^2*f + (2*(a^2*b + a*b^2)*c*d^2*f^2 - (a*b^2 + b^3)*d^3*f)*x)*cosh(f*x + e)^2 - 2*((a^2*b + a*b^2)*d^3*f^2*x^2 + (a^2*b + a*b^2)*c^2*d*f^2 - (a*b^2 + b^3)*c*d^2*f + (2*(a^2*b + a*b^2)*c*d^2*f^2 - (a*b^2 + b^3)*d^3*f)*x)*cosh(f*x + e)*sinh(f*x + e) - ((a^2*b + a*b^2)*d^3*f^2*x^2 + (a^2*b + a*b^2)*c^2*d*f^2 - (a*b^2 + b^3)*c*d^2*f + (2*(a^2*b + a*b^2)*c*d^2*f^2 - (a*b^2 + b^3)*d^3*f)*x)*sinh(f*x + e)^2 + (2*(a^2*b - a*b^2)*c*d^2*f^2 - (a*b^2 - b^3)*d^3*f)*x)*dilog(sqrt((a + b)/(a - b))*(cosh(f*x + e) + sinh(f*x + e))) - 24*((a^2*b - a*b^2)*d^3*f^2*x^2 + (a^2*b - a*b^2)*c^2*d*f^2 - (a*b^2 - b^3)*c*d^2*f - ((a^2*b + a*b^2)*d^3*f^2*x^2 + (a^2*b + a*b^2)*c^2*d*f^2 - (a*b^2 + b^3)*c*d^2*f + (2*(a^2*b + a*b^2)*c*d^2*f^2 - (a*b^2 + b^3)*d^3*f)*x)*cosh(f*x + e)^2 - 2*((a^2*b + a*b^2)*d^3*f^2*x^2 + (a^2*b + a*b^2)*c^2*d*f^2 - (a*b^2 + b^3)*c*d^2*f + (2*(a^2*b + a*b^2)*c*d^2*f^2 - (a*b^2 + b^3)*d^3*f)*x)*cosh(f*x + e)*sinh(f*x + e) - ((a^2*b + a*b^2)*d^3*f^2*x^2 + (a^2*b + a*b^2)*c^2*d*f^2 - (a*b^2 + b^3)*c*d^2*f + (2*(a^2*b + a*b^2)*c*d^2*f^2 - (a*b^2 + b^3)*d^3*f)*x)*sinh(f*x + e)^2 + (2*(a^2*b - a*b^2)*c*d^2*f^2 - (a*b^2 - b^3)*d^3*f)*x)*dilog(-sqrt((a + b)/(a - b))*(cosh(f*x + e) + sinh(f*x + e))) + 4*(2*(a^2*b - a*b^2)*d^3*e^3 - 2*(a^2*b - a*b^2)*c^3*f^3 + 3*(a*b^2 - b^3)*d^3*e^2 + 3*(2*(a^2*b - a*b^2)*c^2*d*e + (a*b^2 - b^3)*c^2*d)*f^2 - (2*(a^2*b + a*b^2)*d^3*e^3 - 2*(a^2*b + a*b^2)*c^3*f^3 + 3*(a*b^2 + b^3)*d^3*e^2 + 3*(2*(a^2*b + a*b^2)*c^2*d*e + (a*b^2 + b^3)*c^2*d)*f^2 - 6*((a^2*b + a*b^2)*c*d^2*e^2 + (a*b^2 + b^3)*c*d^2*e)*f)*cosh(f*x + e)^2 - 2*(2*(a^2*b + a*b^2)*d^3*e^3 - 2*(a^2*b + a*b^2)*c^3*f^3 + 3*(a*b^2 + b^3)*d^3*e^2 + 3*(2*(a^2*b + a*b^2)*c^2*d*e + (a*b^2 + b^3)*c^2*d)*f^2 - 6*((a^2*b + a*b^2)*c*d^2*e^2 + (a*b^2 + b^3)*c*d^2*e)*f)*cosh(f*x + e)*sinh(f*x + e) - (2*(a^2*b + a*b^2)*d^3*e^3 - 2*(a^2*b + a*b^2)*c^3*f^3 + 3*(a*b^2 + b^3)*d^3*e^2 + 3*(2*(a^2*b + a*b^2)*c^2*d*e + (a*b^2 + b^3)*c^2*d)*f^2 - 6*((a^2*b + a*b^2)*c*d^2*e^2 + (a*b^2 + b^3)*c*d^2*e)*f)*sinh(f*x + e)^2 - 6*((a^2*b - a*b^2)*c*d^2*e^2 + (a*b^2 - b^3)*c*d^2*e)*f)*log(2*(a + b))*cosh(f*x + e) + 2*(a + b)*sinh(f*x + e) + 2*(a - b)*sqrt((a + b)/(a - b))
\end{aligned}$$

$$\begin{aligned}
& d^3 f^3 - (a^2 b^2 + b^3) c d^2 f^2 x \sinh(fx + e)^2 - 6((a^2 b - a b^2) c d^2 e^2 + (a^2 b^2 - b^3) c d^2 e) f + 6((a^2 b - a b^2) c^2 d f^3 - (a^2 b^2 - b^3) c d^2 f^2 x) \log(-\sqrt{(a+b)/(a-b)} (\cosh(fx + e) + \sinh(fx + e)) + 1) \\
& + 48((a^2 b + a b^2) d^3 \cosh(fx + e)^2 + 2(a^2 b + a b^2) d^3 \cosh(fx + e) \sinh(fx + e) + (a^2 b + a b^2) d^3 \sinh(fx + e)^2 - (a^2 b - a b^2) d^3) \operatorname{polylog}(4, \sqrt{(a+b)/(a-b)} (\cosh(fx + e) + \sinh(fx + e))) \\
& + 48((a^2 b + a b^2) d^3 \cosh(fx + e)^2 + 2(a^2 b + a b^2) d^3 \cosh(fx + e) \sinh(fx + e) + (a^2 b + a b^2) d^3 \sinh(fx + e)^2 - (a^2 b - a b^2) d^3) \operatorname{polylog}(4, -\sqrt{(a+b)/(a-b)} (\cosh(fx + e) + \sinh(fx + e))) \\
& + 24(2(a^2 b - a b^2) d^3 f x + 2(a^2 b - a b^2) c d^2 f - (a b^2 - b^3) d^3 - (2(a^2 b + a b^2) d^3 f x + 2(a^2 b + a b^2) c d^2 f - (a b^2 + b^3) d^3) \cosh(fx + e)^2 - 2(2(a^2 b + a b^2) d^3 f x + 2(a^2 b + a b^2) c d^2 f - (a b^2 + b^3) d^3) \cosh(fx + e) \sinh(fx + e) - (2(a^2 b + a b^2) d^3 f x + 2(a^2 b + a b^2) c d^2 f - (a b^2 + b^3) d^3) \sinh(fx + e)^2) \operatorname{polylog}(3, \sqrt{(a+b)/(a-b)} (\cosh(fx + e) + \sinh(fx + e))) \\
& + 24(2(a^2 b - a b^2) d^3 f x + 2(a^2 b - a b^2) c d^2 f - (a b^2 - b^3) d^3 - (2(a^2 b + a b^2) d^3 f x + 2(a^2 b + a b^2) c d^2 f - (a b^2 + b^3) d^3) \cosh(fx + e)^2 - 2(2(a^2 b + a b^2) d^3 f x + 2(a^2 b + a b^2) c d^2 f - (a b^2 + b^3) d^3) \cosh(fx + e) \sinh(fx + e) - (2(a^2 b + a b^2) d^3 f x + 2(a^2 b + a b^2) c d^2 f - (a b^2 + b^3) d^3) \sinh(fx + e)^2) \operatorname{polylog}(3, -\sqrt{(a+b)/(a-b)} (\cosh(fx + e) + \sinh(fx + e))) \\
& / ((a^5 + a^4 b - 2 a^3 b^2 - 2 a^2 b^3 + a b^4 + b^5) f^4 \cosh(fx + e)^2 + 2(a^5 + a^4 b - 2 a^3 b^2 - 2 a^2 b^3 + a b^4 + b^5) f^4 \cosh(fx + e) \sinh(fx + e) + (a^5 + a^4 b - 2 a^3 b^2 - 2 a^2 b^3 + a b^4 + b^5) f^4 \sinh(fx + e)^2 - (a^5 - a^4 b - 2 a^3 b^2 + 2 a^2 b^3 + a b^4 - b^5) f^4)
\end{aligned}$$

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**3/(a+b*coth(f*x+e))**2,x)

[Out] Exception raised: TypeError

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx + c)^3}{(b \coth(fx + e) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+b*coth(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3/(b*coth(f*x + e) + a)^2, x)
```

$$3.58 \quad \int \frac{(c+dx)^2}{(a+b \coth(e+fx))^2} dx$$

Optimal. Leaf size=475

$$\frac{2b^2d(c+dx)\text{PolyLog}\left(2, \frac{(a+b)e^{2e+2fx}}{a-b}\right)}{f^2(a^2-b^2)^2} + \frac{b^2d^2\text{PolyLog}\left(2, \frac{(a+b)e^{2e+2fx}}{a-b}\right)}{f^3(a^2-b^2)^2} - \frac{b^2d^2\text{PolyLog}\left(3, \frac{(a+b)e^{2e+2fx}}{a-b}\right)}{f^3(a^2-b^2)^2} - \frac{2bd(c+dx)\text{PolyLog}\left(2, \frac{(a+b)e^{2e+2fx}}{a-b}\right)}{f^2(a-b)}$$

[Out] $(-2*b^2*(c+d*x)^2)/((a^2-b^2)^2*f) + (2*b^2*(c+d*x)^2)/((a-b)*(a+b)^2*(a-b-(a+b)*E^(2*e+2*f*x))*f) + (c+d*x)^3/(3*(a-b)^2*d) + (2*b^2*d*(c+d*x)*\text{Log}[1-((a+b)*E^(2*e+2*f*x))/(a-b)])/((a^2-b^2)^2*f^2) - (2*b*(c+d*x)^2*\text{Log}[1-((a+b)*E^(2*e+2*f*x))/(a-b)])/((a-b)^2*(a+b)*f) + (2*b^2*(c+d*x)^2*\text{Log}[1-((a+b)*E^(2*e+2*f*x))/(a-b)])/((a^2-b^2)^2*f) + (b^2*d^2*\text{PolyLog}[2, ((a+b)*E^(2*e+2*f*x))/(a-b)])/((a^2-b^2)^2*f^3) - (2*b*d*(c+d*x)*\text{PolyLog}[2, ((a+b)*E^(2*e+2*f*x))/(a-b)])/((a-b)^2*(a+b)*f^2) + (2*b^2*d*(c+d*x)*\text{PolyLog}[2, ((a+b)*E^(2*e+2*f*x))/(a-b)])/((a^2-b^2)^2*f^2) + (b*d^2*\text{PolyLog}[3, ((a+b)*E^(2*e+2*f*x))/(a-b)])/((a-b)^2*(a+b)*f^3) - (b^2*d^2*\text{PolyLog}[3, ((a+b)*E^(2*e+2*f*x))/(a-b)])/((a^2-b^2)^2*f^3)$

Rubi [A] time = 1.71445, antiderivative size = 475, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 11, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.55$, Rules used = {3734, 2254, 2185, 2184, 2190, 2531, 2282, 6589, 2191, 2279, 2391}

$$\frac{2b^2d(c+dx)\text{PolyLog}\left(2, \frac{(a+b)e^{2e+2fx}}{a-b}\right)}{f^2(a^2-b^2)^2} + \frac{b^2d^2\text{PolyLog}\left(2, \frac{(a+b)e^{2e+2fx}}{a-b}\right)}{f^3(a^2-b^2)^2} - \frac{b^2d^2\text{PolyLog}\left(3, \frac{(a+b)e^{2e+2fx}}{a-b}\right)}{f^3(a^2-b^2)^2} - \frac{2bd(c+dx)\text{PolyLog}\left(2, \frac{(a+b)e^{2e+2fx}}{a-b}\right)}{f^2(a-b)}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)^2/(a + b*Coth[e + f*x])^2,x]

[Out] $(-2*b^2*(c+d*x)^2)/((a^2-b^2)^2*f) + (2*b^2*(c+d*x)^2)/((a-b)*(a+b)^2*(a-b-(a+b)*E^(2*e+2*f*x))*f) + (c+d*x)^3/(3*(a-b)^2*d) + (2*b^2*d*(c+d*x)*\text{Log}[1-((a+b)*E^(2*e+2*f*x))/(a-b)])/((a^2-b^2)^2*f^2) - (2*b*(c+d*x)^2*\text{Log}[1-((a+b)*E^(2*e+2*f*x))/(a-b)])/((a-b)^2*(a+b)*f) + (2*b^2*(c+d*x)^2*\text{Log}[1-((a+b)*E^(2*e+2*f*x))/(a-b)])/((a^2-b^2)^2*f) + (b^2*d^2*\text{PolyLog}[2, ((a+b)*E^(2*e+2*f*x))/(a-b)])/((a^2-b^2)^2*f^3) - (2*b*d*(c+d*x)*\text{PolyLog}[2, ((a+b)*E^(2*e+2*f*x))/(a-b)])/((a-b)^2*(a+b)*f^2) + (2*b^2*d*(c+d*x)*\text{PolyLog}[2, ((a+b)*E^(2*e+2*f*x))/(a-b)])/((a^2-b^2)^2*f^2) + (b*d^2*\text{PolyLog}[3, ((a+b)*E^(2*e+2*f*x))/(a-b)])/((a-b)^2*(a+b)*f^3) - (b^2*d^2*\text{PolyLog}[3, ((a+b)*E^(2*e+2*f*x))/(a-b)])/((a^2-b^2)^2*f^3)$

$$\frac{((a + b)E^{(2e + 2fx)})/(a - b)}{(a - b)^2(a + b)f^3} - \frac{(b^2d^2 \text{PolyLog}[3, ((a + b)E^{(2e + 2fx)})/(a - b)])}{(a^2 - b^2)^2f^3}$$

Rule 3734

$$\text{Int}[\frac{(c + d(x))^m((a + b)\tan(e + f(x)))^n}{x}, x] \rightarrow \text{Int}[\text{ExpandIntegrand}[(c + dx)^m, (1/(a - I*b) - (2*I*b)/(a^2 + b^2 + (a - I*b)^2E^{(2*I*(e + fx))))^{-n}], x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \ \&\& \ \text{NeQ}[a^2 + b^2, 0] \ \&\& \ \text{ILtQ}[n, 0] \ \&\& \ \text{IGtQ}[m, 0]$$

Rule 2254

$$\text{Int}[\frac{(a + b(F)^u)^p((c + d(F)^v))^q((e + f(x))^m)}{x}, x] \rightarrow \text{With}\{w = \text{ExpandIntegrand}[(e + fx)^m, (a + bF^u)^p(c + dF^v)^q, x]\}, \text{Int}[w, x] /; \text{SumQ}[w] /; \text{FreeQ}\{F, a, b, c, d, e, f, m\}, x] \ \&\& \ \text{IntegersQ}[p, q] \ \&\& \ \text{LinearQ}\{u, v\}, x] \ \&\& \ \text{RationalQ}[\text{Simplify}[u/v]]$$

Rule 2185

$$\text{Int}[\frac{(a + b(F)^{(g*(e + f(x)))})^n)^p((c + d(x))^m)}{x}, x] \rightarrow \text{Dist}[1/a, \text{Int}[(c + dx)^m(a + b(F^{g*(e + fx)})^n)^{p+1}, x], x] - \text{Dist}[b/a, \text{Int}[(c + dx)^m(F^{g*(e + fx)})^n(a + b(F^{g*(e + fx)})^n)^p, x], x] /; \text{FreeQ}\{F, a, b, c, d, e, f, g, n\}, x] \ \&\& \ \text{ILtQ}[p, 0] \ \&\& \ \text{IGtQ}[m, 0]$$

Rule 2184

$$\text{Int}[\frac{(c + d(x))^m}{(a + b(F)^{(g*(e + f(x)))})^n}, x] \rightarrow \text{Simp}[(c + dx)^{m+1}/(a*d*(m+1)), x] - \text{Dist}[b/a, \text{Int}[(c + dx)^m(F^{g*(e + fx)})^n/(a + b(F^{g*(e + fx)})^n), x], x] /; \text{FreeQ}\{F, a, b, c, d, e, f, g, n\}, x] \ \&\& \ \text{IGtQ}[m, 0]$$

Rule 2190

$$\text{Int}[\frac{(F)^{(g*(e + f(x)))})^n((c + d(x))^m)}{(a + b(F)^{(g*(e + f(x)))})^n}, x] \rightarrow \text{Simp}[(c + dx)^m \text{Log}[1 + (b(F^{g*(e + fx)})^n)/a]/(b*f*g*n \text{Log}[F]), x] - \text{Dist}[(d*m)/(b*f*g*n \text{Log}[F]), \text{Int}[(c + dx)^{m-1} \text{Log}[1 + (b(F^{g*(e + fx)})^n)/a], x], x] /; \text{FreeQ}\{F, a, b, c, d, e, f, g, n\}, x] \ \&\& \ \text{IGtQ}[m, 0]$$

Rule 2531

$$\text{Int}[\text{Log}[1 + (e + f(x))(F)^{(c*(a + b*x))}]^n * (f + g(x))^m, x] \rightarrow -\text{Simp}[(f + gx)^m \text{PolyLog}[2, -(e(F^{c*(a + b*x)}))]]$$

```

)))^n)]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]

```

Rule 2282

```

Int[u_, x_Symbol] :=> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

```

Rule 6589

```

Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :=> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]

```

Rule 2191

```

Int[((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((a_.) + (b_.)*((F_)^((g_.)*
(e_.) + (f_.)*(x_))))^(p_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :=>
Simp[((c + d*x)^m*(a + b*(F^(g*(e + f*x)))^n)^(p + 1))/(b*f*g*n*(p + 1)*Lo
g[F]), x] - Dist[(d*m)/(b*f*g*n*(p + 1)*Log[F]), Int[(c + d*x)^(m - 1)*(a +
b*(F^(g*(e + f*x)))^n)^(p + 1), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m
, n, p}, x] && NeQ[p, -1]

```

Rule 2279

```

Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:=> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

```

Rule 2391

```

Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :=> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]

```

Rubi steps

$$\begin{aligned}
\int \frac{(c+dx)^2}{(a+b \coth(e+fx))^2} dx &= \int \left(\frac{(c+dx)^2}{(a-b)^2} + \frac{4b^2 e^{4e+4fx} (c+dx)^2}{(a-b)^2 \left(a \left(1 - \frac{b}{a} \right) - a \left(1 + \frac{b}{a} \right) e^{2e+2fx} \right)^2} + \frac{4b e^{2e+2fx} (c+dx)^2}{(a-b)^2 \left(a \left(1 - \frac{b}{a} \right) - a \left(1 + \frac{b}{a} \right) e^{2e+2fx} \right)} \right) dx \\
&= \frac{(c+dx)^3}{3(a-b)^2 d} + \frac{(4b) \int \frac{e^{2e+2fx} (c+dx)^2}{a \left(1 - \frac{b}{a} \right) - a \left(1 + \frac{b}{a} \right) e^{2e+2fx}} dx}{(a-b)^2} + \frac{(4b^2) \int \frac{e^{4e+4fx} (c+dx)^2}{\left(a \left(1 - \frac{b}{a} \right) - a \left(1 + \frac{b}{a} \right) e^{2e+2fx} \right)^2} dx}{(a-b)^2} \\
&= \frac{(c+dx)^3}{3(a-b)^2 d} - \frac{2b(c+dx)^2 \log \left(1 - \frac{(a+b)e^{2e+2fx}}{a-b} \right)}{(a-b)^2 (a+b)f} + \frac{(4b^2) \int \left(\frac{(c+dx)^2}{(a+b)^2} + \frac{(a-b)^2 (c+dx)^2}{(a+b)^2 (a-b - (a+b)e^{2e+2fx})^2} \right) dx}{(a-b)^2} \\
&= \frac{(c+dx)^3}{3(a-b)^2 d} + \frac{4b^2 (c+dx)^3}{3(a^2 - b^2)^2 d} - \frac{2b(c+dx)^2 \log \left(1 - \frac{(a+b)e^{2e+2fx}}{a-b} \right)}{(a-b)^2 (a+b)f} - \frac{2bd(c+dx) \text{Li}_2 \left(\frac{(a+b)e^{2e+2fx}}{a-b} \right)}{(a-b)^2 (a+b)f^2} \\
&= \frac{(c+dx)^3}{3(a-b)^2 d} - \frac{4b^2 (c+dx)^3}{3(a^2 - b^2)^2 d} - \frac{2b(c+dx)^2 \log \left(1 - \frac{(a+b)e^{2e+2fx}}{a-b} \right)}{(a-b)^2 (a+b)f} - \frac{2bd(c+dx) \text{Li}_2 \left(\frac{(a+b)e^{2e+2fx}}{a-b} \right)}{(a-b)^2 (a+b)f^2} \\
&= \frac{2b^2 (c+dx)^2}{(a-b)(a+b)^2 (a-b - (a+b)e^{2e+2fx}) f} + \frac{(c+dx)^3}{3(a-b)^2 d} - \frac{2b(c+dx)^2 \log \left(1 - \frac{(a+b)e^{2e+2fx}}{a-b} \right)}{(a-b)^2 (a+b)f} \\
&= -\frac{2b^2 (c+dx)^2}{(a^2 - b^2)^2 f} + \frac{2b^2 (c+dx)^2}{(a-b)(a+b)^2 (a-b - (a+b)e^{2e+2fx}) f} + \frac{(c+dx)^3}{3(a-b)^2 d} - \frac{2b(c+dx)^2 \log \left(1 - \frac{(a+b)e^{2e+2fx}}{a-b} \right)}{(a-b)^2 (a+b)f} \\
&= -\frac{2b^2 (c+dx)^2}{(a^2 - b^2)^2 f} + \frac{2b^2 (c+dx)^2}{(a-b)(a+b)^2 (a-b - (a+b)e^{2e+2fx}) f} + \frac{(c+dx)^3}{3(a-b)^2 d} + \frac{2b^2 d (c+dx) \log \left(1 - \frac{(a+b)e^{2e+2fx}}{a-b} \right)}{(a^2 - b^2)^2 f} \\
&= -\frac{2b^2 (c+dx)^2}{(a^2 - b^2)^2 f} + \frac{2b^2 (c+dx)^2}{(a-b)(a+b)^2 (a-b - (a+b)e^{2e+2fx}) f} + \frac{(c+dx)^3}{3(a-b)^2 d} + \frac{2b^2 d (c+dx) \log \left(1 - \frac{(a+b)e^{2e+2fx}}{a-b} \right)}{(a^2 - b^2)^2 f} \\
&= -\frac{2b^2 (c+dx)^2}{(a^2 - b^2)^2 f} + \frac{2b^2 (c+dx)^2}{(a-b)(a+b)^2 (a-b - (a+b)e^{2e+2fx}) f} + \frac{(c+dx)^3}{3(a-b)^2 d} + \frac{2b^2 d (c+dx) \log \left(1 - \frac{(a+b)e^{2e+2fx}}{a-b} \right)}{(a^2 - b^2)^2 f}
\end{aligned}$$

Mathematica [A] time = 8.29476, size = 509, normalized size = 1.07

$$-6bd(bd - 2acf)\text{PolyLog}\left(2, \frac{(a-b)e^{-2(e+fx)}}{a+b}\right) + 6abd^2\left(2fx\text{PolyLog}\left(2, \frac{(a-b)e^{-2(e+fx)}}{a+b}\right) + \text{PolyLog}\left(3, \frac{(a-b)e^{-2(e+fx)}}{a+b}\right)\right) - \frac{f^2(a-b)(a+b)}{a+b}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)^2/(a + b*Coth[e + f*x])^2,x]

[Out] ((24*(a - b)*b*c*f^2*(-(b*d) + a*c*f)*x)/(a*(-1 + E^(2*e)) + b*(1 + E^(2*e))) + (12*(a - b)*b*d*f^2*(-(b*d) + 2*a*c*f)*x^2)/(a*(-1 + E^(2*e)) + b*(1 + E^(2*e))) + (8*a*(a - b)*b*d^2*f^3*x^3)/(a*(-1 + E^(2*e)) + b*(1 + E^(2*e))) + 12*b*d*f*(b*d - 2*a*c*f)*x*Log[1 + (-a + b)/((a + b)*E^(2*(e + f*x)))] - 12*a*b*d^2*f^2*x^2*Log[1 + (-a + b)/((a + b)*E^(2*(e + f*x)))] + 12*b*c*f*(-(b*d) + a*c*f)*(2*f*x - Log[a - b - (a + b)*E^(2*(e + f*x))]) - 6*b*d*(b*d - 2*a*c*f)*PolyLog[2, (a - b)/((a + b)*E^(2*(e + f*x)))] + 6*a*b*d^2*(2*f*x*PolyLog[2, (a - b)/((a + b)*E^(2*(e + f*x)))] + PolyLog[3, (a - b)/((a + b)*E^(2*(e + f*x)))] - ((a - b)*(a + b)*f^2*((a^2 + b^2)*f*x*(3*c^2 + 3*c*d*x + d^2*x^2)*Cosh[f*x] - (a^2 - b^2)*f*x*(3*c^2 + 3*c*d*x + d^2*x^2)*Cosh[2*e + f*x] + 2*b*(-3*b*(c + d*x)^2 + a*f*x*(3*c^2 + 3*c*d*x + d^2*x^2))*Sinh[f*x]))/(b*Cosh[e] + a*Sinh[e])*(b*Cosh[e + f*x] + a*Sinh[e + f*x]))/(6*(a - b)^2*(a + b)^2*f^3)

Maple [B] time = 0.263, size = 1217, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^2/(a+b*coth(f*x+e))^2,x)

[Out] -4*b^2/(a+b)^2/(a-b)^2/f^2*d^2*e*x+8*b/(a+b)^2/(a-b)^2/f*a*c*d*e*x+1/3/(a^2+2*a*b+b^2)*d^2*x^3+1/(a^2+2*a*b+b^2)*c^2*x-2*b/(a+b)^2/(a-b)^2/f*a*d^2*ln(1-(a+b)*exp(2*f*x+2*e)/(a-b))*x^2+2*b/(a+b)^2/(a-b)^2/f^3*a*d^2*ln(1-(a+b)*exp(2*f*x+2*e)/(a-b))*e^2-2*b/(a+b)^2/(a-b)^2/f^2*a*d^2*polylog(2, (a+b)*exp(2*f*x+2*e)/(a-b))*x+4*b/(a+b)^2/(a-b)^2/f^3*a*d^2*e^2*ln(exp(f*x+e))-2*b/(a+b)^2/(a-b)^2/f^3*a*d^2*e^2*ln(a*exp(2*f*x+2*e)+b*exp(2*f*x+2*e)-a+b)+4*b/(a+b)^2/(a-b)^2*a*c*d*x^2+4*b/(a+b)^2/(a-b)^2/f^2*a*c*d*e^2-2*b/(a+b)^2/(a-b)^2/f^2*a*c*d*polylog(2, (a+b)*exp(2*f*x+2*e)/(a-b))-4*b/(a+b)^2/(a-b)^2/f^2*a*d^2*e^2*x-2*b^2/(a+b)^2/(a-b)^2/f*d^2*x^2-2*b^2/(a+b)^2/(a-b)^2/f^3*d^2*e^2+b^2/(a+b)^2/(a-b)^2/f^3*d^2*polylog(2, (a+b)*exp(2*f*x+2*e)/(a-b))-2/(a

$$\begin{aligned}
& -b)/f/(a^2+2ab+b^2)*(d^2x^2+2cdx+c^2)*b^2/(a\exp(2fx+2e)+b\exp(2fx+2e)-a+b)+4/3*b/(a+b)^2/(a-b)^2*a*d^2*x^3-8/3*b/(a+b)^2/(a-b)^2/f^3*a*d^2*e^3+2*b^2/(a+b)^2/(a-b)^2/f^2*d^2*\ln(1-(a+b)*\exp(2fx+2e)/(a-b))*x+2*b^2/(a+b)^2/(a-b)^2/f^3*d^2*\ln(1-(a+b)*\exp(2fx+2e)/(a-b))*e+4*b/(a+b)^2/(a-b)^2/f*a*c^2*\ln(\exp(fx+e))-2*b/(a+b)^2/(a-b)^2/f*a*c^2*\ln(a\exp(2fx+2e)+b\exp(2fx+2e)-a+b)+b/(a+b)^2/(a-b)^2/f^3*a*d^2*\text{polylog}(3,(a+b)*\exp(2fx+2e)/(a-b))-4*b^2/(a+b)^2/(a-b)^2/f^2*c*d*\ln(\exp(fx+e))-4*b/(a+b)^2/(a-b)^2/f*a*c*d*\ln(1-(a+b)*\exp(2fx+2e)/(a-b))*x-4*b/(a+b)^2/(a-b)^2/f^2*a*c*d*\ln(1-(a+b)*\exp(2fx+2e)/(a-b))*e-8*b/(a+b)^2/(a-b)^2/f^2*a*c*d*e*\ln(\exp(fx+e))+4*b/(a+b)^2/(a-b)^2/f^2*a*c*d*e*\ln(a\exp(2fx+2e)+b\exp(2fx+2e)-a+b)+2*b^2/(a+b)^2/(a-b)^2/f^2*c*d*\ln(a\exp(2fx+2e)+b\exp(2fx+2e)-a+b)+4*b^2/(a+b)^2/(a-b)^2/f^3*d^2*e*\ln(\exp(fx+e))-2*b^2/(a+b)^2/(a-b)^2/f^3*d^2*e*\ln(a\exp(2fx+2e)+b\exp(2fx+2e)-a+b)+1/(a^2+2ab+b^2)*c*d*x^2
\end{aligned}$$

Maxima [A] time = 2.79082, size = 1014, normalized size = 2.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((dx+c)^2/(a+b*coth(f*x+e))^2,x, algorithm="maxima")

[Out]
$$\begin{aligned}
& -4*b^2*c*d*f*x/(a^4*f^2 - 2*a^2*b^2*f^2 + b^4*f^2) - (2*f^2*x^2*\log(-(a*e^{2e} + b*e^{2e})*e^{2fx}/(a - b) + 1) + 2*f*x*\text{dilog}((a*e^{2e} + b*e^{2e})*e^{2fx}/(a - b)) - \text{polylog}(3, (a*e^{2e} + b*e^{2e})*e^{2fx}/(a - b))) * a*b*d^2/(a^4*f^3 - 2*a^2*b^2*f^3 + b^4*f^3) + 2*b^2*c*d*\log((a*e^{2e} + b*e^{2e})*e^{2fx} - a + b)/(a^4*f^2 - 2*a^2*b^2*f^2 + b^4*f^2) - c^2*(2*a*b*\log(-(a - b)*e^{-2fx - 2e} + a + b)/((a^4 - 2*a^2*b^2 + b^4)*f) + 2*b^2/((a^4 - 2*a^2*b^2 + b^4 - (a^4 - 2*a^3*b + 2*a*b^3 - b^4)*e^{-2fx - 2e}))*f) - (f*x + e)/((a^2 + 2*a*b + b^2)*f) - (2*a*b*c*d*f - b^2*d^2)*(2*f*x*\log(-(a*e^{2e} + b*e^{2e})*e^{2fx}/(a - b) + 1) + \text{dilog}((a*e^{2e} + b*e^{2e})*e^{2fx}/(a - b)))/(a^4*f^3 - 2*a^2*b^2*f^3 + b^4*f^3) + 2/3*(2*a*b*d^2*f^3*x^3 + 3*(2*a*b*c*d*f - b^2*d^2)*f^2*x^2)/(a^4*f^3 - 2*a^2*b^2*f^3 + b^4*f^3) + 1/3*(12*b^2*c*d*x + (a^2*d^2*f - 2*a*b*d^2*f + b^2*d^2*f)*x^3 + 3*(a^2*c*d*f - 2*a*b*c*d*f + (c*d*f + 2*d^2)*b^2)*x^2 - ((a^2*d^2*f*e^{2e} - b^2*d^2*f*e^{2e})*x^3 + 3*(a^2*c*d*f*e^{2e} - b^2*c*d*f*e^{2e})*x^2)*e^{2fx})/(a^4*f - 2*a^2*b^2*f + b^4*f - (a^4*f*e^{2e} + 2*a^3*b*f*e^{2e} - 2*a*b^3*f*e^{2e} - b^4*f*e^{2e}))*e^{2fx})
\end{aligned}$$

Fricas [C] time = 2.90485, size = 7876, normalized size = 16.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2/(a+b*coth(f*x+e))^2,x, algorithm="fricas")

[Out]
$$\begin{aligned} & -1/3*((a^3 + a^2*b - a*b^2 - b^3)*d^2*f^3*x^3 + 3*(a^3 + a^2*b - a*b^2 - b^3)*c*d*f^3*x^2 + 3*(a^3 + a^2*b - a*b^2 - b^3)*c^2*f^3*x + 4*(a^2*b - a*b^2) \\ & *d^2*e^3 + 6*(a*b^2 - b^3)*d^2*e^2 + 6*(2*(a^2*b - a*b^2)*c^2*e + (a*b^2 - b^3)*c^2)*f^2 - ((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d^2*f^3*x^3 + 4*(a^2*b + a*b^2)*d^2*e^3 + 12*(a^2*b + a*b^2)*c^2*e*f^2 + 6*(a*b^2 + b^3)*d^2*e^2 + 3 \\ & *((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*c*d*f^3 - 2*(a*b^2 + b^3)*d^2*f^2)*x^2 - 12*((a^2*b + a*b^2)*c*d*e^2 + (a*b^2 + b^3)*c*d*e)*f + 3*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*c^2*f^3 - 4*(a*b^2 + b^3)*c*d*f^2)*x)*\cosh(f*x + e)^2 - 2*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d^2*f^3*x^3 + 4*(a^2*b + a*b^2)*d^2*e^3 + 12 \\ & *(a^2*b + a*b^2)*c^2*e*f^2 + 6*(a*b^2 + b^3)*d^2*e^2 + 3*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*c*d*f^3 - 2*(a*b^2 + b^3)*d^2*f^2)*x^2 - 12*((a^2*b + a*b^2) \\ & *c*d*e^2 + (a*b^2 + b^3)*c*d*e)*f + 3*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*c^2*f^3 - 4*(a*b^2 + b^3)*c*d*f^2)*x)*\cosh(f*x + e)*\sinh(f*x + e) - ((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d^2*f^3*x^3 + 4*(a^2*b + a*b^2)*d^2*e^3 + 12*(a^2*b + a*b^2)*c^2*e*f^2 + 6*(a*b^2 + b^3)*d^2*e^2 + 3*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*c*d*f^3 - 2*(a*b^2 + b^3)*d^2*f^2)*x^2 - 12*((a^2*b + a*b^2) \\ & *c*d*e^2 + (a*b^2 + b^3)*c*d*e)*f + 3*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*c^2*f^3 - 4*(a*b^2 + b^3)*c*d*f^2)*x)*\sinh(f*x + e)^2 - 12*((a^2*b - a*b^2)*c*d*e^2 + (a \\ & *b^2 - b^3)*c*d*e)*f - 6*(2*(a^2*b - a*b^2)*d^2*f*x + 2*(a^2*b - a*b^2)*c*d*f - (a*b^2 - b^3)*d^2 - (2*(a^2*b + a*b^2)*d^2*f*x + 2*(a^2*b + a*b^2)*c*d*f - (a*b^2 + b^3)*d^2)*\cosh(f*x + e)^2 - 2*(2*(a^2*b + a*b^2)*d^2*f*x + 2*(a^2*b + a*b^2)*c*d*f - (a*b^2 + b^3)*d^2)*\cosh(f*x + e)*\sinh(f*x + e) - (2 \\ & *(a^2*b + a*b^2)*d^2*f*x + 2*(a^2*b + a*b^2)*c*d*f - (a*b^2 + b^3)*d^2)*\sinh(f*x + e)^2*dilog(\sqrt{(a + b)/(a - b)}*(\cosh(f*x + e) + \sinh(f*x + e))) \\ & - 6*(2*(a^2*b - a*b^2)*d^2*f*x + 2*(a^2*b - a*b^2)*c*d*f - (a*b^2 - b^3)*d^2 - (2*(a^2*b + a*b^2)*d^2*f*x + 2*(a^2*b + a*b^2)*c*d*f - (a*b^2 + b^3)*d^2)*\cosh(f*x + e)^2 - 2*(2*(a^2*b + a*b^2)*d^2*f*x + 2*(a^2*b + a*b^2)*c*d*f - (a*b^2 + b^3)*d^2)*\cosh(f*x + e)*\sinh(f*x + e) - (2*(a^2*b + a*b^2)*d^2*f*x + 2*(a^2*b + a*b^2)*c*d*f - (a*b^2 + b^3)*d^2)*\sinh(f*x + e)^2*dilog(-\sqrt{(a + b)/(a - b)}*(\cosh(f*x + e) + \sinh(f*x + e))) - 6*((a^2*b - a*b^2) \\ & *d^2*e^2 + (a^2*b - a*b^2)*c^2*f^2 + (a*b^2 - b^3)*d^2*e - ((a^2*b + a*b^2) \\ & *d^2*e^2 + (a^2*b + a*b^2)*c^2*f^2 + (a*b^2 + b^3)*d^2*e - (2*(a^2*b + a*b^2)*c*d*e + (a*b^2 + b^3)*c*d)*f)*\cosh(f*x + e)^2 - 2*((a^2*b + a*b^2)*d^2*e^2 + (a^2*b + a*b^2)*c^2*f^2 + (a*b^2 + b^3)*d^2*e - (2*(a^2*b + a*b^2)*c*d*e + (a*b^2 + b^3)*c*d)*f)*\cosh(f*x + e)*\sinh(f*x + e) - ((a^2*b + a*b^2)*d^2*e^2 + (a^2*b + a*b^2)*c^2*f^2 + (a*b^2 + b^3)*d^2*e - (2*(a^2*b + a*b^2) \end{aligned}$$

$$\begin{aligned}
& *c*d*e + (a*b^2 + b^3)*c*d*f)*\sinh(f*x + e)^2 - (2*(a^2*b - a*b^2)*c*d*e + \\
& (a*b^2 - b^3)*c*d*f)*\log(2*(a + b)*\cosh(f*x + e) + 2*(a + b)*\sinh(f*x + e) \\
&) + 2*(a - b)*\sqrt{(a + b)/(a - b)}) - 6*((a^2*b - a*b^2)*d^2*e^2 + (a^2*b \\
& - a*b^2)*c^2*f^2 + (a*b^2 - b^3)*d^2*e - ((a^2*b + a*b^2)*d^2*e^2 + (a^2*b \\
& + a*b^2)*c^2*f^2 + (a*b^2 + b^3)*d^2*e - (2*(a^2*b + a*b^2)*c*d*e + (a*b^2 \\
& + b^3)*c*d*f)*\cosh(f*x + e)^2 - 2*((a^2*b + a*b^2)*d^2*e^2 + (a^2*b + a*b^2) \\
&)*c^2*f^2 + (a*b^2 + b^3)*d^2*e - (2*(a^2*b + a*b^2)*c*d*e + (a*b^2 + b^3) \\
&)*c*d*f)*\cosh(f*x + e)*\sinh(f*x + e) - ((a^2*b + a*b^2)*d^2*e^2 + (a^2*b + \\
& a*b^2)*c^2*f^2 + (a*b^2 + b^3)*d^2*e - (2*(a^2*b + a*b^2)*c*d*e + (a*b^2 + \\
& b^3)*c*d*f)*\sinh(f*x + e)^2 - (2*(a^2*b - a*b^2)*c*d*e + (a*b^2 - b^3)*c*d \\
&)*f)*\log(2*(a + b)*\cosh(f*x + e) + 2*(a + b)*\sinh(f*x + e) - 2*(a - b)*\sqrt{ \\
& ((a + b)/(a - b))}) - 6*((a^2*b - a*b^2)*d^2*f^2*x^2 - (a^2*b - a*b^2)*d^2*e \\
& ^2 + 2*(a^2*b - a*b^2)*c*d*e*f - (a*b^2 - b^3)*d^2*e - ((a^2*b + a*b^2)*d^2 \\
& *f^2*x^2 - (a^2*b + a*b^2)*d^2*e^2 + 2*(a^2*b + a*b^2)*c*d*e*f - (a*b^2 + b \\
& ^3)*d^2*e + (2*(a^2*b + a*b^2)*c*d*f^2 - (a*b^2 + b^3)*d^2*f)*x)*\cosh(f*x + \\
& e)^2 - 2*((a^2*b + a*b^2)*d^2*f^2*x^2 - (a^2*b + a*b^2)*d^2*e^2 + 2*(a^2*b \\
& + a*b^2)*c*d*e*f - (a*b^2 + b^3)*d^2*e + (2*(a^2*b + a*b^2)*c*d*f^2 - (a*b \\
& ^2 + b^3)*d^2*f)*x)*\cosh(f*x + e)*\sinh(f*x + e) - ((a^2*b + a*b^2)*d^2*f^2*x \\
& ^2 - (a^2*b + a*b^2)*d^2*e^2 + 2*(a^2*b + a*b^2)*c*d*e*f - (a*b^2 + b^3)*d \\
& ^2*e + (2*(a^2*b + a*b^2)*c*d*f^2 - (a*b^2 + b^3)*d^2*f)*x)*\sinh(f*x + e)^2 \\
& + (2*(a^2*b - a*b^2)*c*d*f^2 - (a*b^2 - b^3)*d^2*f)*x)*\log(\sqrt{(a + b)/(a \\
& - b)}*(\cosh(f*x + e) + \sinh(f*x + e)) + 1) - 6*((a^2*b - a*b^2)*d^2*f^2*x^ \\
& 2 - (a^2*b - a*b^2)*d^2*e^2 + 2*(a^2*b - a*b^2)*c*d*e*f - (a*b^2 - b^3)*d^2 \\
& *e - ((a^2*b + a*b^2)*d^2*f^2*x^2 - (a^2*b + a*b^2)*d^2*e^2 + 2*(a^2*b + a* \\
& b^2)*c*d*e*f - (a*b^2 + b^3)*d^2*e + (2*(a^2*b + a*b^2)*c*d*f^2 - (a*b^2 + \\
& b^3)*d^2*f)*x)*\cosh(f*x + e)^2 - 2*((a^2*b + a*b^2)*d^2*f^2*x^2 - (a^2*b + \\
& a*b^2)*d^2*e^2 + 2*(a^2*b + a*b^2)*c*d*e*f - (a*b^2 + b^3)*d^2*e + (2*(a^2* \\
& b + a*b^2)*c*d*f^2 - (a*b^2 + b^3)*d^2*f)*x)*\cosh(f*x + e)*\sinh(f*x + e) - \\
& ((a^2*b + a*b^2)*d^2*f^2*x^2 - (a^2*b + a*b^2)*d^2*e^2 + 2*(a^2*b + a*b^2)* \\
& c*d*e*f - (a*b^2 + b^3)*d^2*e + (2*(a^2*b + a*b^2)*c*d*f^2 - (a*b^2 + b^3)* \\
& d^2*f)*x)*\sinh(f*x + e)^2 + (2*(a^2*b - a*b^2)*c*d*f^2 - (a*b^2 - b^3)*d^2* \\
& f)*x)*\log(-\sqrt{(a + b)/(a - b)}*(\cosh(f*x + e) + \sinh(f*x + e)) + 1) - 12* \\
& ((a^2*b + a*b^2)*d^2*\cosh(f*x + e)^2 + 2*(a^2*b + a*b^2)*d^2*\cosh(f*x + e)* \\
& \sinh(f*x + e) + (a^2*b + a*b^2)*d^2*\sinh(f*x + e)^2 - (a^2*b - a*b^2)*d^2)* \\
& \text{polylog}(3, \sqrt{(a + b)/(a - b)}*(\cosh(f*x + e) + \sinh(f*x + e))) - 12*((a^ \\
& 2*b + a*b^2)*d^2*\cosh(f*x + e)^2 + 2*(a^2*b + a*b^2)*d^2*\cosh(f*x + e)*\sinh \\
& (f*x + e) + (a^2*b + a*b^2)*d^2*\sinh(f*x + e)^2 - (a^2*b - a*b^2)*d^2)*\text{poly} \\
& \text{log}(3, -\sqrt{(a + b)/(a - b)}*(\cosh(f*x + e) + \sinh(f*x + e)))/((a^5 + a^4 \\
& *b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5)*f^3*\cosh(f*x + e)^2 + 2*(a^5 + a^ \\
& 4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5)*f^3*\cosh(f*x + e)*\sinh(f*x + e) \\
& + (a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5)*f^3*\sinh(f*x + e)^2 - \\
& (a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*f^3)
\end{aligned}$$

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**2/(a+b*coth(f*x+e))**2,x)

[Out] Exception raised: TypeError

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx + c)^2}{(b \coth(fx + e) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2/(a+b*coth(f*x+e))^2,x, algorithm="giac")

[Out] integrate((d*x + c)^2/(b*coth(f*x + e) + a)^2, x)

$$3.59 \quad \int \frac{c+dx}{(a+b \coth(e+fx))^2} dx$$

Optimal. Leaf size=196

$$\frac{abd \operatorname{PolyLog}\left(2, \frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{f^2(a^2-b^2)^2} + \frac{b(-2acf - 2adfx + bd) \log\left(1 - \frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{f^2(a^2-b^2)^2} + \frac{b(c+dx)}{f(a^2-b^2)(a+b \coth(e+fx))} - \frac{(c+dx)}{2d(a+b \coth(e+fx))}$$

```
[Out] -(c + d*x)^2/(2*(a^2 - b^2)*d) + (b*d - 2*a*c*f - 2*a*d*f*x)^2/(4*a*(a - b)
*(a + b)^2*d*f^2) + (b*(c + d*x))/((a^2 - b^2)*f*(a + b*Coth[e + f*x])) + (
b*(b*d - 2*a*c*f - 2*a*d*f*x)*Log[1 - (a - b)/((a + b)*E^(2*(e + f*x)))]/(
(a^2 - b^2)^2*f^2) + (a*b*d*PolyLog[2, (a - b)/((a + b)*E^(2*(e + f*x)))]/
((a^2 - b^2)^2*f^2)
```

Rubi [A] time = 0.295785, antiderivative size = 196, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {3733, 3731, 2190, 2279, 2391}

$$\frac{abd \operatorname{PolyLog}\left(2, \frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{f^2(a^2-b^2)^2} + \frac{b(-2acf - 2adfx + bd) \log\left(1 - \frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{f^2(a^2-b^2)^2} + \frac{b(c+dx)}{f(a^2-b^2)(a+b \coth(e+fx))} - \frac{(c+dx)}{2d(a+b \coth(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a + b*Coth[e + f*x])^2, x]
```

```
[Out] -(c + d*x)^2/(2*(a^2 - b^2)*d) + (b*d - 2*a*c*f - 2*a*d*f*x)^2/(4*a*(a - b)
*(a + b)^2*d*f^2) + (b*(c + d*x))/((a^2 - b^2)*f*(a + b*Coth[e + f*x])) + (
b*(b*d - 2*a*c*f - 2*a*d*f*x)*Log[1 - (a - b)/((a + b)*E^(2*(e + f*x)))]/(
(a^2 - b^2)^2*f^2) + (a*b*d*PolyLog[2, (a - b)/((a + b)*E^(2*(e + f*x)))]/
((a^2 - b^2)^2*f^2)
```

Rule 3733

```
Int[((c_.) + (d_.)*(x_))/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^2, x_Symbol
] :> -Simp[(c + d*x)^2/(2*d*(a^2 + b^2)), x] + (Dist[1/(f*(a^2 + b^2)), Int
[(b*d + 2*a*c*f + 2*a*d*f*x)/(a + b*Tan[e + f*x]), x], x] - Simp[(b*(c + d*
x))/(f*(a^2 + b^2)*(a + b*Tan[e + f*x])), x]) /; FreeQ[{a, b, c, d, e, f},
x] && NeQ[a^2 + b^2, 0]
```

Rule 3731

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(d*(m + 1)*(a + I*b)), x] + Dist[2*I*b, Int[((c + d*x)^m*E^(2*I*k*Pi)*E^Simp[2*I*(e + f*x), x])/((a + I*b)^2 + (a^2 + b^2)*E^(2*I*k*Pi)*E^Simp[2*I*(e + f*x), x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[4*k] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol] :> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{c + dx}{(a + b \coth(e + fx))^2} dx &= -\frac{(c + dx)^2}{2(a^2 - b^2)d} + \frac{b(c + dx)}{(a^2 - b^2)f(a + b \coth(e + fx))} - \frac{i \int \frac{-ibd + 2iacf + 2iadfx}{a + b \coth(e + fx)} dx}{(a^2 - b^2)f} \\
&= -\frac{(c + dx)^2}{2(a^2 - b^2)d} + \frac{(bd - 2acf - 2adfx)^2}{4a(a - b)(a + b)^2df^2} + \frac{b(c + dx)}{(a^2 - b^2)f(a + b \coth(e + fx))} + \frac{(2ib) \int \frac{e^{-2(e+fx)}}{(a+b \coth(e+fx))} dx}{(a^2 - b^2)f} \\
&= -\frac{(c + dx)^2}{2(a^2 - b^2)d} + \frac{(bd - 2acf - 2adfx)^2}{4a(a - b)(a + b)^2df^2} + \frac{b(c + dx)}{(a^2 - b^2)f(a + b \coth(e + fx))} + \frac{b(bd - 2acf)}{(a^2 - b^2)f} \\
&= -\frac{(c + dx)^2}{2(a^2 - b^2)d} + \frac{(bd - 2acf - 2adfx)^2}{4a(a - b)(a + b)^2df^2} + \frac{b(c + dx)}{(a^2 - b^2)f(a + b \coth(e + fx))} + \frac{b(bd - 2acf)}{(a^2 - b^2)f} \\
&= -\frac{(c + dx)^2}{2(a^2 - b^2)d} + \frac{(bd - 2acf - 2adfx)^2}{4a(a - b)(a + b)^2df^2} + \frac{b(c + dx)}{(a^2 - b^2)f(a + b \coth(e + fx))} + \frac{b(bd - 2acf)}{(a^2 - b^2)f}
\end{aligned}$$

Mathematica [C] time = 6.00419, size = 477, normalized size = 2.43

$$\text{csch}^2(e + fx)(a \sinh(e + fx) + b \cosh(e + fx)) \left(-2ad(a \sinh(e + fx) + b \cosh(e + fx)) \left(b \text{PolyLog} \left(2, e^{-2 \left(\tanh^{-1} \left(\frac{b}{a} \right) + e + fx \right)} \right) \right) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(c + d*x)/(a + b*Coth[e + f*x])^2,x]

[Out] (Csch[e + f*x]^2*(b*Cosh[e + f*x] + a*Sinh[e + f*x])*(2*b*(-a^2 + b^2)*f*(c + d*x)*Sinh[e + f*x] - (a^2 - b^2)*(e + f*x)*(-2*c*f + d*(e - f*x))*(b*Cosh[e + f*x] + a*Sinh[e + f*x]) + 2*b*d*(a*(e + f*x) - b*Log[b*Cosh[e + f*x] + a*Sinh[e + f*x]])*(b*Cosh[e + f*x] + a*Sinh[e + f*x]) + 4*a*d*e*(a*(e + f*x) - b*Log[b*Cosh[e + f*x] + a*Sinh[e + f*x]])*(b*Cosh[e + f*x] + a*Sinh[e + f*x]) - 4*a*c*f*(a*(e + f*x) - b*Log[b*Cosh[e + f*x] + a*Sinh[e + f*x]])*(b*Cosh[e + f*x] + a*Sinh[e + f*x]) - 2*a*d*((a*Sqrt[1 - b^2/a^2]*(e + f*x)^2)/E^ArcTanh[b/a] - I*b*(e + f*x)*(Pi - (2*I)*ArcTanh[b/a]) + I*b*Pi*Log[1 + E^(2*(e + f*x))] - 2*b*(e + f*x + ArcTanh[b/a])*Log[1 - E^(-2*(e + f*x + ArcTanh[b/a]))] - I*b*Pi*Log[Cosh[e + f*x]] + 2*b*ArcTanh[b/a]*Log[I*Sinh[e + f*x + ArcTanh[b/a]]] + b*PolyLog[2, E^(-2*(e + f*x + ArcTanh[b/a]))])*)

$(b \cdot \text{Cosh}[e + f \cdot x] + a \cdot \text{Sinh}[e + f \cdot x]) / (2 \cdot (-a + b) \cdot (a + b) \cdot (a^2 - b^2) \cdot f^2 \cdot (a + b \cdot \text{Coth}[e + f \cdot x])^2)$

Maple [B] time = 0.234, size = 620, normalized size = 3.2

$$\frac{dx^2}{2a^2 + 4ab + 2b^2} + \frac{cx}{a^2 + 2ab + b^2} - 2 \frac{b^2(dx + c)}{(a-b)f(a^2 + 2ab + b^2)(ae^{2fx+2e} + be^{2fx+2e} - a + b)} - 2 \frac{b^2 d \ln(e^{fx+e})}{(a-b)^2 f^2 (a^2 + 2ab)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x+c)/(a+b*coth(f*x+e))^2,x)`

[Out] $1/2/(a^2+2*a*b+b^2)*d*x^2+1/(a^2+2*a*b+b^2)*c*x-2/(a-b)/f/(a^2+2*a*b+b^2)*b^2*(d*x+c)/(a*\exp(2*f*x+2*e)+b*\exp(2*f*x+2*e)-a+b)-2*b^2/(a-b)^2/f^2/(a^2+2*a*b+b^2)*d*\ln(\exp(f*x+e))+b^2/(a-b)^2/f^2/(a^2+2*a*b+b^2)*d*\ln(a*\exp(2*f*x+2*e)+b*\exp(2*f*x+2*e)-a+b)+4*b/(a-b)^2/f/(a^2+2*a*b+b^2)*a*c*\ln(\exp(f*x+e))-2*b/(a-b)^2/f/(a^2+2*a*b+b^2)*a*c*\ln(a*\exp(2*f*x+2*e)+b*\exp(2*f*x+2*e)-a+b)-4*b/(a-b)^2/f^2/(a^2+2*a*b+b^2)*d*a*e*\ln(\exp(f*x+e))+2*b/(a-b)^2/f^2/(a^2+2*a*b+b^2)*d*a*e*\ln(a*\exp(2*f*x+2*e)+b*\exp(2*f*x+2*e)-a+b)-2*b/(a-b)^2/f/(a^2+2*a*b+b^2)*d*a*\ln(1-(a+b)*\exp(2*f*x+2*e)/(a-b))*x-2*b/(a-b)^2/f^2/(a^2+2*a*b+b^2)*d*a*\ln(1-(a+b)*\exp(2*f*x+2*e)/(a-b))*e+2*b/(a-b)^2/(a^2+2*a*b+b^2)*d*a*x^2+4*b/(a-b)^2/f/(a^2+2*a*b+b^2)*d*a*e*x+2*b/(a-b)^2/f^2/(a^2+2*a*b+b^2)*d*a*e^2-b/(a-b)^2/f^2/(a^2+2*a*b+b^2)*d*a*polylog(2,(a+b)*\exp(2*f*x+2*e)/(a-b))$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2} \left(8abf \int \frac{x}{a^4 f e^{2fx+2e} + 2a^3 b f e^{2fx+2e} - 2ab^3 f e^{2fx+2e} - b^4 f e^{2fx+2e} - a^4 f + 2a^2 b^2 f - b^4 f} dx + 2b^2 \left(\frac{2(fx)}{a^4 - 2a^2 b^2} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)/(a+b*coth(f*x+e))^2,x, algorithm="maxima")`

[Out] $-1/2*(8*a*b*f*\integrate(x/(a^4*f*e^{2*f*x+2*e} + 2*a^3*b*f*e^{2*f*x+2*e} - 2*a*b^3*f*e^{2*f*x+2*e} - b^4*f*e^{2*f*x+2*e} - a^4*f + 2*a^2*b^2*f - b^4*f), x) + 2*b^2*(2*(f*x + e)/((a^4 - 2*a^2*b^2 + b^4)*f^2) - \log((a + b)*e^{2*f*x + 2*e} - a + b)/((a^4 - 2*a^2*b^2 + b^4)*f^2)) + ((a^2*f*e^{2*f*x+2*e} - a + b)/((a^4 - 2*a^2*b^2 + b^4)*f^2) + ((a^2*f*e^{2*f*x+2*e} - a + b)/((a^4 - 2*a^2*b^2 + b^4)*f^2))$

$$e) - b^2 f e^{(2e)} x^2 e^{(2fx)} - 4b^2 x - (a^2 f - 2abf + b^2 f) x^2 \\) / (a^4 f - 2a^2 b^2 f + b^4 f - (a^4 f e^{(2e)} + 2a^3 b f e^{(2e)} - 2a b^3 f e^{(2e)} - b^4 f e^{(2e)}) e^{(2fx)}) d - c(2ab \log(-(a-b)e^{(-2fx - 2e)} + a + b) / ((a^4 - 2a^2 b^2 + b^4) f) + 2b^2 / ((a^4 - 2a^2 b^2 + b^4 - (a^4 - 2a^3 b + 2ab^3 - b^4) e^{(-2fx - 2e)}) f) - (fx + e) / ((a^2 + 2ab + b^2) f))$$

Fricas [B] time = 2.55933, size = 4058, normalized size = 20.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((dx+c)/(a+b*coth(f*x+e))^2,x, algorithm="fricas")

[Out] $-1/2*((a^3 + a^2 b - a b^2 - b^3) d f^2 x^2 + 2(a^3 + a^2 b - a b^2 - b^3) c f^2 x - 4(a^2 b - a b^2) d e^2 - 4(a b^2 - b^3) d e - ((a^3 + 3a^2 b + 3a b^2 + b^3) d f^2 x^2 - 4(a^2 b + a b^2) d e^2 + 8(a^2 b + a b^2) c e f - 4(a b^2 + b^3) d e + 2((a^3 + 3a^2 b + 3a b^2 + b^3) c f^2 - 2(a b^2 + b^3) d f) x) \cosh(fx + e)^2 - 2((a^3 + 3a^2 b + 3a b^2 + b^3) d f^2 x^2 - 4(a^2 b + a b^2) d e^2 + 8(a^2 b + a b^2) c e f - 4(a b^2 + b^3) d e + 2((a^3 + 3a^2 b + 3a b^2 + b^3) c f^2 - 2(a b^2 + b^3) d f) x) \cosh(fx + e) \sinh(fx + e) - ((a^3 + 3a^2 b + 3a b^2 + b^3) d f^2 x^2 - 4(a^2 b + a b^2) d e^2 + 8(a^2 b + a b^2) c e f - 4(a b^2 + b^3) d e + 2((a^3 + 3a^2 b + 3a b^2 + b^3) c f^2 - 2(a b^2 + b^3) d f) x) \sinh(fx + e)^2 + 4(2(a^2 b - a b^2) c e + (a b^2 - b^3) c) f + 4((a^2 b + a b^2) d \cosh(fx + e)^2 + 2(a^2 b + a b^2) d \cosh(fx + e) \sinh(fx + e) + (a^2 b + a b^2) d \sinh(fx + e)^2 - (a^2 b - a b^2) d) \operatorname{dilog}(\sqrt{(a+b)/(a-b)}) (\cosh(fx + e) + \sinh(fx + e)) + 4((a^2 b + a b^2) d \cosh(fx + e)^2 + 2(a^2 b + a b^2) d \cosh(fx + e) \sinh(fx + e) + (a^2 b + a b^2) d \sinh(fx + e)^2 - (a^2 b - a b^2) d) \operatorname{dilog}(-\sqrt{(a+b)/(a-b)}) (\cosh(fx + e) + \sinh(fx + e)) + 2(2(a^2 b - a b^2) d e - 2(a^2 b - a b^2) c f - (2(a^2 b + a b^2) d e - 2(a^2 b + a b^2) c f + (a b^2 + b^3) d) \cosh(fx + e)^2 - 2(2(a^2 b + a b^2) d e - 2(a^2 b + a b^2) c f + (a b^2 + b^3) d) \cosh(fx + e) \sinh(fx + e) - (2(a^2 b + a b^2) d e - 2(a^2 b + a b^2) c f + (a b^2 + b^3) d) \sinh(fx + e)^2 + (a b^2 - b^3) d) \log(2(a+b) \cosh(fx + e) + 2(a+b) \sinh(fx + e) + 2(a-b) \sqrt{(a+b)/(a-b)}) + 2(2(a^2 b - a b^2) d e - 2(a^2 b - a b^2) c f - (2(a^2 b + a b^2) d e - 2(a^2 b + a b^2) c f + (a b^2 + b^3) d) \cosh(fx + e)^2 - 2(2(a^2 b + a b^2) d e - 2(a^2 b + a b^2) c f + (a b^2 + b^3) d) \cosh(fx + e) \sinh(fx + e) - (2(a^2 b + a b^2) d e - 2(a^2 b + a b^2) c f + (a b^2 + b^3) d) \sinh(fx + e)^2 + (a b^2 - b^3) d) \log(2(a+b) \cosh(fx + e) + 2(a+b) \sinh$

```

h(f*x + e) - 2*(a - b)*sqrt((a + b)/(a - b))) - 4*((a^2*b - a*b^2)*d*f*x +
(a^2*b - a*b^2)*d*e - ((a^2*b + a*b^2)*d*f*x + (a^2*b + a*b^2)*d*e)*cosh(f*
x + e)^2 - 2*((a^2*b + a*b^2)*d*f*x + (a^2*b + a*b^2)*d*e)*cosh(f*x + e)*si
nh(f*x + e) - ((a^2*b + a*b^2)*d*f*x + (a^2*b + a*b^2)*d*e)*sinh(f*x + e)^2
)*log(sqrt((a + b)/(a - b))*(cosh(f*x + e) + sinh(f*x + e)) + 1) - 4*((a^2*
b - a*b^2)*d*f*x + (a^2*b - a*b^2)*d*e - ((a^2*b + a*b^2)*d*f*x + (a^2*b +
a*b^2)*d*e)*cosh(f*x + e)^2 - 2*((a^2*b + a*b^2)*d*f*x + (a^2*b + a*b^2)*d*
e)*cosh(f*x + e)*sinh(f*x + e) - ((a^2*b + a*b^2)*d*f*x + (a^2*b + a*b^2)*d
*e)*sinh(f*x + e)^2)*log(-sqrt((a + b)/(a - b))*(cosh(f*x + e) + sinh(f*x +
e)) + 1))/((a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5)*f^2*cosh(f*
x + e)^2 + 2*(a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5)*f^2*cosh(f
*x + e)*sinh(f*x + e) + (a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5)
*f^2*sinh(f*x + e)^2 - (a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*
f^2)

```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+b*coth(f*x+e))**2,x)
```

```
[Out] Exception raised: TypeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx + c}{(b \coth(fx + e) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+b*coth(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)/(b*coth(f*x + e) + a)^2, x)
```

$$3.60 \quad \int \frac{1}{(c+dx)(a+b \coth(e+fx))^2} dx$$

Optimal. Leaf size=22

$$\text{Unintegrable} \left(\frac{1}{(c+dx)(a+b \coth(e+fx))^2}, x \right)$$

[Out] Unintegrable[1/((c + d*x)*(a + b*Coth[e + f*x])^2), x]

Rubi [A] time = 0.0619273, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{1}{(c+dx)(a+b \coth(e+fx))^2} dx$$

Verification is Not applicable to the result.

[In] Int[1/((c + d*x)*(a + b*Coth[e + f*x])^2), x]

[Out] Defer[Int][1/((c + d*x)*(a + b*Coth[e + f*x])^2), x]

Rubi steps

$$\int \frac{1}{(c+dx)(a+b \coth(e+fx))^2} dx = \int \frac{1}{(c+dx)(a+b \coth(e+fx))^2} dx$$

Mathematica [A] time = 144.035, size = 0, normalized size = 0.

$$\int \frac{1}{(c+dx)(a+b \coth(e+fx))^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/((c + d*x)*(a + b*Coth[e + f*x])^2), x]

[Out] Integrate[1/((c + d*x)*(a + b*Coth[e + f*x])^2), x]

Maple [A] time = 0.752, size = 0, normalized size = 0.

$$\int \frac{1}{(dx+c)(a+b\coth(fx+e))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(d*x+c)/(a+b*coth(f*x+e))^2,x)

[Out] int(1/(d*x+c)/(a+b*coth(f*x+e))^2,x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$2b^2$$

$$a^4cf - 2a^2b^2cf + b^4cf + (a^4df - 2a^2b^2df + b^4df)x - (a^4cfe^{(2e)} + 2a^3bcfe^{(2e)} - 2ab^3cfe^{(2e)} - b^4cfe^{(2e)} + (a^4dfe^{(2e)} +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d*x+c)/(a+b*coth(f*x+e))^2,x, algorithm="maxima")

[Out] $2*b^2/(a^4*c*f - 2*a^2*b^2*c*f + b^4*c*f + (a^4*d*f - 2*a^2*b^2*d*f + b^4*d*f)*x - (a^4*c*f*e^{(2*e)} + 2*a^3*b*c*f*e^{(2*e)} - 2*a*b^3*c*f*e^{(2*e)} - b^4*c*f*e^{(2*e)} + (a^4*d*f*e^{(2*e)} + 2*a^3*b*d*f*e^{(2*e)} - 2*a*b^3*d*f*e^{(2*e)} - b^4*d*f*e^{(2*e)})*x)*e^{(2*f*x)}) + \log(dx+c)/(a^2*d + 2*a*b*d + b^2*d) - \int (-2*(2*a*b*d*f*x + 2*a*b*c*f + b^2*d)/(a^4*c^2*f - 2*a^2*b^2*c^2*f + b^4*c^2*f + (a^4*d^2*f - 2*a^2*b^2*d^2*f + b^4*d^2*f)*x^2 + 2*(a^4*c*d*f - 2*a^2*b^2*c*d*f + b^4*c*d*f)*x - (a^4*c^2*f*e^{(2*e)} + 2*a^3*b*c^2*f*e^{(2*e)} - 2*a*b^3*c^2*f*e^{(2*e)} - b^4*c^2*f*e^{(2*e)} + (a^4*d^2*f*e^{(2*e)} + 2*a^3*b*d^2*f*e^{(2*e)} - 2*a*b^3*d^2*f*e^{(2*e)} - b^4*d^2*f*e^{(2*e)})*x^2 + 2*(a^4*c*d*f*e^{(2*e)} + 2*a^3*b*c*d*f*e^{(2*e)} - 2*a*b^3*c*d*f*e^{(2*e)} - b^4*c*d*f*e^{(2*e)})*x)*e^{(2*f*x)}, x)$

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{1}{a^2 dx + a^2 c + (b^2 dx + b^2 c) \coth(fx + e)^2 + 2(ab dx + abc) \coth(fx + e)}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+b*coth(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral(1/(a^2*d*x + a^2*c + (b^2*d*x + b^2*c)*coth(f*x + e)^2 + 2*(a*b*d*x + a*b*c)*coth(f*x + e)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a + b \coth(e + fx))^2 (c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+b*coth(f*x+e))^2,x)
```

```
[Out] Integral(1/((a + b*coth(e + f*x))^2*(c + d*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dx + c)(b \coth(fx + e) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+b*coth(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((d*x + c)*(b*coth(f*x + e) + a)^2), x)
```

$$3.61 \quad \int \frac{1}{(c+dx)^2(a+b \coth(e+fx))^2} dx$$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{1}{(c+dx)^2(a+b \coth(e+fx))^2}, x\right)$$

[Out] Unintegrable[1/((c + d*x)^2*(a + b*Coth[e + f*x])^2), x]

Rubi [A] time = 0.0586905, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{1}{(c+dx)^2(a+b \coth(e+fx))^2} dx$$

Verification is Not applicable to the result.

[In] Int[1/((c + d*x)^2*(a + b*Coth[e + f*x])^2), x]

[Out] Defer[Int][1/((c + d*x)^2*(a + b*Coth[e + f*x])^2), x]

Rubi steps

$$\int \frac{1}{(c+dx)^2(a+b \coth(e+fx))^2} dx = \int \frac{1}{(c+dx)^2(a+b \coth(e+fx))^2} dx$$

Mathematica [A] time = 115.397, size = 0, normalized size = 0.

$$\int \frac{1}{(c+dx)^2(a+b \coth(e+fx))^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/((c + d*x)^2*(a + b*Coth[e + f*x])^2), x]

[Out] Integrate[1/((c + d*x)^2*(a + b*Coth[e + f*x])^2), x]

Maple [A] time = 0.697, size = 0, normalized size = 0.

$$\int \frac{1}{(dx+c)^2 (a+b\coth(fx+e))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(d*x+c)^2/(a+b*coth(f*x+e))^2,x)

[Out] int(1/(d*x+c)^2/(a+b*coth(f*x+e))^2,x)

Maxima [A] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d*x+c)^2/(a+b*coth(f*x+e))^2,x, algorithm="maxima")

[Out] $-(a^2*c*f - 2*a*b*c*f + (c*f - 2*d)*b^2 + (a^2*d*f - 2*a*b*d*f + b^2*d*f)*x - (a^2*c*f*e^{(2*e)} - b^2*c*f*e^{(2*e)} + (a^2*d*f*e^{(2*e)} - b^2*d*f*e^{(2*e)})*x)*e^{(2*f*x)})/(a^4*c^2*d*f - 2*a^2*b^2*c^2*d*f + b^4*c^2*d*f + (a^4*d^3*f - 2*a^2*b^2*d^3*f + b^4*d^3*f)*x^2 + 2*(a^4*c*d^2*f - 2*a^2*b^2*c*d^2*f + b^4*c*d^2*f)*x - (a^4*c^2*d*f*e^{(2*e)} + 2*a^3*b*c^2*d*f*e^{(2*e)} - 2*a*b^3*c^2*d*f*e^{(2*e)} - b^4*c^2*d*f*e^{(2*e)} + (a^4*d^3*f*e^{(2*e)} + 2*a^3*b*d^3*f*e^{(2*e)} - 2*a*b^3*d^3*f*e^{(2*e)} - b^4*d^3*f*e^{(2*e)}))*x^2 + 2*(a^4*c*d^2*f*e^{(2*e)} + 2*a^3*b*c*d^2*f*e^{(2*e)} - 2*a*b^3*c*d^2*f*e^{(2*e)} - b^4*c*d^2*f*e^{(2*e)})*x)*e^{(2*f*x)}) - \text{integrate}(-4*(a*b*d*f*x + a*b*c*f + b^2*d)/(a^4*c^3*f - 2*a^2*b^2*c^3*f + b^4*c^3*f + (a^4*d^3*f - 2*a^2*b^2*d^3*f + b^4*d^3*f)*x^3 + 3*(a^4*c*d^2*f - 2*a^2*b^2*c*d^2*f + b^4*c*d^2*f)*x^2 + 3*(a^4*c^2*d*f - 2*a^2*b^2*c^2*d*f + b^4*c^2*d*f)*x - (a^4*c^3*f*e^{(2*e)} + 2*a^3*b*c^3*f*e^{(2*e)} - 2*a*b^3*c^3*f*e^{(2*e)} - b^4*c^3*f*e^{(2*e)} + (a^4*d^3*f*e^{(2*e)} + 2*a^3*b*d^3*f*e^{(2*e)} - 2*a*b^3*d^3*f*e^{(2*e)} - b^4*d^3*f*e^{(2*e)}))*x^3 + 3*(a^4*c*d^2*f*e^{(2*e)} + 2*a^3*b*c*d^2*f*e^{(2*e)} - 2*a*b^3*c*d^2*f*e^{(2*e)} - b^4*c*d^2*f*e^{(2*e)}))*x^2 + 3*(a^4*c^2*d*f*e^{(2*e)} + 2*a^3*b*c^2*d*f*e^{(2*e)} - 2*a*b^3*c^2*d*f*e^{(2*e)} - b^4*c^2*d*f*e^{(2*e)}))*x)*e^{(2*f*x)}, x)$

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{1}{a^2 d^2 x^2 + 2 a^2 c d x + a^2 c^2 + (b^2 d^2 x^2 + 2 b^2 c d x + b^2 c^2) \coth(fx + e)^2 + 2 (abd^2 x^2 + 2 abcdx + abc^2) \coth(fx + e)}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d*x+c)^2/(a+b*coth(f*x+e))^2,x, algorithm="fricas")

[Out] integral(1/(a^2*d^2*x^2 + 2*a^2*c*d*x + a^2*c^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*coth(f*x + e)^2 + 2*(a*b*d^2*x^2 + 2*a*b*c*d*x + a*b*c^2)*coth(f*x + e)), x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a + b \coth(e + fx))^2 (c + dx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d*x+c)**2/(a+b*coth(f*x+e))**2,x)

[Out] Integral(1/((a + b*coth(e + f*x))**2*(c + d*x)**2), x)

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dx + c)^2 (b \coth(fx + e) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d*x+c)^2/(a+b*coth(f*x+e))^2,x, algorithm="giac")

[Out] integrate(1/((d*x + c)^2*(b*coth(f*x + e) + a)^2), x)

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
1 (* Original version thanks to Albert Rich emailed on 03/21/2017 *)
2 (* ::Package:: *)
3
4 (* ::Subsection:: *)
5 (*GradeAntiderivative[result,optimal]*)
6
7
8 (* ::Text:: *)
9 (*If result and optimal are mathematical expressions, *)
10 (*      GradeAntiderivative[result,optimal] returns*)
11 (* "F" if the result fails to integrate an expression that*)
12 (*      is integrable*)
13 (* "C" if result involves higher level functions than necessary*)
14 (* "B" if result is more than twice the size of the optimal*)
15 (*      antiderivative*)
16 (* "A" if result can be considered optimal*)
17
18
19 GradeAntiderivative[result_,optimal_] :=
20   If[ExpnType[result]<=ExpnType[optimal],
21     If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
```

```

22     If[LeafCount[result]<=2*LeafCount[optimal],
23         "A",
24         "B"],
25     "C"],
26 If[FreeQ[result,Integrate] && FreeQ[result,Int],
27     "C",
28     "F"]]
29
30
31 (* ::Text:: *)
32 (*The following summarizes the type number assigned an *)
33 (*expression based on the functions it involves*)
34 (*1 = rational function*)
35 (*2 = algebraic function*)
36 (*3 = elementary function*)
37 (*4 = special function*)
38 (*5 = hyperpergeometric function*)
39 (*6 = appell function*)
40 (*7 = rootsum function*)
41 (*8 = integrate function*)
42 (*9 = unknown function*)
43
44
45 ExpnType[expn_] :=
46     If[AtomQ[expn],
47         1,
48     If[ListQ[expn],
49         Max[Map[ExpnType,expn]],
50     If[Head[expn]===Power,
51         If[IntegerQ[expn[[2]]],
52             ExpnType[expn[[1]]],
53         If[Head[expn[[2]]]===Rational,
54             If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
55                 1,
56                 Max[ExpnType[expn[[1]],2]],
57             Max[ExpnType[expn[[1]],ExpnType[expn[[2]],3]],
58     If[Head[expn]===Plus || Head[expn]===Times,
59         Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
60     If[ElementaryFunctionQ[Head[expn]],
61         Max[3,ExpnType[expn[[1]]],
62     If[SpecialFunctionQ[Head[expn]],
63         Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
64     If[HypergeometricFunctionQ[Head[expn]],
65         Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
66     If[AppellFunctionQ[Head[expn]],
67         Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
68     If[Head[expn]===RootSum,

```

```

69   Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 7]],
70   If[Head[expn]===Integrate || Head[expn]===Int,
71     Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 8]],
72   9]]]]]]]]]]
73
74
75 ElementaryFunctionQ[func_] :=
76   MemberQ[{
77     Exp, Log,
78     Sin, Cos, Tan, Cot, Sec, Csc,
79     ArcSin, ArcCos, ArcTan, ArcCot, ArcSec, ArcCsc,
80     Sinh, Cosh, Tanh, Coth, Sech, Csch,
81     ArcSinh, ArcCosh, ArcTanh, ArcCoth, ArcSech, ArcCsch
82   }, func]
83
84
85 SpecialFunctionQ[func_] :=
86   MemberQ[{
87     Erf, Erfc, Erfi,
88     FresnelS, FresnelC,
89     ExpIntegralE, ExpIntegralEi, LogIntegral,
90     SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
91     Gamma, LogGamma, PolyGamma,
92     Zeta, PolyLog, ProductLog,
93     EllipticF, EllipticE, EllipticPi
94   }, func]
95
96
97 HypergeometricFunctionQ[func_] :=
98   MemberQ[{Hypergeometric1F1, Hypergeometric2F1, HypergeometricPFQ}, func]
99
100
101 AppellFunctionQ[func_] :=
102   MemberQ[{AppellF1}, func]

```

4.0.2 Maple grading function

```

1 # File: GradeAntiderivative.mpl
2 # Original version thanks to Albert Rich emailed on 03/21/2017
3
4 #Nasser 03/22/2017 Use Maple leaf count instead since buildin
5 #Nasser 03/23/2017 missing 'ln' for ElementaryFunctionQ added
6 #Nasser 03/24/2017 corrected the check for complex result
7 #Nasser 10/27/2017 check for leafsize and do not call ExpnType()
8 #
9 #Nasser 12/22/2019 Added debug flag, added 'dilog' to special functions
10 #
11 see problem 156, file Apostol_Problems

```

```

11
12 GradeAntiderivative := proc(result,optimal)
13 local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;
14
15     leaf_count_result:=leafcount(result);
16     #do NOT call ExpnType() if leaf size is too large. Recursion problem
17     if leaf_count_result > 500000 then
18         return "B";
19     fi;
20
21     leaf_count_optimal:=leafcount(optimal);
22
23     ExpnType_result:=ExpnType(result);
24     ExpnType_optimal:=ExpnType(optimal);
25
26     if debug then
27         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
    ExpnType_optimal);
28     fi;
29
30 # If result and optimal are mathematical expressions,
31 # GradeAntiderivative[result,optimal] returns
32 #   "F" if the result fails to integrate an expression that
33 #     is integrable
34 #   "C" if result involves higher level functions than necessary
35 #   "B" if result is more than twice the size of the optimal
36 #     antiderivative
37 #   "A" if result can be considered optimal
38
39 #This check below actually is not needed, since I only
40 #call this grading only for passed integrals. i.e. I check
41 #for "F" before calling this. But no harm of keeping it here.
42 #just in case.
43
44
45 if not type(result,freeof('int')) then
46     return "F";
47 end if;
48
49
50 if ExpnType_result<=ExpnType_optimal then
51     if debug then
52         print("ExpnType_result<=ExpnType_optimal");
53     fi;
54     if is_contains_complex(result) then
55         if is_contains_complex(optimal) then

```

```

56     if debug then
57         print("both result and optimal complex");
58     fi;
59     #both result and optimal complex
60     if leaf_count_result<=2*leaf_count_optimal then
61         return "A";
62     else
63         return "B";
64     end if
65     else #result contains complex but optimal is not
66         if debug then
67             print("result contains complex but optimal is not");
68         fi;
69         return "C";
70     end if
71     else # result do not contain complex
72         # this assumes optimal do not as well
73         if debug then
74             print("result do not contain complex, this assumes optimal do not
as well");
75         fi;
76         if leaf_count_result<=2*leaf_count_optimal then
77             if debug then
78                 print("leaf_count_result<=2*leaf_count_optimal");
79             fi;
80             return "A";
81         else
82             if debug then
83                 print("leaf_count_result>2*leaf_count_optimal");
84             fi;
85             return "B";
86         end if
87     end if
88     else #ExpnType(result) > ExpnType(optimal)
89         if debug then
90             print("ExpnType(result) > ExpnType(optimal)");
91         fi;
92         return "C";
93     end if
94
95 end proc:
96
97 #
98 # is_contains_complex(result)
99 # takes expressions and returns true if it contains "I" else false
100 #
101 #Nasser 032417

```

```

102 is_contains_complex:= proc(expression)
103   return (has(expression,I));
104 end proc:
105
106 # The following summarizes the type number assigned an expression
107 # based on the functions it involves
108 # 1 = rational function
109 # 2 = algebraic function
110 # 3 = elementary function
111 # 4 = special function
112 # 5 = hyperpergeometric function
113 # 6 = appell function
114 # 7 = rootsum function
115 # 8 = integrate function
116 # 9 = unknown function
117
118 ExpnType := proc(expn)
119   if type(expn,'atomic') then
120     1
121   elif type(expn,'list') then
122     apply(max,map(ExpnType,expn))
123   elif type(expn,'sqrt') then
124     if type(op(1,expn),'rational') then
125       1
126     else
127       max(2,ExpnType(op(1,expn)))
128     end if
129   elif type(expn,'^^') then
130     if type(op(2,expn),'integer') then
131       ExpnType(op(1,expn))
132     elif type(op(2,expn),'rational') then
133       if type(op(1,expn),'rational') then
134         1
135       else
136         max(2,ExpnType(op(1,expn)))
137       end if
138     else
139       max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
140     end if
141   elif type(expn,'^+^') or type(expn,'`*`') then
142     max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
143   elif ElementaryFunctionQ(op(0,expn)) then
144     max(3,ExpnType(op(1,expn)))
145   elif SpecialFunctionQ(op(0,expn)) then
146     max(4,apply(max,map(ExpnType,[op(expn)])))
147   elif HypergeometricFunctionQ(op(0,expn)) then
148     max(5,apply(max,map(ExpnType,[op(expn)])))

```



```

149   elif AppellFunctionQ(op(0,expn)) then
150       max(6,apply(max,map(ExpnType,[op(expn)])))
151   elif op(0,expn)='int' then
152       max(8,apply(max,map(ExpnType,[op(expn)]))) else
153       9
154   end if
155 end proc:
156
157
158 ElementaryFunctionQ := proc(func)
159     member(func,[
160         exp,log,ln,
161         sin,cos,tan,cot,sec,csc,
162         arcsin,arccos,arctan,arccot,arcsec,arccsc,
163         sinh,cosh,tanh,coth,sech,csch,
164         arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
165 end proc:
166
167 SpecialFunctionQ := proc(func)
168     member(func,[
169         erf,erfc,erfi,
170         FresnelS,FresnelC,
171         Ei,Ei,Li,Si,Ci,Shi,Chi,
172         GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
173         EllipticF,EllipticE,EllipticPi])
174 end proc:
175
176 HypergeometricFunctionQ := proc(func)
177     member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
178 end proc:
179
180 AppellFunctionQ := proc(func)
181     member(func,[AppellF1])
182 end proc:
183
184 # u is a sum or product. rest(u) returns all but the
185 # first term or factor of u.
186 rest := proc(u) local v;
187     if nops(u)=2 then
188         op(2,u)
189     else
190         apply(op(0,u),op(2..nops(u),u))
191     end if
192 end proc:
193
194 #leafcount(u) returns the number of nodes in u.
195 #Nasser 3/23/17 Replaced by build-in leafCount from package in Maple

```

```

196 leafcount := proc(u)
197     MmaTranslator[Mma][LeafCount](u);
198 end proc:

```

4.0.3 Sympy grading function

```

1 #Dec 24, 2019. Nasser M. Abbasi:
2 #           Port of original Maple grading function by
3 #           Albert Rich to use with Sympy/Python
4 #Dec 27, 2019 Nasser. Added `RootSum`. See problem 177, Timofeev file
5 #           added 'exp_polar'
6 from sympy import *
7
8 def leaf_count(expr):
9     #sympy do not have leaf count function. This is approximation
10    return round(1.7*count_ops(expr))
11
12 def is_sqrt(expr):
13     if isinstance(expr,Pow):
14         if expr.args[1] == Rational(1,2):
15             return True
16         else:
17             return False
18     else:
19         return False
20
21 def is_elementary_function(func):
22     return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
23                    asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
24                    asinh,acosh,atanh,acoth,asech,acsch
25                    ]
26
27 def is_special_function(func):
28     return func in [ erf,erfc,erfi,
29                    fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
30                    gamma,loggamma,digamma,zeta,polylog,LambertW,
31                    elliptic_f,elliptic_e,elliptic_pi,exp_polar
32                    ]
33
34 def is_hypergeometric_function(func):
35     return func in [hyper]
36
37 def is_appell_function(func):
38     return func in [appellf1]
39
40 def is_atom(expn):
41     try:

```

```

42     if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
43         return True
44     else:
45         return False
46
47     except AttributeError as error:
48         return False
49
50 def expnType(expn):
51     debug=False
52     if debug:
53         print("expn=",expn,"type(expn)=",type(expn))
54
55     if is_atom(expn):
56         return 1
57     elif isinstance(expn,list):
58         return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
59     elif is_sqrt(expn):
60         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
61             return 1
62         else:
63             return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
64     elif isinstance(expn,Pow): #type(expn,'^^')
65         if isinstance(expn.args[1],Integer): #type(op(2,expn),'integer')
66             return expnType(expn.args[0]) #ExpnType(op(1,expn))
67         elif isinstance(expn.args[1],Rational): #type(op(2,expn),'rational')
68             if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
69                 return 1
70             else:
71                 return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)
72 ))
73     else:
74         return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
75 ExpnType(op(1,expn)),ExpnType(op(2,expn)))
76     elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'+' or type
77 (expn,'*')
78         m1 = expnType(expn.args[0])
79         m2 = expnType(list(expn.args[1:]))
80         return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
81     elif is_elementary_function(expn.func): #ElementaryFunctionQ(op(0,expn))
82         return max(3,expnType(expn.args[0])) #max(3,ExpnType(op(1,expn)))
83     elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
84         m1 = max(map(expnType, list(expn.args)))
85         return max(4,m1) #max(4,apply(max,map(ExpnType,[op(expn)])))
86     elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
87 expn))
88         m1 = max(map(expnType, list(expn.args)))

```

```

85     return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
86 elif is_appell_function(expn.func):
87     m1 = max(map(expnType, list(expn.args)))
88     return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
89 elif isinstance(expn,RootSum):
90     m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,
Apply[List,expn]],7]],
91     return max(7,m1)
92 elif str(expn).find("Integral") != -1:
93     m1 = max(map(expnType, list(expn.args)))
94     return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
95 else:
96     return 9
97
98 #main function
99 def grade_antiderivative(result,optimal):
100
101     leaf_count_result  = leaf_count(result)
102     leaf_count_optimal = leaf_count(optimal)
103
104     expnType_result  = expnType(result)
105     expnType_optimal = expnType(optimal)
106
107     if str(result).find("Integral") != -1:
108         return "F"
109
110     if expnType_result <= expnType_optimal:
111         if result.has(I):
112             if optimal.has(I): #both result and optimal complex
113                 if leaf_count_result <= 2*leaf_count_optimal:
114                     return "A"
115                 else:
116                     return "B"
117             else: #result contains complex but optimal is not
118                 return "C"
119         else: # result do not contain complex, this assumes optimal do not as
well
120             if leaf_count_result <= 2*leaf_count_optimal:
121                 return "A"
122             else:
123                 return "B"
124     else:
125         return "C"

```

4.0.4 SageMath grading function

1 #Dec 24, 2019. Nasser: Ported original Maple grading function by

```

2 #           Albert Rich to use with Sagemath. This is used to
3 #           grade Fracas, Giac and Maxima results.
4 #Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
5 #           'arctan2','floor','abs','log_integral'
6
7 from sage.all import *
8 from sage.symbolic.operators import add_vararg, mul_vararg
9
10 def tree(expr):
11     debug=False;
12     if debug:
13         print ("Enter tree(expr), expr=",expr)
14         print ("expr.operator()=",expr.operator())
15         print ("expr.operands()=",expr.operands())
16         print ("map(tree, expr.operands()=",map(tree, expr.operands()))
17
18     if expr.operator() is None:
19         return expr
20     else:
21         return [expr.operator()+list(map(tree, expr.operands()))
22
23 def leaf_count(anti):
24     debug=False;
25
26     if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))
27
28     if len(anti) == 0: #special check for optimal being 0 for some test cases.
29         if debug: print ("len(anti) == 0")
30         return 1
31     else:
32         if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len(
33         flatten(tree(anti))))
34         return round(1.35*len(flatten(tree(anti)))) #fudge factor
35         #since this estimate of leaf count is bit lower than
36         #what it should be compared to Mathematica's
37
38 def is_sqrt(expr):
39     debug=False;
40     if expr.operator() == operator.pow: #isinstance(expr,Pow):
41         if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
42             if debug: print ("expr is sqrt")
43             return True
44         else:
45             return False
46     else:
47         return False

```

```

48 def is_elementary_function(func):
49     debug = False
50
51     m = func.name() in ['exp','log','ln',
52         'sin','cos','tan','cot','sec','csc',
53         'arcsin','arccos','arctan','arccot','arcsec','arccsc',
54         'sinh','cosh','tanh','coth','sech','csch',
55         'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
56         'arctan2','floor','abs'
57     ]
58     if debug:
59         if m:
60             print ("func ", func , " is elementary_function")
61         else:
62             print ("func ", func , " is NOT elementary_function")
63
64
65     return m
66
67 def is_special_function(func):
68     debug = False
69
70     if debug: print ("type(func)=", type(func))
71
72     m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
73         'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
74     sinh_integral'
75         'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
76         'polylog','lambert_w','elliptic_f','elliptic_e',
77         'elliptic_pi','exp_integral_e','log_integral']
78
79     if debug:
80         print ("m=",m)
81         if m:
82             print ("func ", func ," is special_function")
83         else:
84             print ("func ", func ," is NOT special_function")
85
86     return m
87
88
89 def is_hypergeometric_function(func):
90     return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U
91     ']
92
93 def is_appell_function(func):

```

```

93     return func.name() in ['hypergeometric']    #[appellf1] can't find this in
          sagemath
94
95 def is_atom(expn):
96
97     #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
98     try:
99         if expn.parent() is SR:
100             return expn.operator() is None
101         if expn.parent() in (ZZ, QQ, AA, QQbar):
102             return expn in expn.parent() # Should always return True
103         if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
104             return expn in expn.parent().base_ring() or expn in expn.parent().
          gens()
105         return False
106
107     except AttributeError as error:
108         return False
109
110
111 def expnType(expn):
112     debug=False
113
114     if debug:
115         print(">>>>Enter expnType, expn=", expn)
116         print(">>>>is_atom(expn)=", is_atom(expn))
117
118     if is_atom(expn):
119         return 1
120     elif type(expn)==list:    #isinstance(expn,list):
121         return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
122     elif is_sqrt(expn):
123         if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
          Rational):
124             return 1
125         else:
126             return max(2,expnType(expn.operands()[0]))    #max(2,expnType(expn.
          args[0]))
127     elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
128         if type(expn.operands()[1])==Integer:    #isinstance(expn.args[1],Integer)
129             return expnType(expn.operands()[0])    #expnType(expn.args[0])
130         elif type(expn.operands()[1])==Rational:    #isinstance(expn.args[1],
          Rational)
131             if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
          Rational)
132                 return 1

```

```

133         else:
134             return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.
args[0]))
135         else:
136             return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
137         elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
isinstance(expn,Add) or isinstance(expn,Mul)
138             m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
139             m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
140             return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
141         elif is_elementary_function(expn.operator()): #is_elementary_function(expn.
func)
142             return max(3,expnType(expn.operands()[0]))
143         elif is_special_function(expn.operator()): #is_special_function(expn.func)
144             m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
145             return max(4,m1) #max(4,m1)
146         elif is_hypergeometric_function(expn.operator()): #
is_hypergeometric_function(expn.func)
147             m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
148             return max(5,m1) #max(5,m1)
149         elif is_appell_function(expn.operator()):
150             m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
151             return max(6,m1) #max(6,m1)
152         elif str(expn).find("Integral") != -1: #this will never happen, since it
153             #is checked before calling the grading function that is passed.
154             #but kept it here.
155             m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
156             return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
157         else:
158             return 9
159
160 #main function
161 def grade_antiderivative(result,optimal):
162     debug = False;
163
164     if debug: print ("Enter grade_antiderivative for sagemath")
165
166     leaf_count_result = leaf_count(result)
167     leaf_count_optimal = leaf_count(optimal)
168
169     if debug: print ("leaf_count_result=", leaf_count_result, "
leaf_count_optimal=",leaf_count_optimal)

```



```
170
171
172     expnType_result = expnType(result)
173     expnType_optimal = expnType(optimal)
174
175     if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
176                     expnType_optimal)
177
178     if expnType_result <= expnType_optimal:
179         if result.has(I):
180             if optimal.has(I): #both result and optimal complex
181                 if leaf_count_result <= 2*leaf_count_optimal:
182                     return "A"
183             else:
184                 return "B"
185         else: #result contains complex but optimal is not
186             return "C"
187     else: # result do not contain complex, this assumes optimal do not as
188         well
189         if leaf_count_result <= 2*leaf_count_optimal:
190             return "A"
191         else:
192             return "B"
193     else:
194         return "C"
```